Structural Reason for Locality of Macroscopic Correlations

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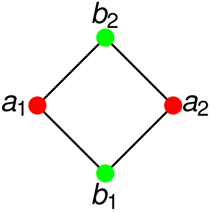
Introduction

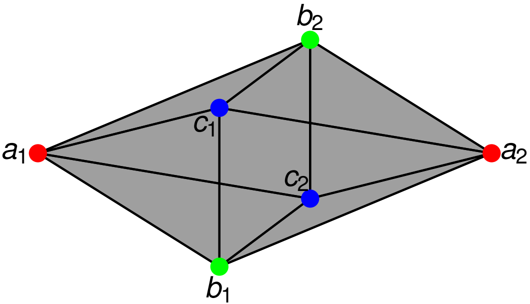
- Macroscopic correlations arising from microscopic models (Ramanathan et al. 2011: QM models)
- Monogamy of violation of Bell inequalities from the non-signalling condition (Pawłowski, Brukner 2009: bipartite models).
- Use the general framework of Abramsky and Brandenburger (2011) and provide a structural reason using Vorob'ev's theorem (1962).

Measurement Scenarios

Abramsky-Brandenburger framework

- a finite set of measurements X;
- a cover $\mathcal{U}$ of X (or an abstract simplicial complex Σ on X), indicating the **compatibility** of measurements.





Examples: Bell-type scenarios, KS configurations, and more.

E.g. Z and X measurements on the W state:

	000	001	010	011	100	101	110	111
$a_1 b_1 c_1$	9	1	1	1	1	1	1	9
$a_1 b_1 c_2$	8	2	0	2	0	2	8	2
$a_1 b_2 c_1$	8	0	2	2	0	8	2	2
$a_1 b_2 c_2$	4	4	4	0	4	4	4	0
$a_2 b_1 c_1$	8	0	0	8	2	2	2	2
$a_2 b_1 c_2$	4	4	4	4	4	0	4	0
$a_2 b_2 c_1$	4	4	4	4	4	4	0	0
$a_2 b_2 c_2$	0	8	8	0	8	0	0	0

(every entry should be divided by 24)

Vorob'ev's theorem

For which measurement compatibility structures $\mathcal{U}$ (or Σ) is it so that **any** nosignalling empirical model admits a global extension, i.e. is local/non-contextual?

Macroscopic Correlations and Monogamy

Emergent macroscopic correlations

A **macroscopic scenario** is obtained from an underlying microscopic scenario by **lumping together** certain measurements (e.g. spins in a given direction of several particles give rise to a magnetisation measurement in that direction).

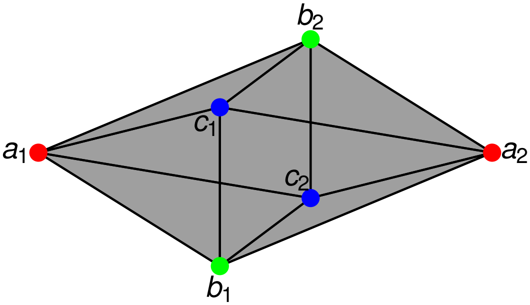
The merged measurements must be 'symmetric' in some sense. E.g. consider a multipartite scenario where several parties are considered as being on the same macroscopic site (several Bobs). For this identification to be possible, there must be a symmetry between these Bobs, i.e. all the Bobs must allow the same measurements, and these must have the same compatibility relations with those of Alice, Claire, etc.

Macroscopic correlations: Tripartite example

Consider a tripartite scenario:

$$X = \{a_1, a_2, b_1, b_2, c_1, c_2\}$$

$$\mathcal{U} = \{\{a_i, b_j, c_k\} \mid i, j, k \in \{1, 2\}\}$$



Macroscopic correlations: Tripartite example

- Empirical model: no signalling probabilities

$$p(a_i, b_j, c_k = x, y, z)$$

where x, y, z are possible outcomes.

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- Consider the subsystem composed of A and B only, given by marginalisation (in QM, partial trace):

$$p(a_i, b_j = x, y) = \sum_z p(a_i, b_j, c_k = x, y, z)$$

(this is independent of c_k due to no-signalling).

Similarly define $p(a_i, c_k = x, y)$.

Macroscopic correlations: Tripartite example

- Consider B and C to be in the same 'macroscopic' site. The symmetry identifies the measurements $b_1 \sim c_1$ and $b_2 \sim c_2$, giving rise to macroscopic measurements m_1 and m_2.

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- The emergent 'macroscopic' probabilities are given as an **average**:

$$p(a_i, m_j = x, y) = \frac{1}{2}\Big(p(a_i, b_j = x, y) + p(a_i, c_j = x, y) \Big)$$

Macroscopic locality and micrscopic monogamy

Consider any **(general) Bell inequality** for a bipartite scenario: a set of coefficients $\alpha(i, j, x, y)$ and a bound R.

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$$\sum_{i,j,x,y} \alpha(i,j,x,y) p(a_i, m_j = x, y) \leq R$$

$$\Leftrightarrow$$

$$\sum_{i,j,x,y} \frac{1}{2} \alpha(i,j,x,y) \Big(p(a_i, b_j = x, y) + p(a_i, c_j = x, y) \Big) \leq R$$

$$\Leftrightarrow$$

$$\sum_{i,j,x,y} \alpha(i,j,x,y) p(a_i, b_j = x, y) + \sum_{i,j,x,y} \alpha(i,j,x,y) p(a_i, c_j = x, y) \leq 2R$$

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$$\Leftrightarrow$$

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$$\sum_{i,j,x,y} \alpha(i,j,x,y) p(a_i, b_j = x, y) + \sum_{i,j,x,y} \alpha(i,j,x,y) p(a_i, c_j = x, y) \leq 2R$$

The macroscopic model $p(a_i, m_j = \cdots)$ **satisfies the inequality** if and only if the microscopic model is **monogamous** with respect to violating it.

Example: W-state

	00	01	10	11
$a_1 m_1$	10	2	2	10
$a_1 m_2$	8	4	8	4
$a_2 m_1$	8	8	4	4
$a_2 m_2$	8	8	8	0

(every entry should be divided by 24)

This is **local**! This is general for any empirical model.

Another example model

	000	001	010	011	100	101	110	111
$a_1 b_1 c_1$	1	1	0	0	0	0	1	1
$a_1 b_1 c_2$	1	1	0	0	0	0	1	1
$a_1 b_2 c_1$	1	1	0	0	0	0	1	1
$a_1 b_2 c_2$	1	1	0	0	0	0	1	1
$a_2 b_1 c_1$	1	1	0	0	0	0	1	1
$a_2 b_1 c_2$	1	1	0	0	0	0	1	1
$a_2 b_2 c_1$	0	0	1	1	1	1	0	0
$a_2 b_2 c_2$	0	0	1	1	1	1	0	0

(every entry should be divided by 4)

Another example model

	00	01	10	11
$a_1 b_1$	2	0	0	2
$a_1 b_2$	2	0	0	2
$a_2 b_1$	2	0	0	2
$a_2 b_2$	0	2	2	0

(divided by 4)

	00	01	10	11
$a_1 c_1$	1	1	1	1
$a_1 c_2$	1	1	1	1
$a_2 c_1$	1	1	1	1
$a_2 c_2$	1	1	1	1

(divided by 4)

left: maximally non-local, right: local

	00	01	10	11
$a_1 m_1$	3	1	1	3
$a_1 m_1$	3	1	1	3
$a_1 m_1$	3	1	1	3
$a_1 m_1$	1	3	3	1

(every entry should be divided by 8)

Again, this is **local**!

Structural Reason

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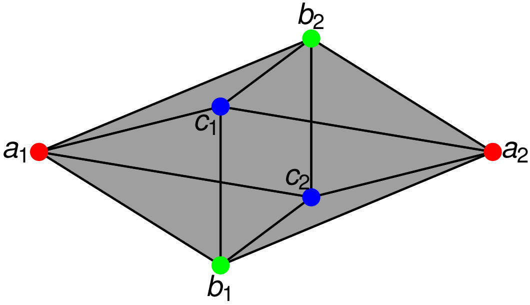
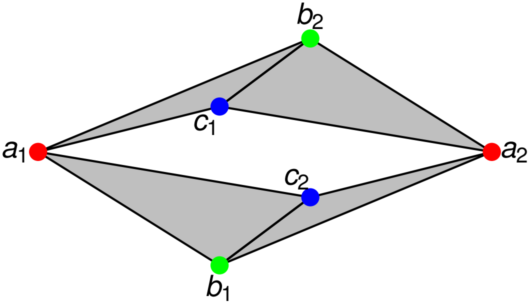


- Microscopic scenario: simplicial complex $\mathfrak{D}_2 * \mathfrak{D}_2 * \mathfrak{D}_2$.
- We identify B and C: $b_1 \sim c_1$, $b_2 \sim c_2$.
- The macroscopic scenario arises as the quotient.

Structural Reason



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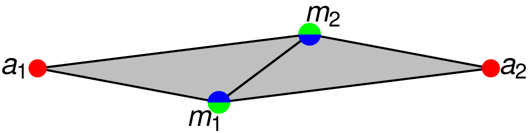

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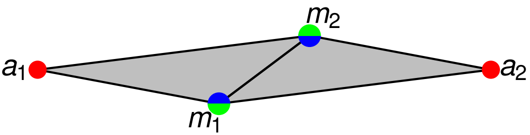

- This quotient complex satisfies the **Vorob'ev condition.**
- Therefore, no matter which micro model $p(a_i, b_j, c_k = \cdots)$ we start from, the macro model $p(a_i, m_j = \cdots)$ is local!
- In particular, it satisfies any Bell inequality. Hence, the original tripartite model also satisfies a monogamy relation for any Bell inequality.

Summary/Conclusions

A macro model arises as a quotient/average of its micro model.
New structural insights stem from our approach:

1. If the quotient (macro) scenario of some (micro) scenario is Vorob'ev-regular, then the emergent macroscopic model will be extendable (i.e. local/non-contextual), **whatever no-signalling** microscopic empirical model it arises from.
2. A finer analysis reveals that an emergent macroscopic model satisfies a given Bell inequality if and only if its underlying microscopic model satisfies a corresponding **monogamy** inequality.
3. In particular, (1) is the case for multipartite scenarios provided there are enough particles on each macro site.
   So our approach highlights the **reason why** the result of Ramanathan et al. holds holds, and generalises it from QM to any no-signalling theory.
   Moreover, it also shows that monogamy relations for violation of general multipartite Bell inequalities follow from the no-signalling condition alone, generalising the result of Pawłowski and Brukner (2009) from bipartite to multipartite.

Questions...

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