Lógica Quântica

Group project

2022–2023

This assignment should be completed in group of e.g. two or three students.

The objective of this assignment is to explore topics related to the content of the course beyond what was covered in lectures and closer to current research or applications. Each group will pick one topic to focus on. A list of suggested topics is provided below, together with pointers to references. The list is not meant to be exhaustive – you may also suggest a different one.

The deliverable consists in a short report (say, 3–5 pages) or presentation in a different format. You can use Portuguese or English. It should present the key ideas and concepts to an informed audience (i.e. someone who knows the contents of the course but has no additional knowledge of the chosen topic), illustrated by some simple examples when appropriate.

List of suggested topics

- **Applications of ZX calculus** The ZX calculus is a graphical language based on string diagrams with X and Z spiders (corresponding to qubit's Z and X bases). There are many introductory references of varying sizes and different focus:
  - See, for example, `https://zxcalculus.com/` or the Wikipedia page for very brief introductions.
  - Chapter 9 of reference book: B. Coecke & A. Kissinger, 'Picturing quantum processes: A first course in quantum theory and diagrammatic reasoning', Cambridge University Press, 2017.
  - arXiv:2012.13966 [quant-ph] is a very extensive survey, introducing the ZX calculus, its underlying theory, and many applications in quantum computation (the report could focus on one of these; see below).
  - arXiv:2102.10984 [quant-ph] also introduces the ZX calculus and more briefly surveys applications, including to natural language.

  Some possible topics are applications of the ZX calculus (n.b.: the report is only supposed to focus on a simple aspect of one of these, and could be more theoretical, or more focused on applications illustrated by simple examples):
  - Circuit optimisation, including support by the Python tool PyZX; see arXiv:1904.04735 [quant-ph]
  - Measurement-based quantum computation and graph states; see e.g. arXiv:1203.6242 [quant-ph], arXiv.org:2003.01664 [quant-ph], or section 6 of the above-mentioned arXiv: 2012.13966 [quant-ph].
  - Error-correcting codes; see e.g. arXiv:1306.4532 [quant-ph], arXiv:1611.08012 [quant-ph]
- **Classical and quantum wires: basic theory or applications** We have used diagrams to represent pure quantum maps, or processes of closed quantum systems. This language can be extended to deal with open quantum systems: mixed states, decoherence, etc. as well as

processes that mix classical and quantum information, including classical control. This uses a 'doubling' construction. An early reference for this construction is arXiv:1605.08617 (see also section 10 of the above-mentioned arXiv: 2012.13966 [quant-ph] for a short description, or the apropriate chapters of Coecke & Kissinger's book).
The project may focus on how this construction works to represent quantum processes. Alternatively, it could focus on a particular application where these tools are used. One suggestion is modelling and security proof of quantum key distribution (QKD) found in arXiv:1704.08668 (nb: this reference also includes a summary of the relevant aspects of diagrammatic language).

- **Diagrammatic reasoning to other application domains** Diagrammatic reasoning à la categorical quantum mechanics has also been aplied to some other (purely classical) domains. Some suggestions are:
  - Natural language modelling and processing. See arXiv:1003.4394 [cs.CL] for the earliest reference. More recently there is also the tool lambeq, see arXiv:2110.04236 [cs.CL]
  - A compositional approach to classical cryptography. See arXiv:2105.05949 [cs.CR] (e.g. the one-time pad in section 5).

- **Graphical languages for photonic quantum computation** A graphical language for linear-optical quantum circuits was proposed in arXiv:2204.11787 [quant-ph], with a rather different flavour. Despite this apparent mismatch, this calculus was subsequently used to derive the first complete equational theory for quantum circuits arXiv:2206.10577 [quant-ph].
  The report could focus on either the LO_v-calculus and its used in photonic computation, or on the main ideas of this important subsequent development.

- **Linear lambda calculus and quantum programming languages** As the lambda-calculus is to (the fragment with conjunction and implication of) intuitionistic logic and to cartesian closed categories, the linear lambda-calculus is to (a fragment of) the resource-sensitive linear logic and to symmetric monoidal categories. See section 7 of arXiv:1102.1313 [math.CT]. The report may focus on the basic theory and subsequent developments (e.g. various more complicated quantum lamba calculi, see e.g. arXiv:cs/0404056 [cs.LO]) and how they underlie functional quantum programming languages like Quipper arXiv:1304.3390 [cs.PL].