Closing Bell

Boxing black box transformations in the resource theory of contextuality

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This talk

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Quantum Physics

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Closing Bell: Boxing black box simulations in the resource theory of contextuality

Rui Soares Barbosa, Martti Karvonen, Shane Mansfield

This chapter contains an exposition of the sheaf-theoretic framework for contextuality emphasising resource-theoretic aspects, as well as some original results on this topic. In particular, we consider functions that transform empirical models on a scenario S to empirical models on another scenario T, and characterise those that are induced by classical procedures between S and T corresponding to 'free' operations in the (non-adaptive) resource theory of contextuality. We proceed by expressing such functions as empirical models themselves, on a new scenario built from S and T. Our characterisation then boils down to the non-contextuality of these models. We also show that this construction on scenarios provides a closed structure in the category of measurement scenarios.

Comments: 36 pages. To appear as part of a volume dedicated to Samson Abramsky in Springer's Outstanding Contributions to Logic series

Subjects: **Quantum Physics (quant-ph)**; Logic in Computer Science (cs.LO); Category Theory (math.CT)

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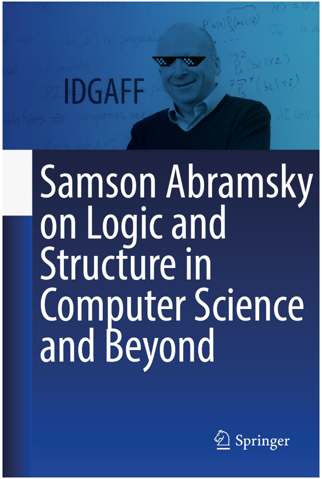

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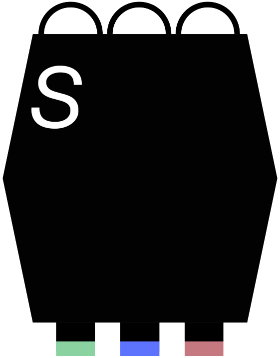
  - $[-, -]$ provides a **closed structure** on the category of measurement scenarios (rather: on a variant of it)

Contextuality

Type or interface: measurement scenario



- Interaction with system: perform measurements and observe respective outcomes

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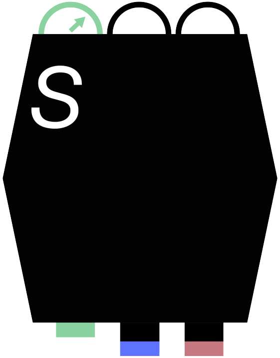

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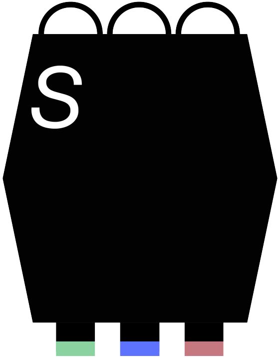

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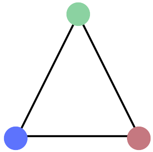

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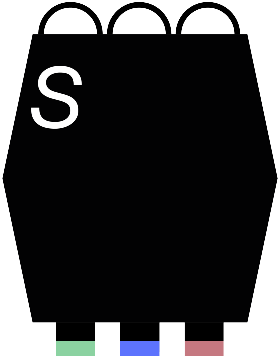

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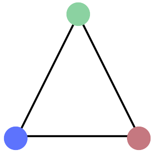

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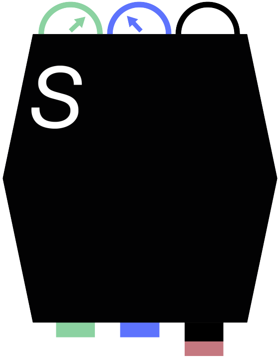

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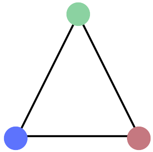

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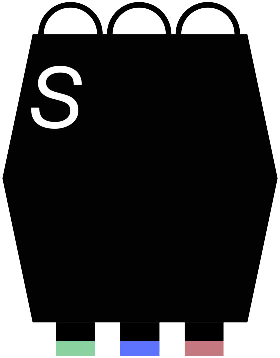

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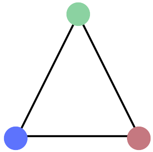

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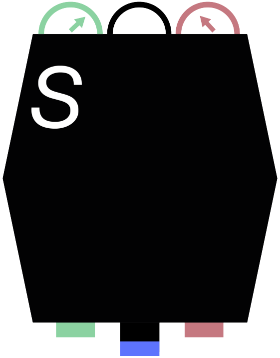

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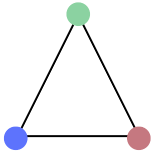

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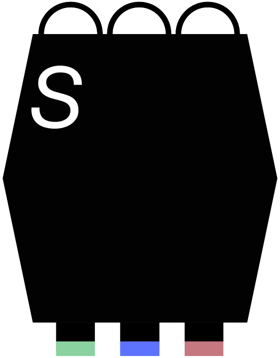

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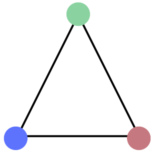

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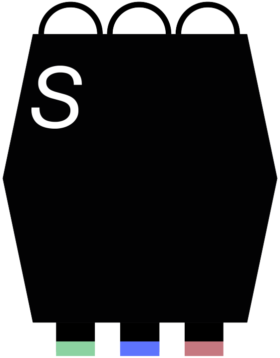

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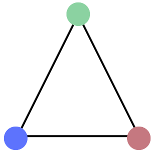

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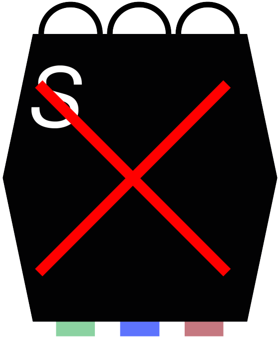

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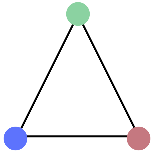

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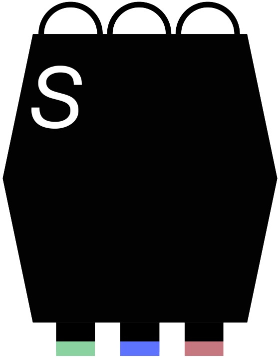

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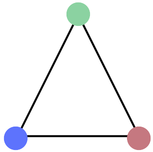

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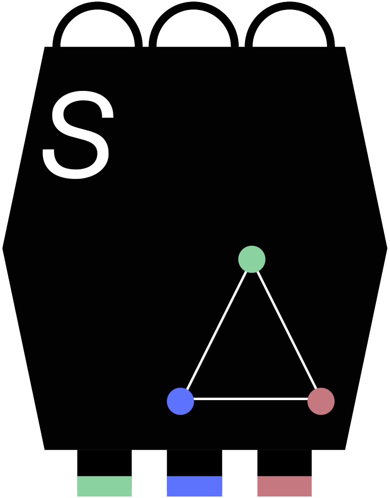
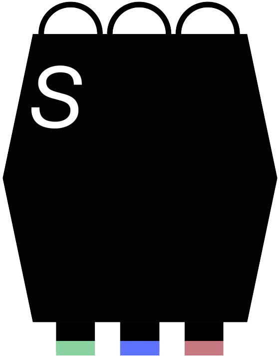


Behaviour: empirical model



- Behaviour of system is described by measurement statistics

		$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$
x	y				
y	z				
x	z				

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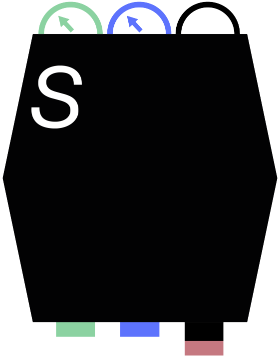

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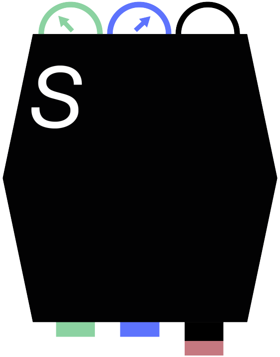

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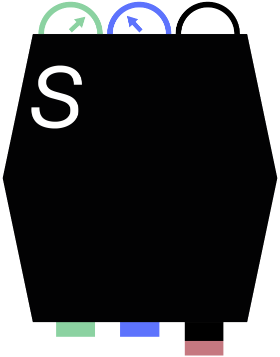

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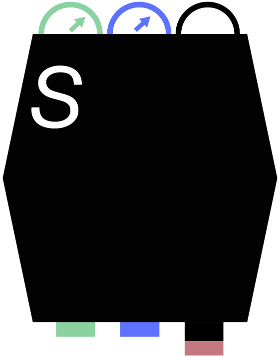

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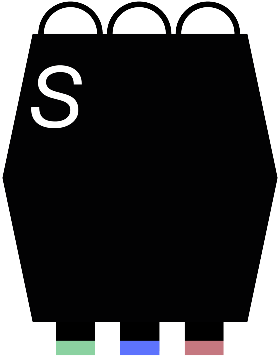

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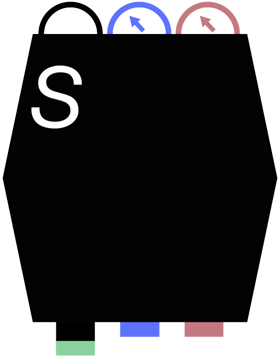

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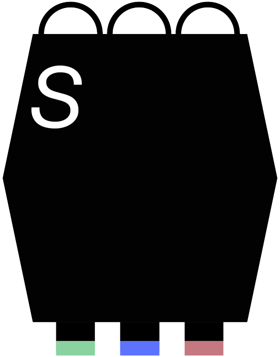

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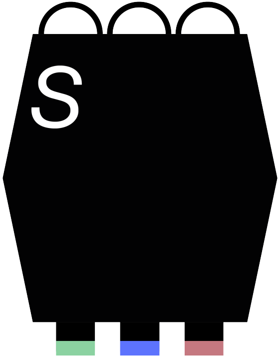

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No-signalling / no-disturbance

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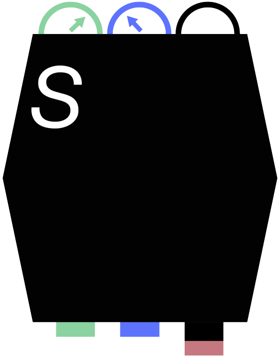

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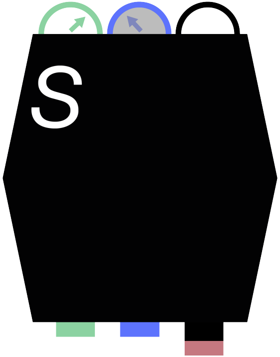

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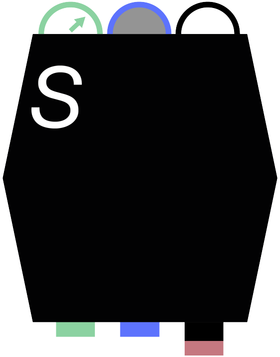

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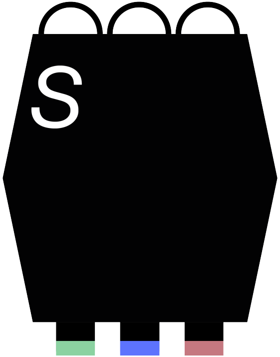
$$\sum_b P(\mathbf{x}, \mathbf{y} \mapsto a, b)$$

Behaviour: empirical model



- Behaviour of system is described by measurement statistics

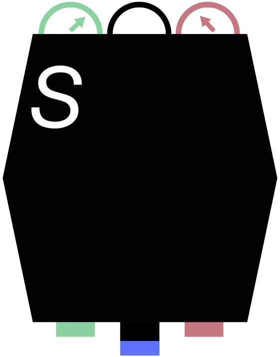
		$(\mathbf{0},\mathbf{0})$	$(\mathbf{0},\mathbf{1})$	$(\mathbf{1},\mathbf{0})$	$(\mathbf{1},\mathbf{1})$
x	y	$3/8$	$1/8$	$1/8$	$3/8$
y	z	$3/8$	$1/8$	$1/8$	$3/8$
x	z	$1/8$	$3/8$	$3/8$	$1/8$

No-signalling / no-disturbance

- Marginal distributions agree

$$\sum_{b} P(\mathbf{x}, \mathbf{y} \mapsto a, b)$$

Behaviour: empirical model



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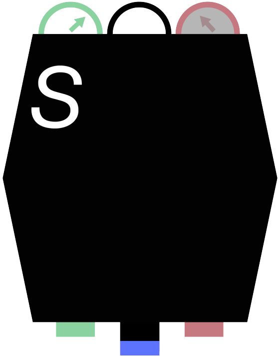
		$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$
x	y	$3/8$	$1/8$	$1/8$	$3/8$
y	z	$3/8$	$1/8$	$1/8$	$3/8$
x	z	$1/8$	$3/8$	$3/8$	$1/8$

No-signalling / no-disturbance

- Marginal distributions agree

$$\sum_b P(\mathbf{x}, \mathbf{y} \mapsto a, b) \qquad\qquad P(\mathbf{x}, \mathbf{z} \mapsto a, c)$$

Behaviour: empirical model



- Behaviour of system is described by measurement statistics

		$(\mathbf{0},\mathbf{0})$	$(\mathbf{0},\mathbf{1})$	$(\mathbf{1},\mathbf{0})$	$(\mathbf{1},\mathbf{1})$
x	y	$3/8$	$1/8$	$1/8$	$3/8$
y	z	$3/8$	$1/8$	$1/8$	$3/8$
x	z	$1/8$	$3/8$	$3/8$	$1/8$

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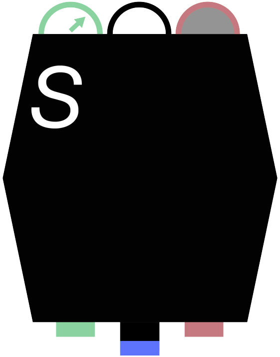
Behaviour: empirical model



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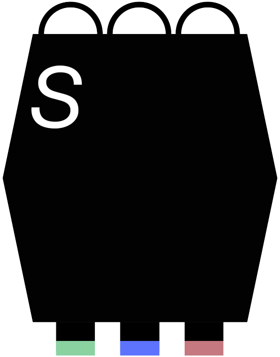
No-signalling / no-disturbance

- Marginal distributions agree

$$\sum_{b} P(\mathsf{x}, \mathsf{y} \mapsto a, b) \qquad \sum_{c} P(\mathsf{x}, \mathsf{z} \mapsto a, c)$$

Behaviour: empirical model



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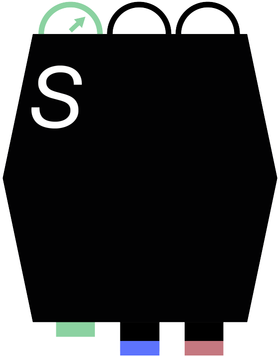
x	z	$1/8$	$3/8$	$3/8$	$1/8$

No-signalling / no-disturbance

- Marginal distributions agree

$$\sum_b P(\mathsf{x}, \mathsf{y} \mapsto a, b) \quad = \quad \sum_c P(\mathsf{x}, \mathsf{z} \mapsto a, c)$$

Behaviour: empirical model



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No-signalling / no-disturbance

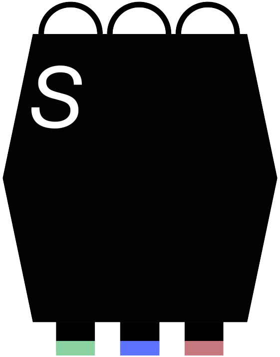
- Marginal distributions agree

$$\sum_b P(\mathsf{x},\mathsf{y} \mapsto a, b) \qquad \sum_c P(\mathsf{x},\mathsf{z} \mapsto a, c)$$

$$=$$

$$P(\mathsf{x} \mapsto a)$$

Behaviour: empirical model



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$$=$$

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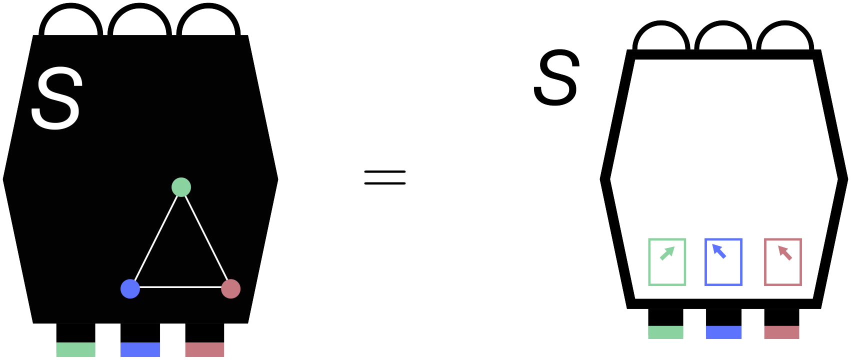
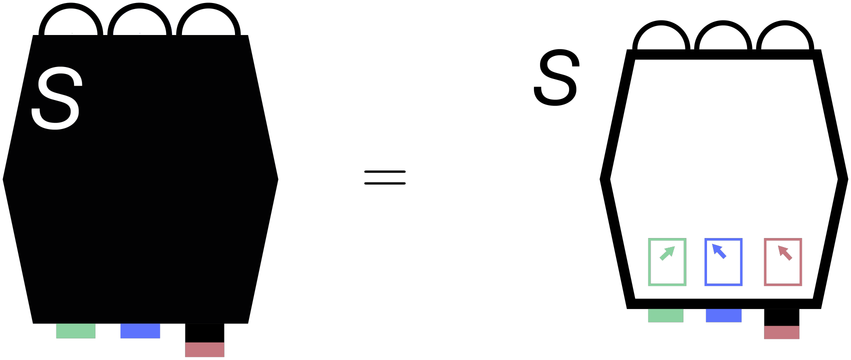
Contextuality

Deterministic model



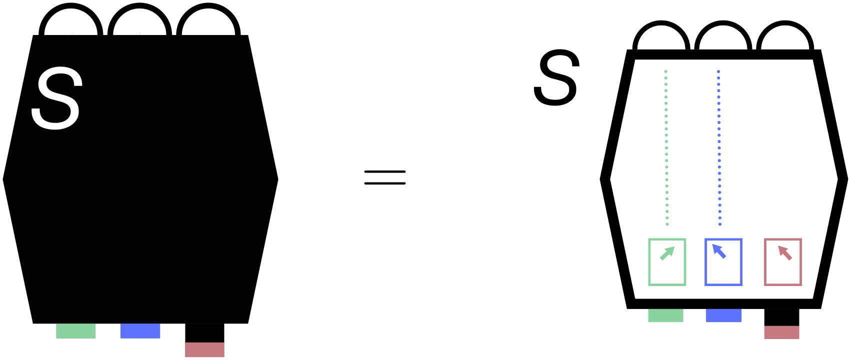
Contextuality

Deterministic model



Contextuality

Deterministic model



Contextuality

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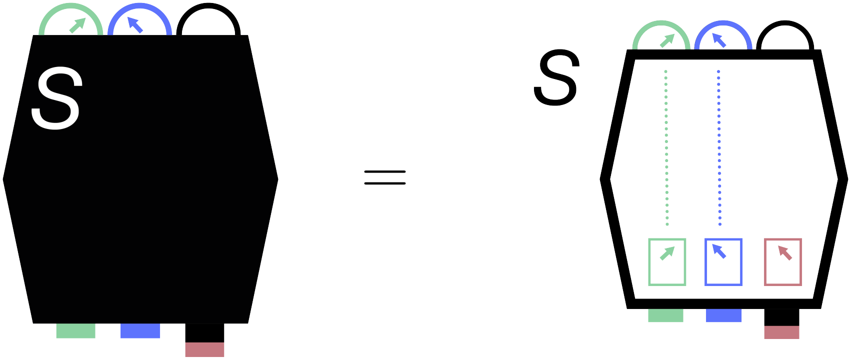
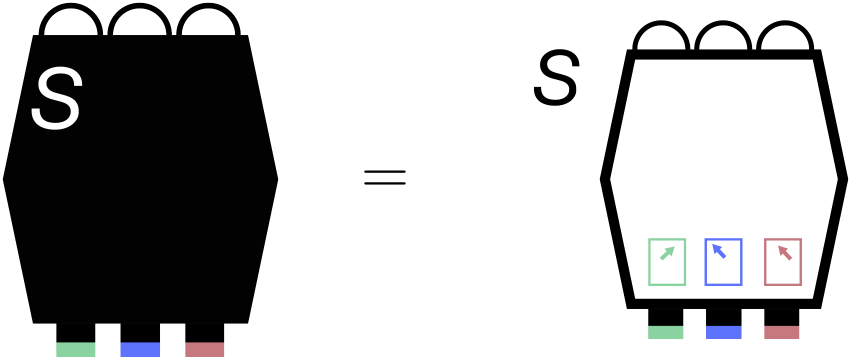


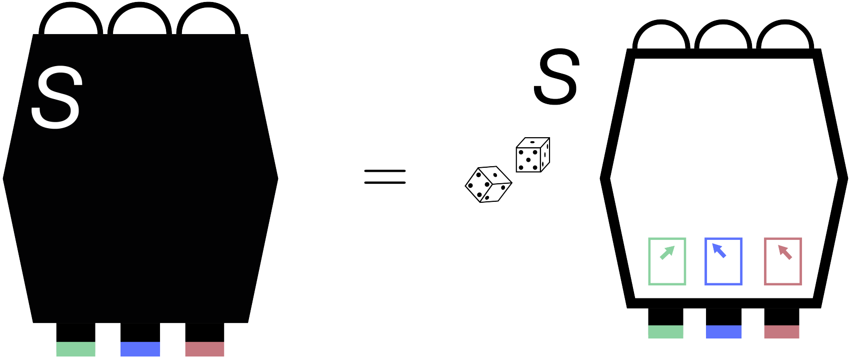
Contextuality

Deterministic model



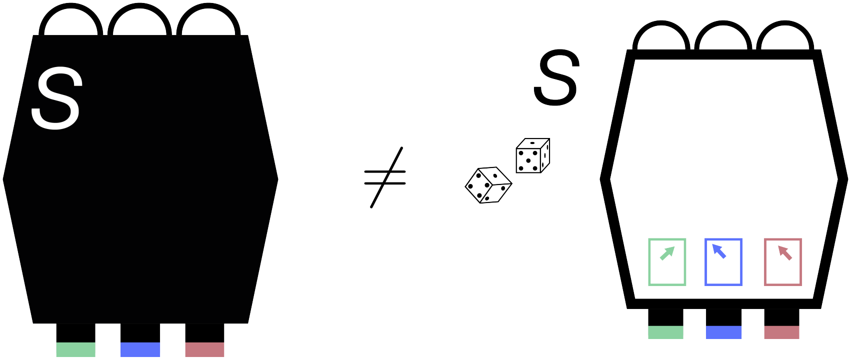
Contextuality

Non-contextual model



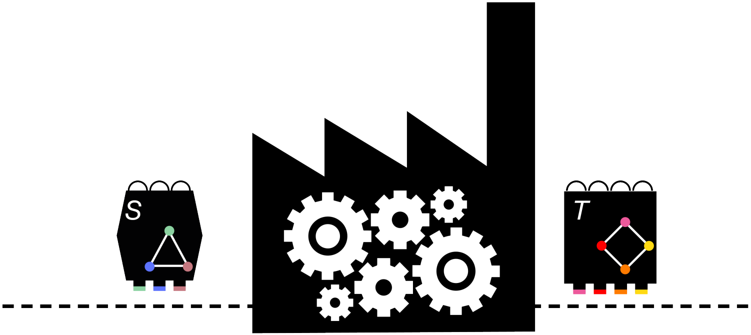
Contextuality

Contextual model

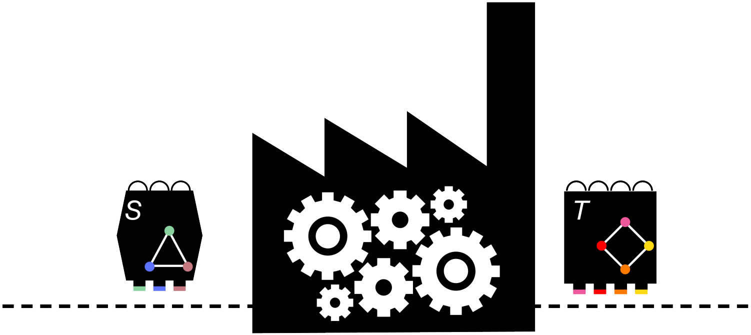


Resource theory of contextuality

Resource theories

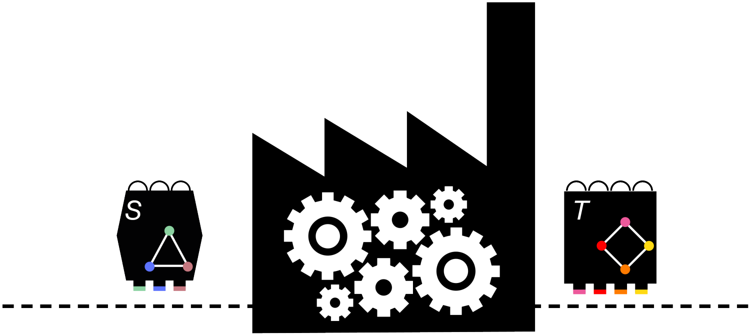


Resource theories



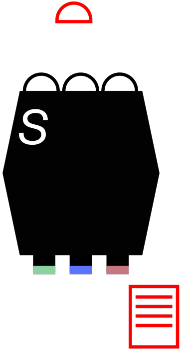
- Consider ‘free’ (i.e. classical) operations:

Resource theories



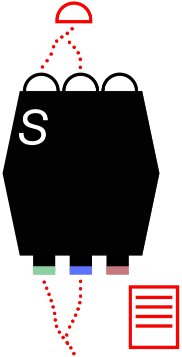
- Consider 'free' (i.e. classical) operations:
  (classical) procedures that use a box of type S to simulate a box of type T

Experiments and procedures



- An S-**experiment** is a protocol for an interaction with the box S:
  - which measurements to perform;
  - how to interpret their joint outcome into an outcome of the intended type.

Experiments and procedures



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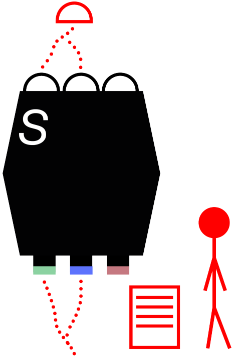

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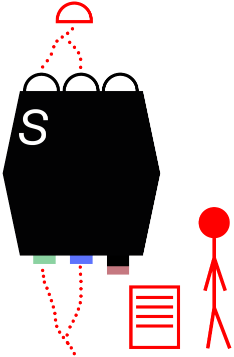
Experiments and procedures



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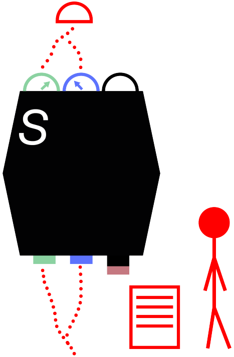

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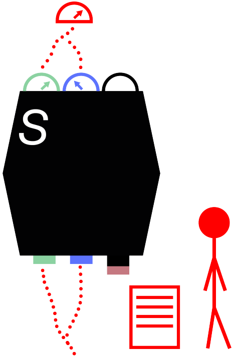
Experiments and procedures



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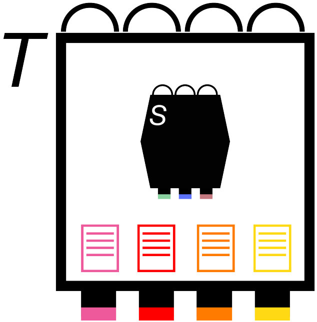
Experiments and procedures



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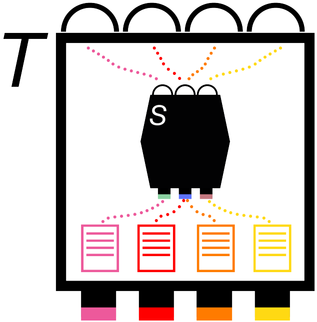
- A **deterministic procedure** $S \longrightarrow T$ specifies an S-experiment for each measurement of T

Experiments and procedures



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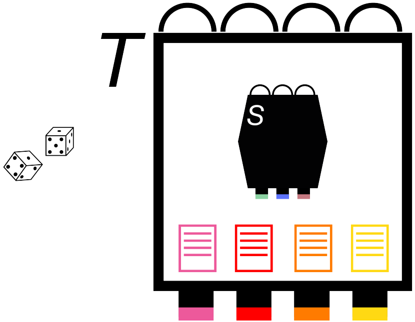

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  - which measurements to perform;
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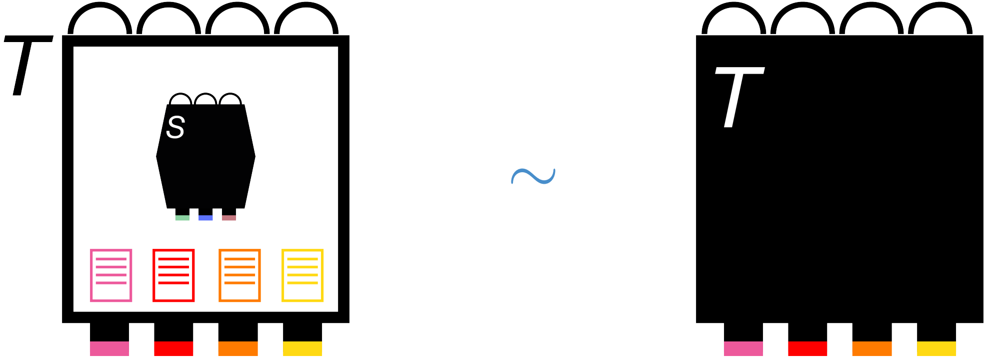
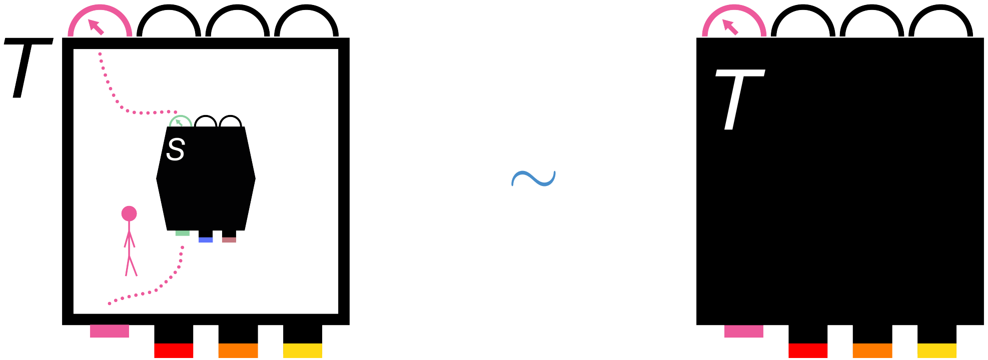
- A **deterministic procedure** $S \longrightarrow T$ specifies an S-experiment for each measurement of T
- A **classical procedure** is a probabilistic mixture of deterministic procedures.

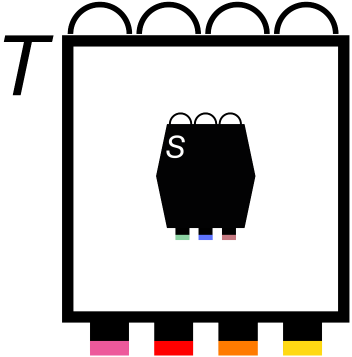
Classical procedures and simulations



Classical procedures and simulations



Classical procedures and simulations



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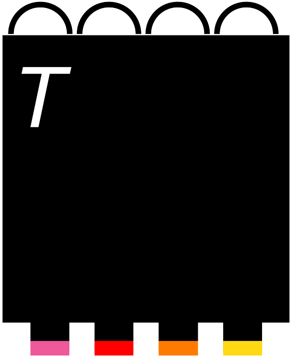

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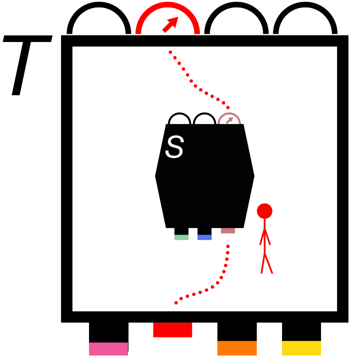

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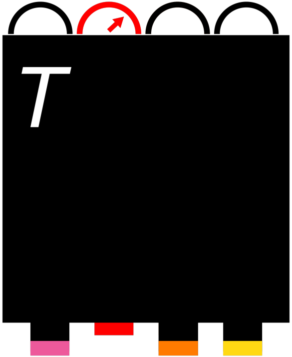

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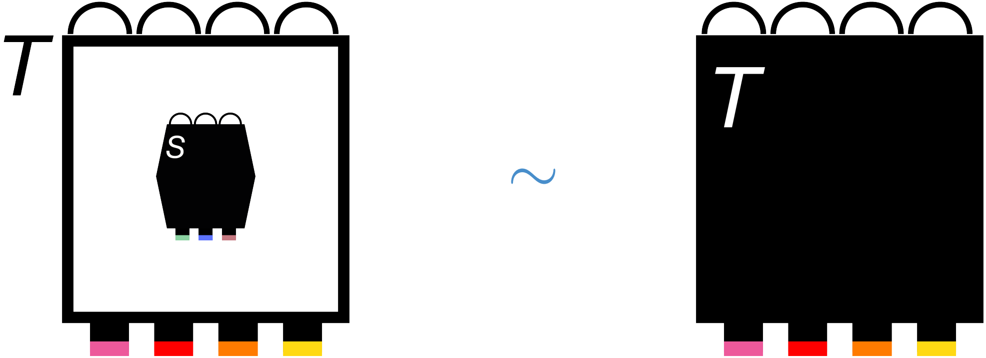

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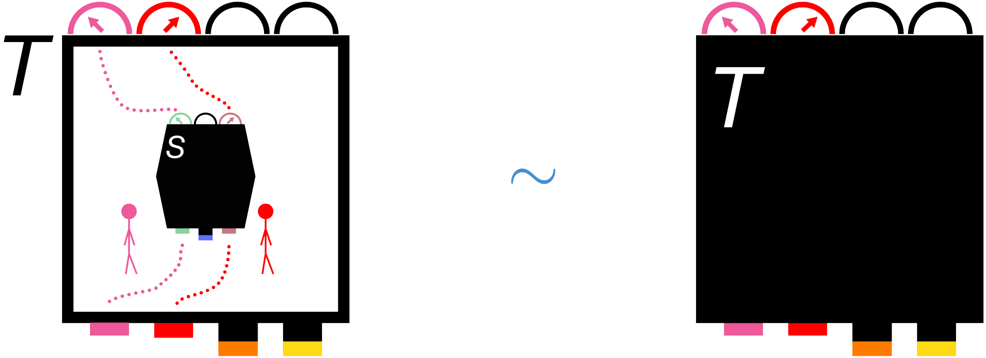

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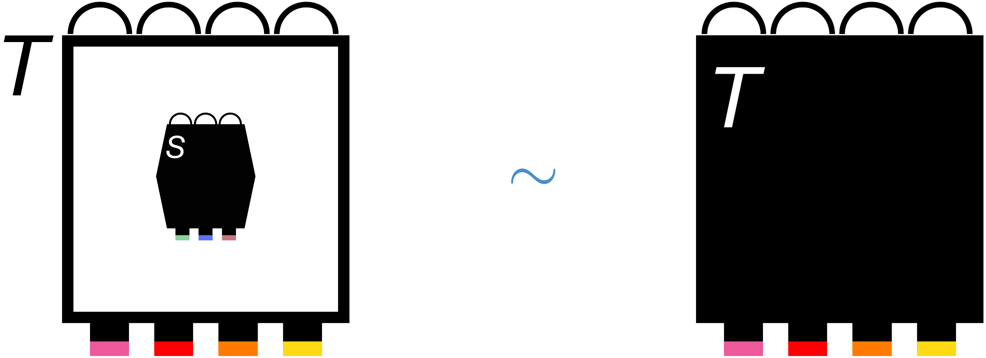

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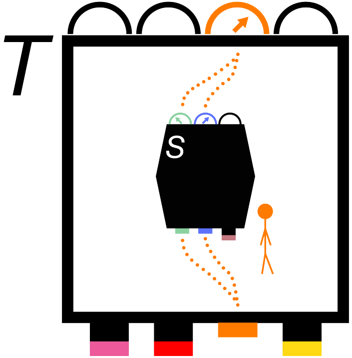

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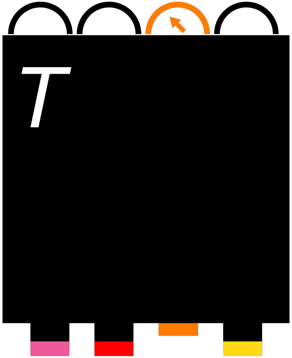
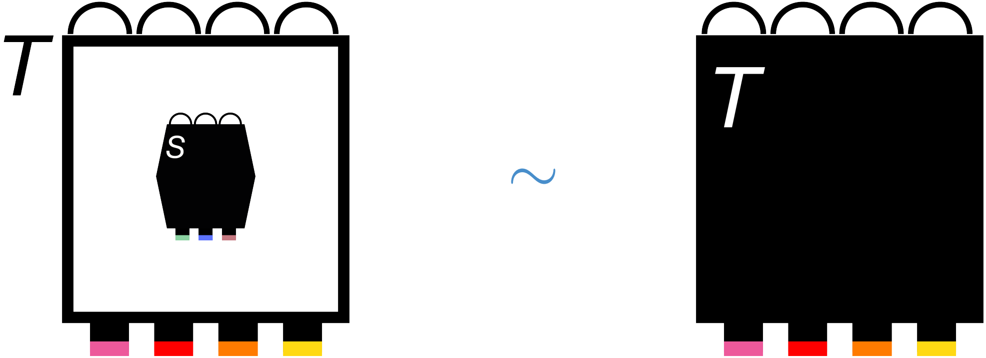


Classical procedures and simulations



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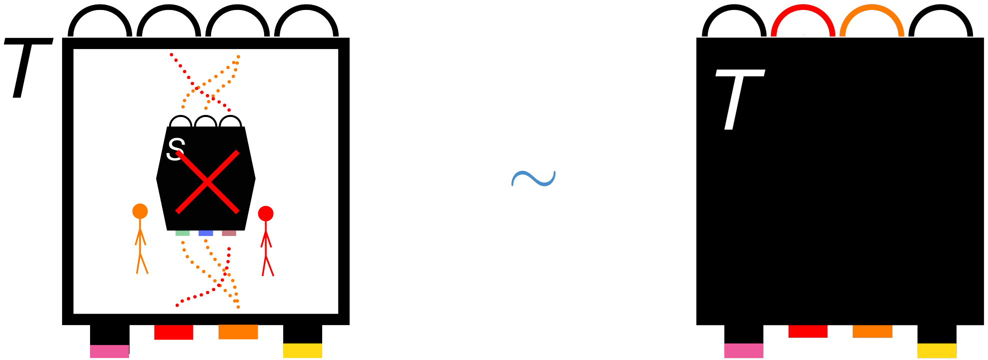

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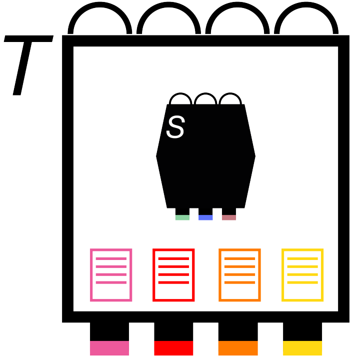

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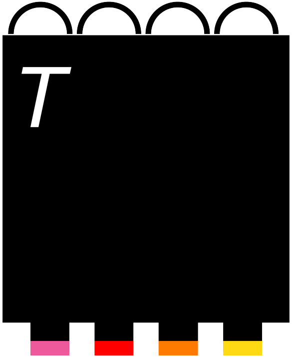
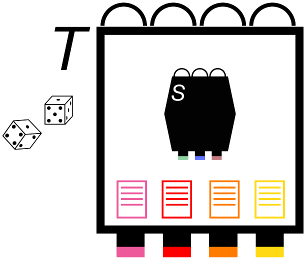


Classical simulations

- A classical procedure induces a (convex-preserving) map between empirical models:

$$f : S \longrightarrow T \qquad\qquad \mathbf{Emp}(f) : \mathbf{Emp}(S) \longrightarrow \mathbf{Emp}(T)$$



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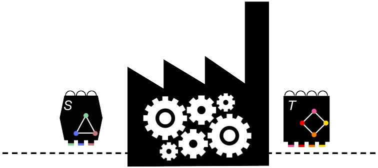

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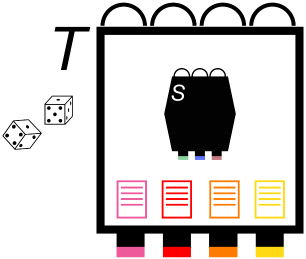

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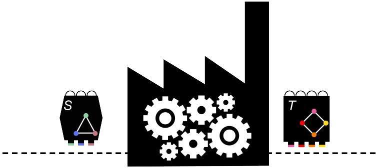
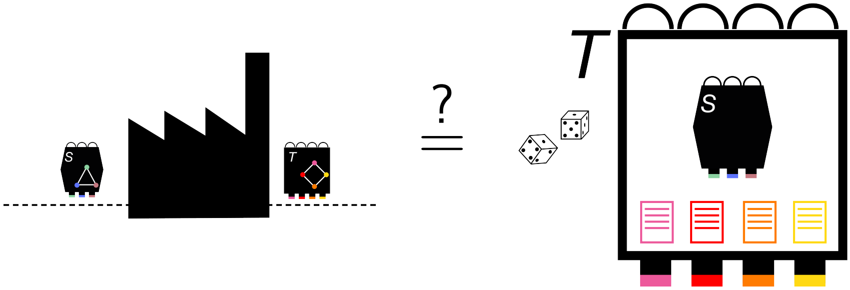


- Which black-box transformations arise in this fashion?

Main question and sketch of the answer

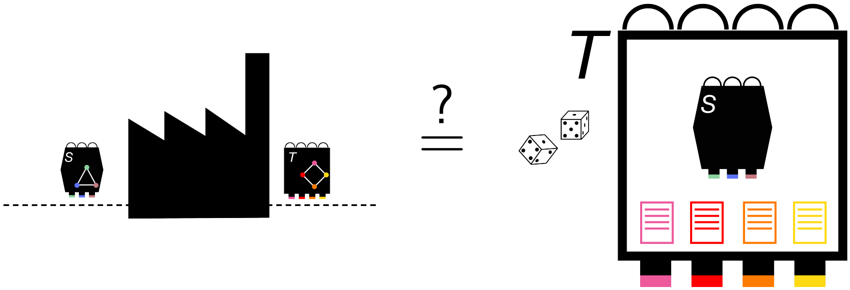
Main question

Given $F : \mathbf{Emp}(S) \longrightarrow \mathbf{Emp}(T)$, can it be realised by an experimental procedure? I.e. is there a procedure $f : S \longrightarrow T$ s.t. $F = \mathbf{Emp}(f)$?



Relativising contextuality

Given $F : \mathbf{Emp}(S) \longrightarrow \mathbf{Emp}(T)$, can it be realised by an experimental procedure? I.e. is there a procedure $f : S \longrightarrow T$ s.t. $F = \mathbf{Emp}(f)$?



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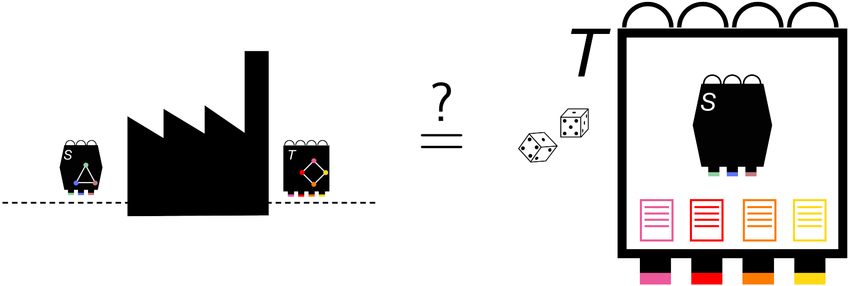
Special case $S = I$



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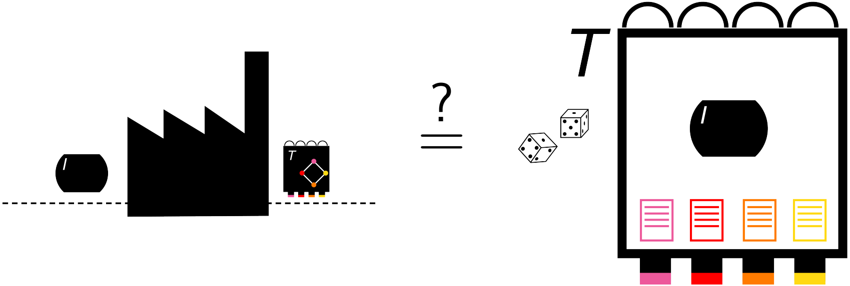
Special case $S = I$

Given $F : \mathbf{Emp}(I) \longrightarrow \mathbf{Emp}(T)$, can it be realised by an classical procedure? I.e. is there a procedure $f : I \longrightarrow T$ s.t. $F = \mathbf{Emp}(f)$?



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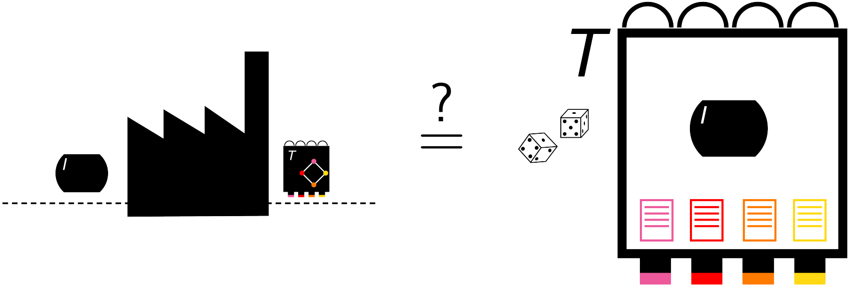

Relativising contextuality

Given $F : \mathbf{Emp}(S) \longrightarrow \mathbf{Emp}(T)$, can it be realised by an experimental procedure? I.e. is there a procedure $f : S \longrightarrow T$ s.t. $F = \mathbf{Emp}(f)$?

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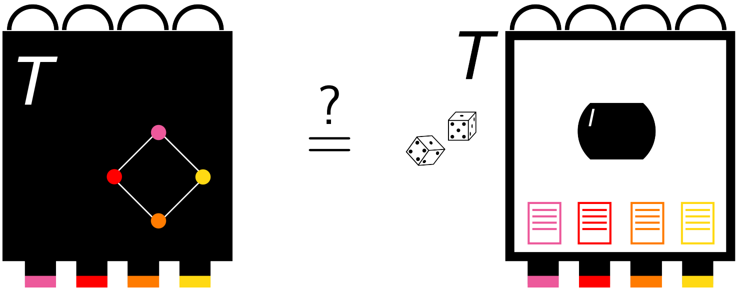

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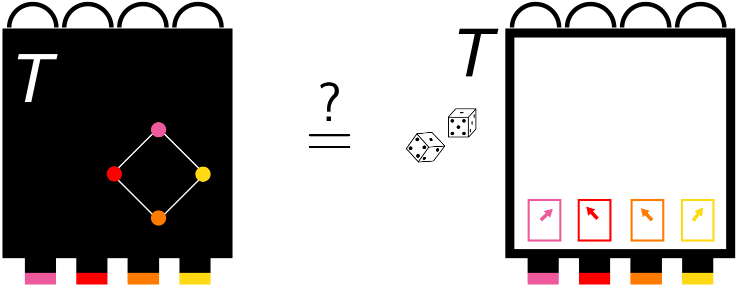

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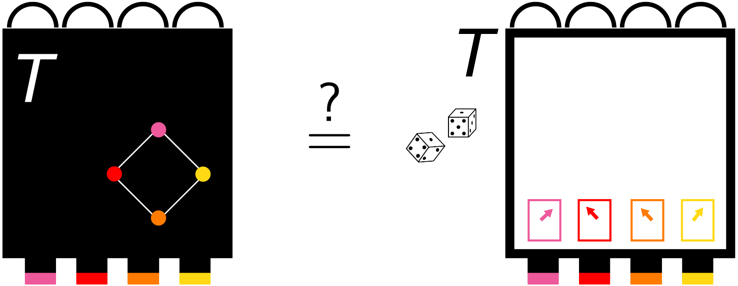
(Non-contextual models are those which can be simulated from nothing.)



From objects to morphisms ...

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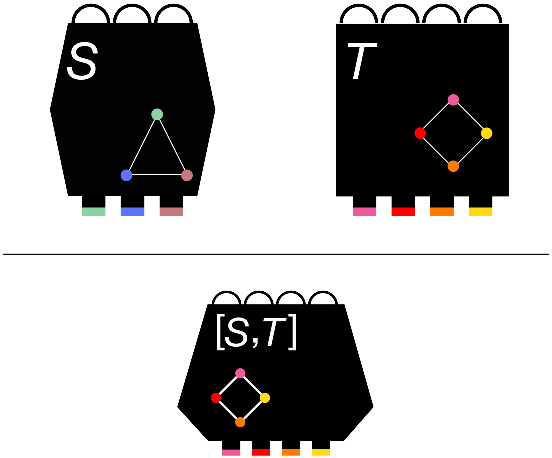
Answering the question by internalisation



From two scenarios S and T, we build a new scenario $[S, T]$.

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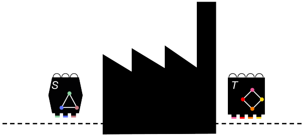

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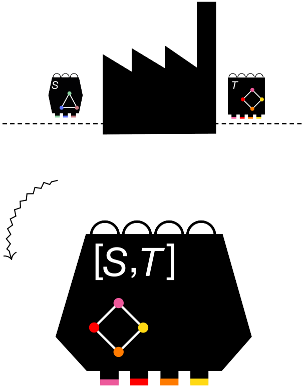

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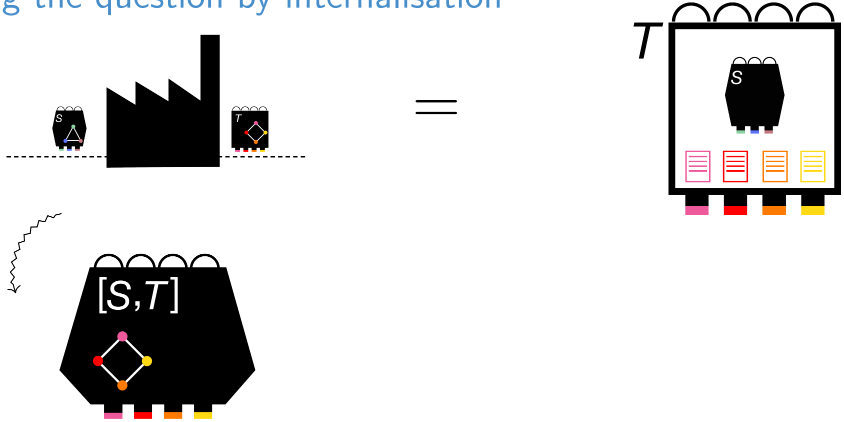

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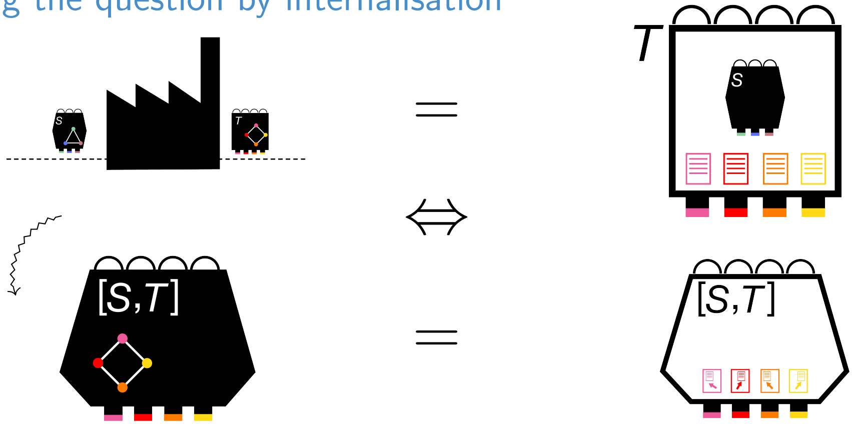

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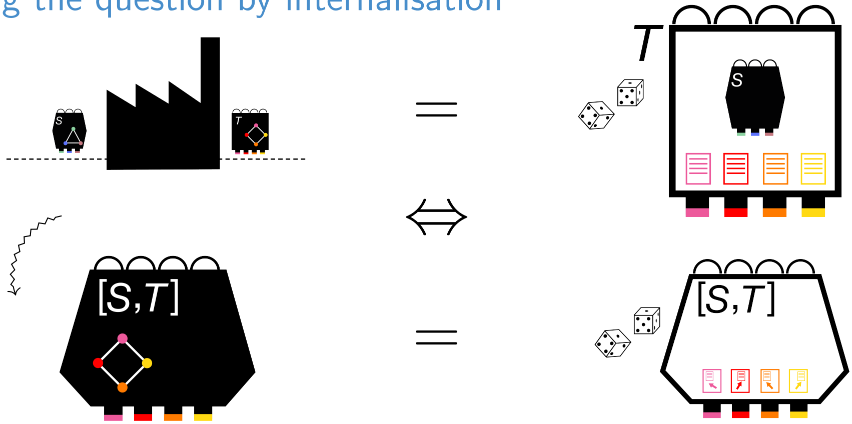

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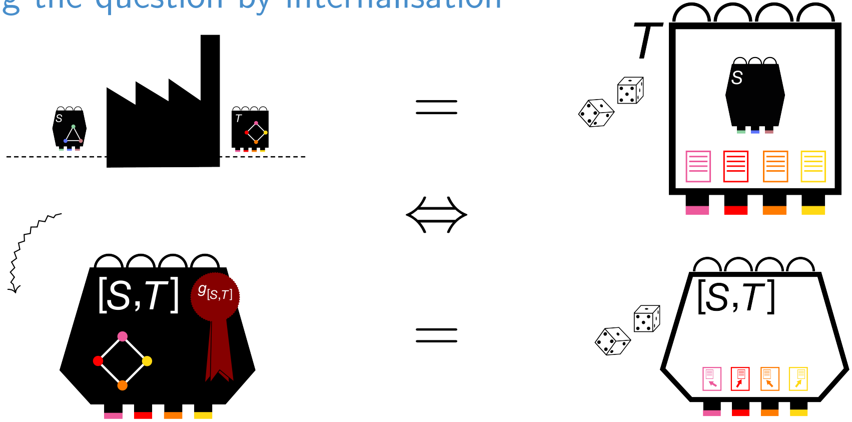

A convex preserving $F : \mathbf{Emp}(S) \longrightarrow \mathbf{Emp}(T)$ induces a canonical model $e_F : [S, T]$.

F is realised by a deterministic procedure iff e_F is deterministic and satisfies $g_{[S,T]}$.

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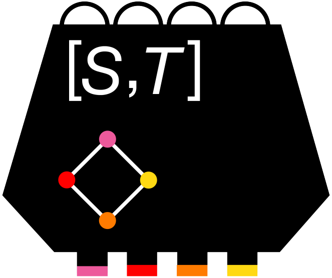
Further details

The hom scenario [S,T]



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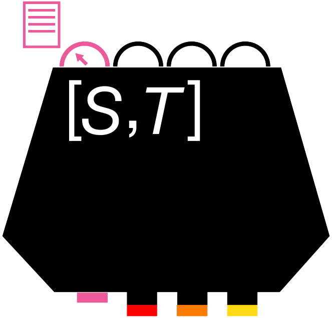

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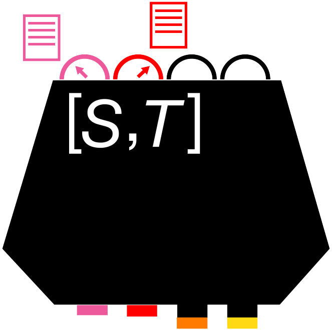

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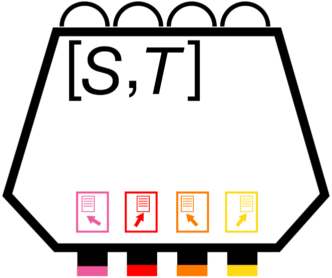

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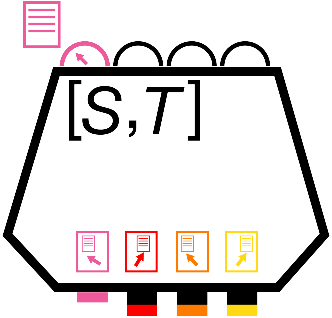

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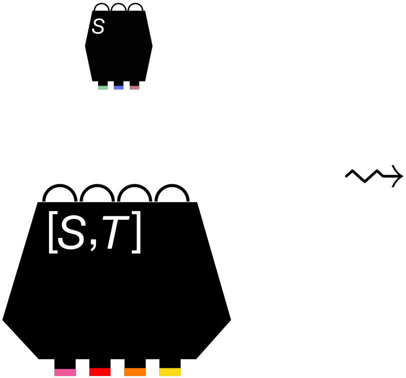

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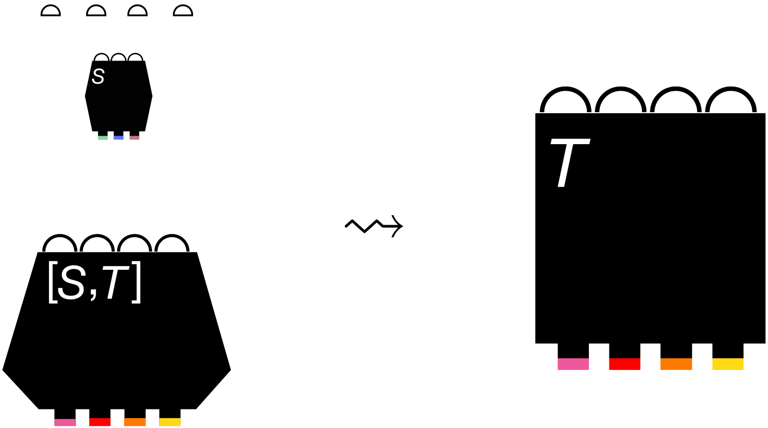

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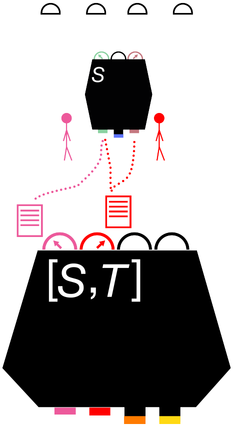
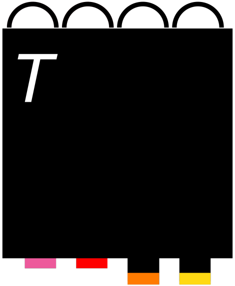


$\rightsquigarrow$



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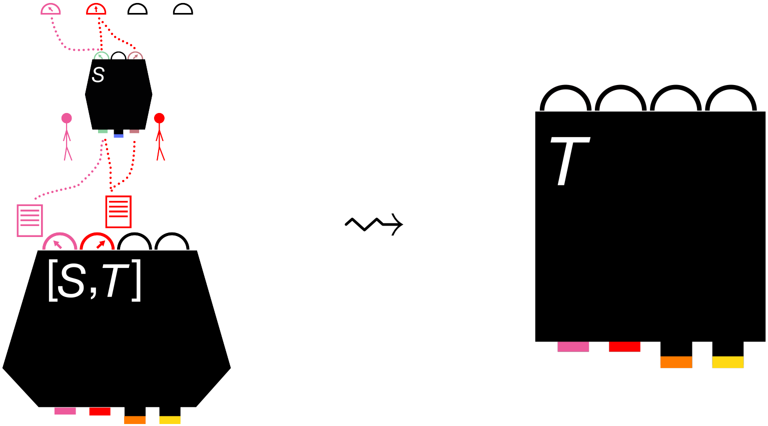

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Similarly, $\sum r_i f_i$ is induced by an experiment if each U_{f_i} is a compatible set of measurements.

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Lemma

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Theorem
$[-,-]$ *(appropriately modified) makes this category into a closed category.*

Outlook

Further questions

- External characterisation of adaptive procedures?
  Note that $[S, T]$ can be defined in the adaptive case, but there is no obvious way of building a canonical adaptive empirical model out of a convex-preserving function $\mathbf{Emp}(S) \longrightarrow \mathbf{Emp}(T)$.

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- Does the set of all predicates on S generalise partial Boolean algebras to arbitrary measurement compatibility structures?

- Examining the closed structure?
  Note that it's not monoidal wrt. the usual monoidal structure, but seems closed wrt a 'directed' tensor product.

Questions...

?