Causal contextuality and adaptive MBQC

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(joint work with Cihan Okay)

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INTERNATIONAL IBERIAN
NANOTECHNOLOGY
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Joint work with Cihan Okay

Funded by
the European Union

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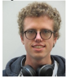
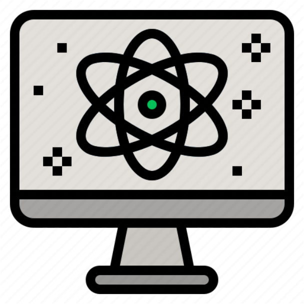
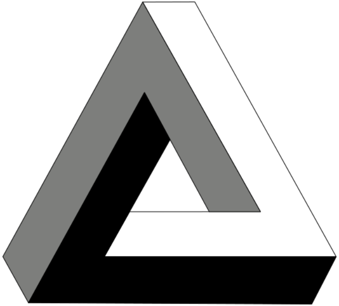
- Related to talks by Samson & Amy, but only using a particular type of models.

- May have some relation to upcoming talk by Sivert.



Introduction



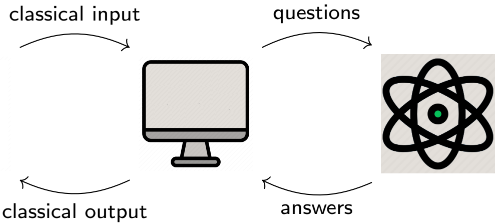


Quantum advantage

Contextuality / Nonclassicality

Contextuality in MBQC

'*Contextuality in measurement-based quantum computation*', Raussendorf, PRA 2013. 



MBQC: Classical control computer with access to quantum resources

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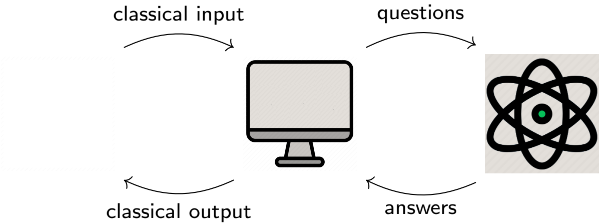

ℓ_2-MBQC: Classical control computer with access to quantum resources

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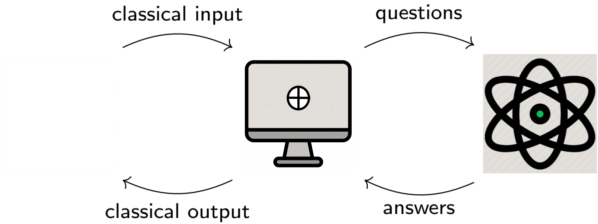

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- Classical control restricted to $\mathbb{Z}_2$-**linear** computation

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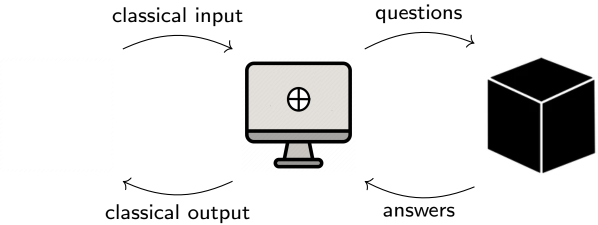

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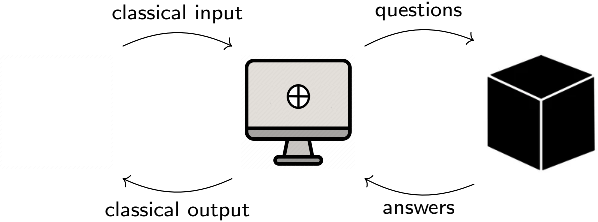

ℓ_2-MBQC: Classical control computer with access to quantum resources

- Classical control restricted to $\mathbb{Z}_2$-**linear** computation
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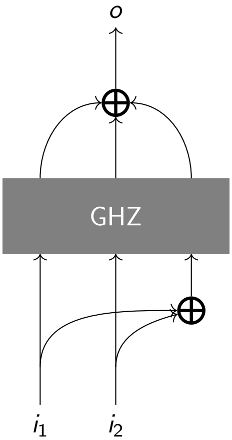
Theorem

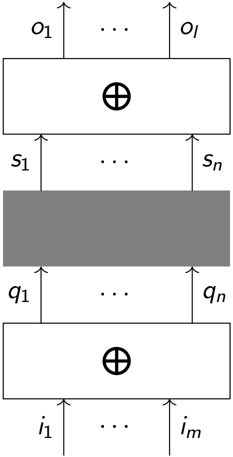
If an ℓ_2-MBQC **deterministically** *computes a* **nonlinear** *Boolean function then the resource is* **strongly contextual**.

The AND function

'*Computational power of correlations*', Anders & Browne, PRL 2009.



Adaptive MBQC



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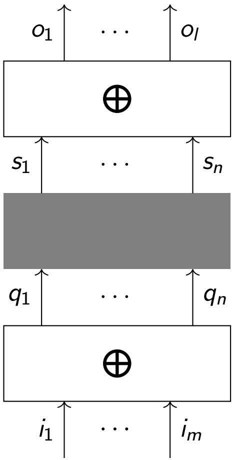

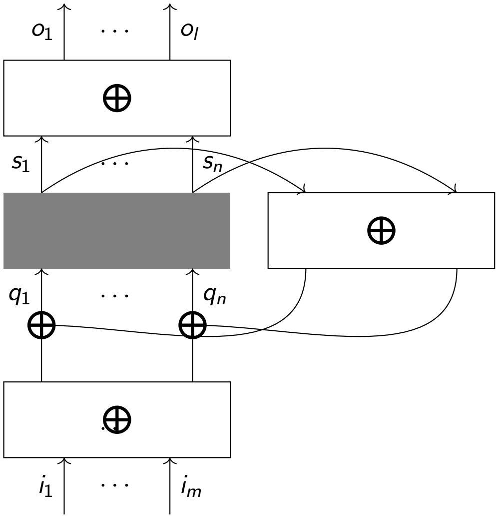

Question

In adaptive MBQC:

- For a given computation, the black box is used in a given (partial) order.

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- Why should the classical benchmark be so restrictive?
- We could think of a classical model that exploits this (causal) knowledge.

$\boxed{\text{Can we find conditions on the computed functions that exclude even such classical HV models?}}$

Non-locality

Bell scenarios

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- a set Ω of **sites** or parties

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$$\mathcal{Q}_{S \subset T} : \mathcal{Q}_T \longrightarrow \mathcal{Q}_S \qquad \text{and} \qquad \mathcal{A}_{S \subset T} : \mathcal{A}_T \longrightarrow \mathcal{A}_S$$

Deterministic local models

A **deterministic local** model is given by a family of functions

$$f_\omega : \mathcal{Q}_\omega \longrightarrow \mathcal{A}_\omega \qquad (\omega \in \Omega).$$

E.g. bipartite scenario: $(\mathcal{Q}_A \longrightarrow \mathcal{A}_A) \times (\mathcal{Q}_B \longrightarrow \mathcal{A}_B)$.

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Locality and no-signalling

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Causal contextuality

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- A causal (partial) order between sites

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- A causal (partial) order between sites
- Classical models are allowed to use information from the causal past
- i.e. the answer at a given site may depend on the questions asked at sites in its past.
- Correspondingly, no-signalling gets relaxed, permitting signalling to the future.

NB: a special class of scenarios within the formalism presented by Samson & Amy.

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Notation: $\downarrow \omega := \{\omega' \in \Omega \mid \omega' \leq \omega\}$ $\qquad$ $\downarrow S := \bigcup_{\omega \in S} \downarrow \omega = \{\omega' \in \Omega \mid \exists \omega \in S.\ \omega' \leq \omega\}$

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  This yields models that are **no-signalling except from the past**.

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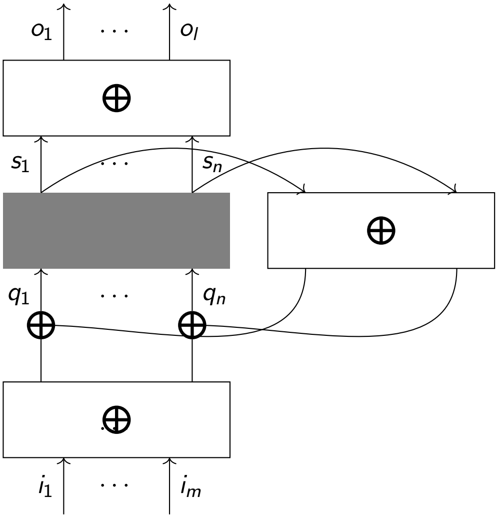
Measurement-based quantum computation

Adaptive ℓ_2-MBQC



- input size m
- output size l
- adaptive structure $(\Omega, \leq)$ with $n = |\Omega|$

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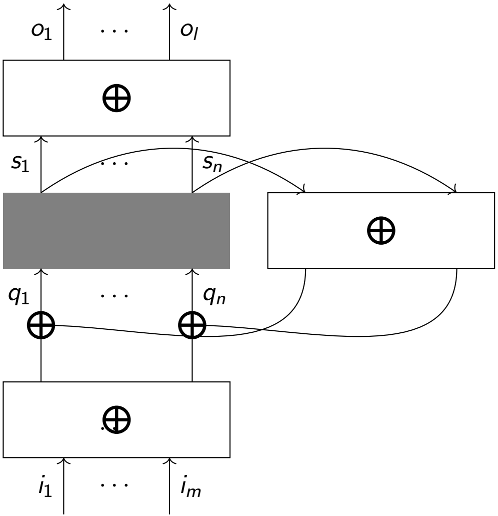

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- $Q : \mathbb{Z}_2^m \longrightarrow \mathbb{Z}_2^n$
- $T : \mathbb{Z}_2^n \longrightarrow \mathbb{Z}_2^n$
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such that $T_{\omega,\omega'} \neq 0 \Rightarrow \omega \leq \omega'$

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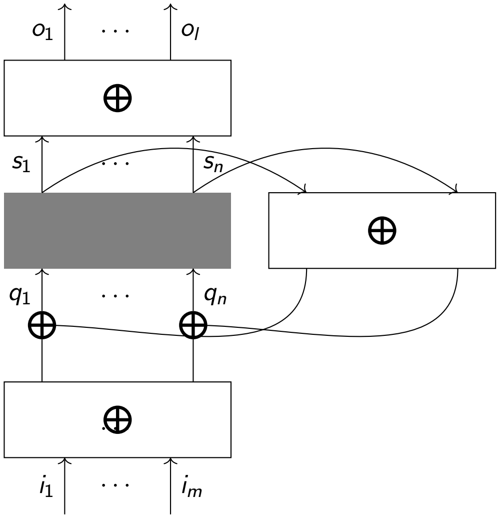

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$$\begin{aligned} \mathbf{q} &= Q\mathbf{i} + T\mathbf{s} \\ \mathbf{s} &\leftarrow e(\mathbf{q}) \\ \mathbf{o} &= Z\mathbf{s} \end{aligned}$$

implements a function $\mathbb{Z}_2^m \longrightarrow D(\mathbb{Z}_2^l)$.

Causal contextuality and adaptive MBQC

Main result

- Functions $g : \mathbb{Z}_2^m \longrightarrow \mathbb{Z}_2$ can be represented as m-variable polynomials in $\mathbb{Z}_2$, $\pi(g)$.
- Functions $g : \mathbb{Z}_2^m \longrightarrow \mathbb{Z}_2^l$ are represented by l-tuples of m-variable polynomials $\pi(g) = \langle \pi(g)_1, \ldots \pi(g)_l \rangle$.

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Theorem

Let (e, Q, T, Z) be an Ω-adaptive ℓ_2-MBQC protocol that **deterministically** *computes a function $g : \mathbb{Z}_2^m \longrightarrow \mathbb{Z}_2^l$. If e is* **causally classical** *then each $\pi(g)_j$ is a polynomial with degree* **at most the height of Ω**, *where the height of a poset is the maximum length of a chain in it.*

Main result

- Functions $g : \mathbb{Z}_2^m \longrightarrow \mathbb{Z}_2$ can be represented as m-variable polynomials in $\mathbb{Z}_2$, $\pi(g)$.
- Functions $g : \mathbb{Z}_2^m \longrightarrow \mathbb{Z}_2^l$ are represented by l-tuples of m-variable polynomials $\pi(g) = \langle \pi(g)_1, \ldots \pi(g)_l \rangle$.

Theorem

Let (e, Q, T, Z) be an Ω-adaptive ℓ_2-MBQC protocol that **deterministically** *computes a function $g : \mathbb{Z}_2^m \longrightarrow \mathbb{Z}_2^l$. If e is* **causally classical** *then each $\pi(g)_j$ is a polynomial with degree* **at most the height of Ω**, *where the height of a poset is the maximum length of a chain in it.*

NB: If Ω is flat, i.e. has heigth 1, one recovers Raussendorf's result about nonlinear functions.

Questions...

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