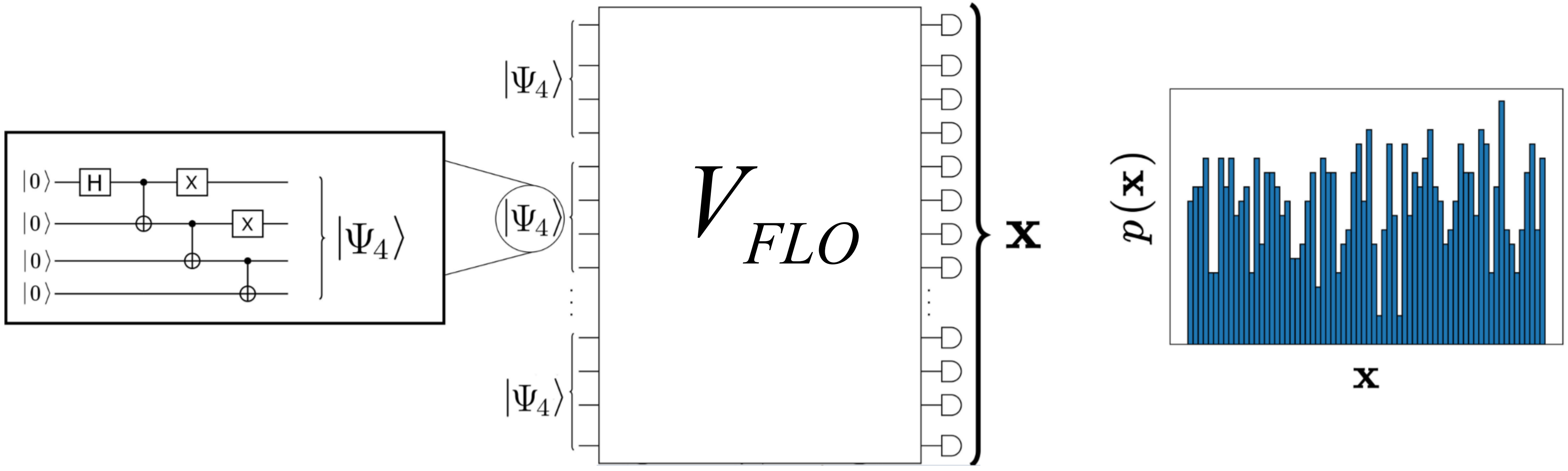
Fermion Sampling: a robust quantum advantage scheme using fermionic linear optics and magic input states

Michał Oszmaniec, Ninnat Dangniam, Mauro Morales, Zoltán Zimborás

arxiv:2012.15825

Fermion Sampling with magic input states



Proposal for quantum computational advantage/supremacy: sample random FLO circuits

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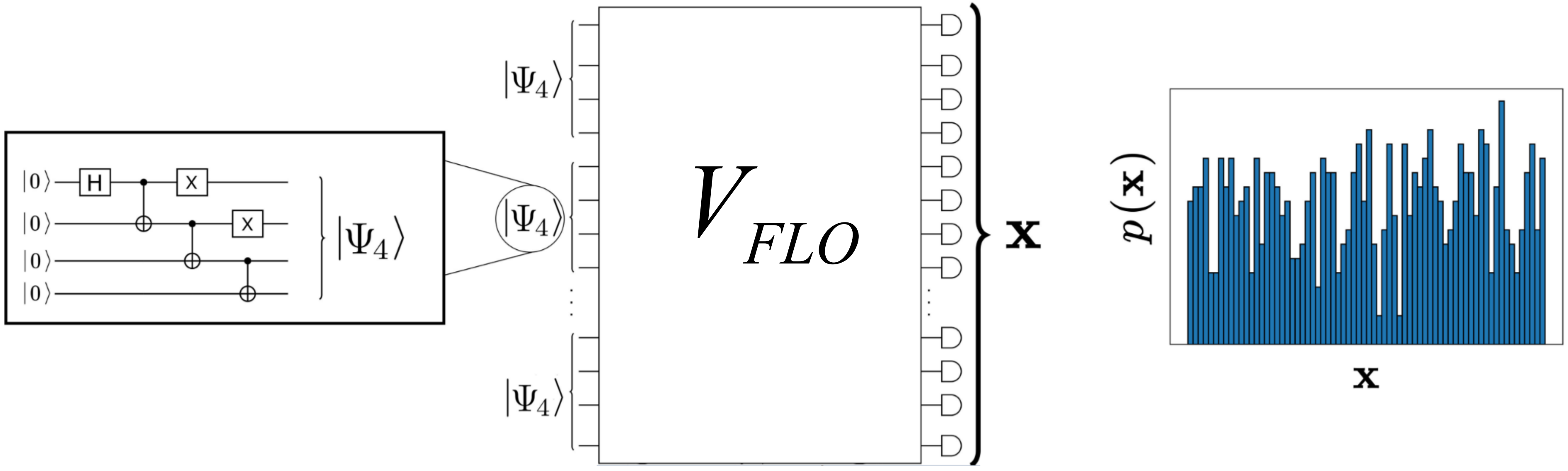

Proposal for quantum computational advantage/supremacy: sample random FLO circuits

- Fermionic analogue of Boson Sampling
- Feasible in near-term architectures
- Hardness guarantees matching Random Circuit Sampling

Near-term quantum computers

- Present-day quantum computers are noisy, imperfect and not scalable.
- Implementation of complicated quantum algorithms (like Shor algorithm)

in the near-term is **science-fiction**

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How to factor 2048 bit RSA integers in 8 hours using 20 million noisy qubits

Craig Gidney[1,*] and Martin Ekerå[2]
[1] *Google Inc., Santa Barbara, California 93117, USA*
[2] *KTH Royal Institute of Technology, SE-100 44 Stockholm, Sweden*
Swedish NCSA, Swedish Armed Forces, SE-107 85 Stockholm, Sweden
(Dated: December 6, 2019)

	Physical assumptions				Approach		Estimated costs		
Historical cost estimate at $n = 2048$	Physical gate error rate	Cycle time (microseconds)	Reaction time (microseconds)	Physical connectivity	Distillation strategy	Execution strategy	Physical qubits (millions)	Expected runtime (days)	Expected volume (megaqubitdays)
Fowler et al. 2012 [9]	0.1%	1	0.1	planar	1200 T	single threaded	1000	1.1	1100
O'Gorman et al. 2017 [18]	0.1%	10	1	arbitrary	block CCZ	single threaded	230	3.7	850
Gheorghiu et al. 2019 [19]	0.1%	0.2	0.1	planar	1100 T	single threaded	170	1	170
(ours) 2019 (1 factory)	0.1%	1	10	planar	1 CCZ	serial distillation	16	6	90
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- Still, we hope that near-term quantum computers will be useful for **something** [Preskill, 2018]

Near-term quantum computers (II)

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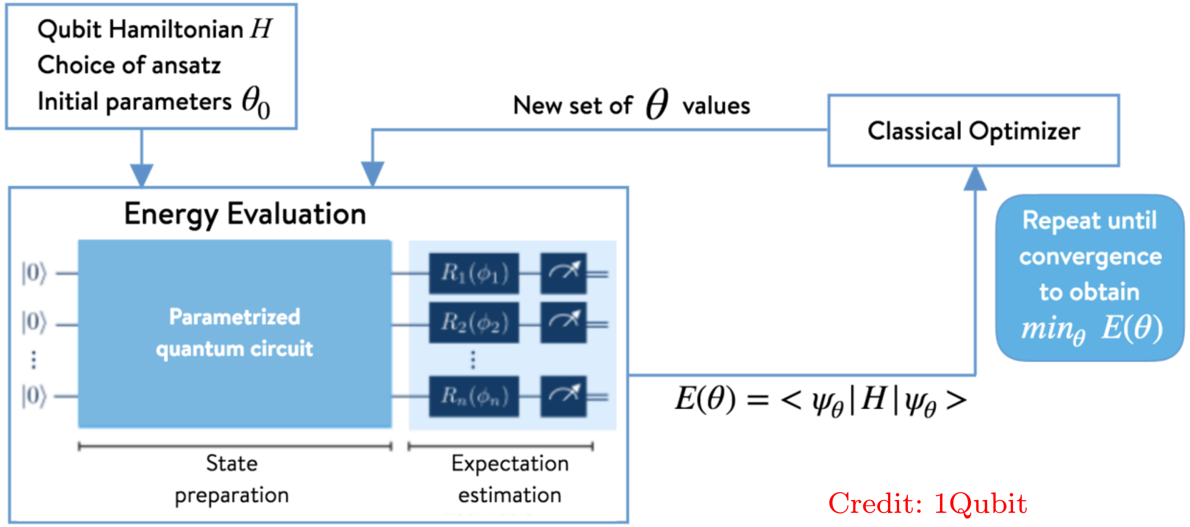

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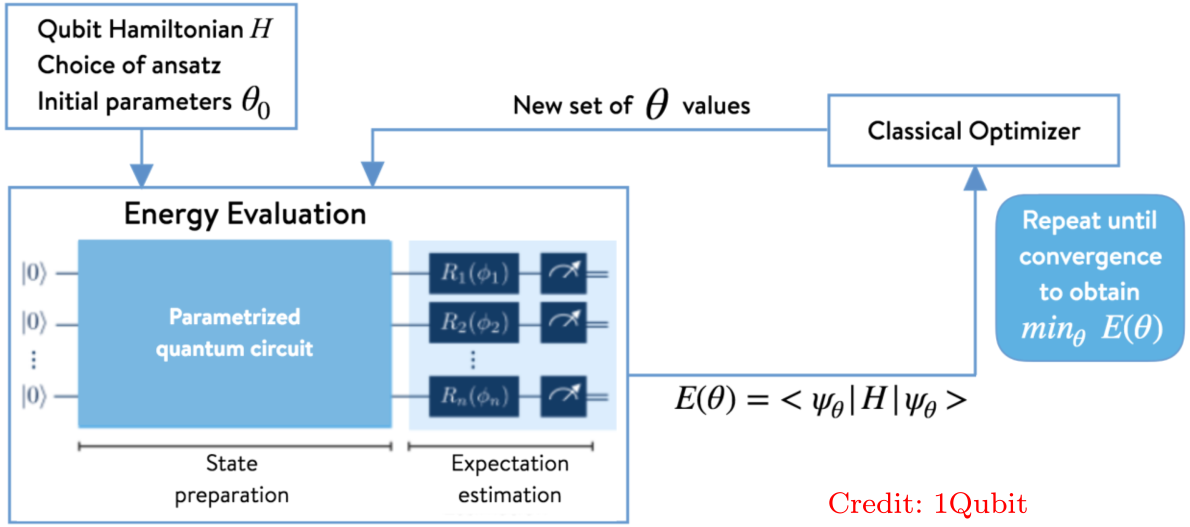

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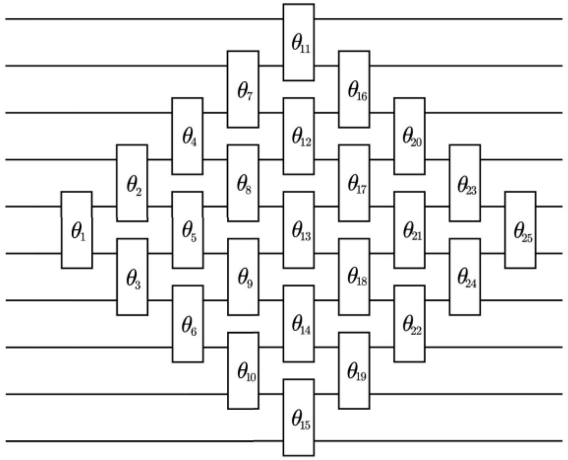
Exemplary parametric circuit



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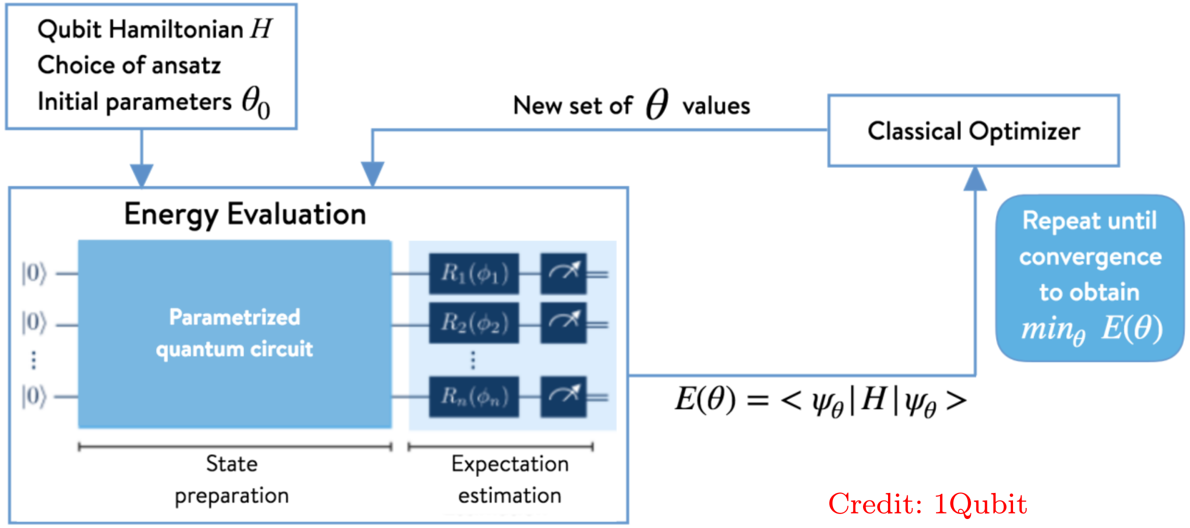

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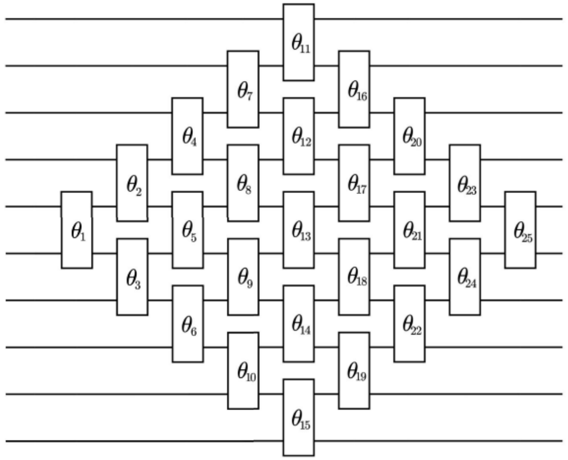

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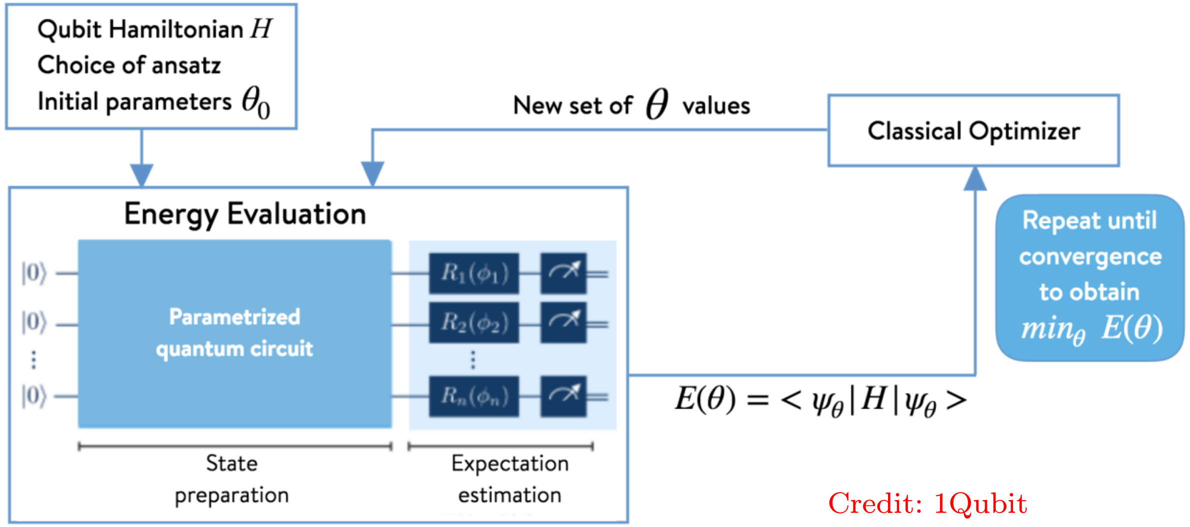

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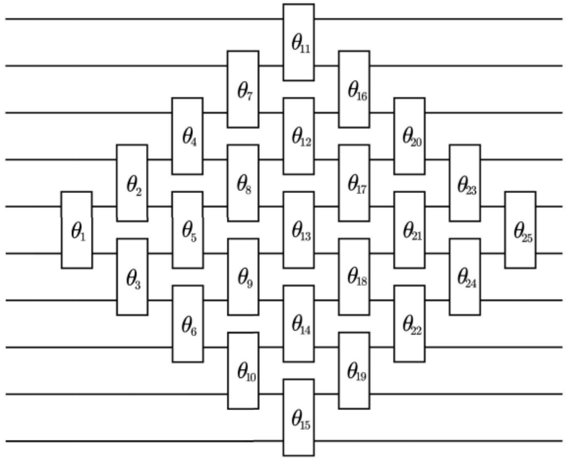

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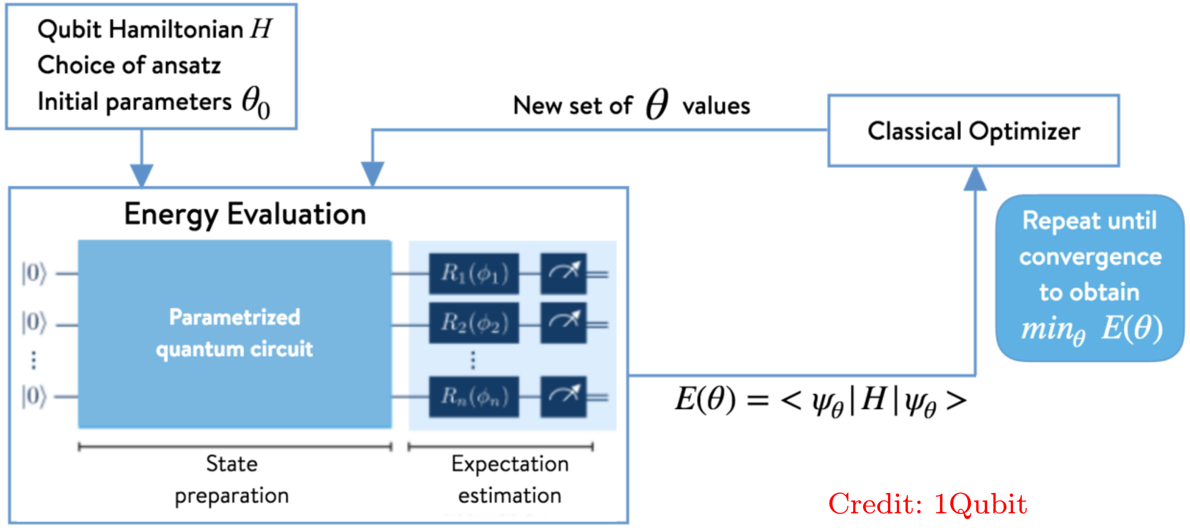

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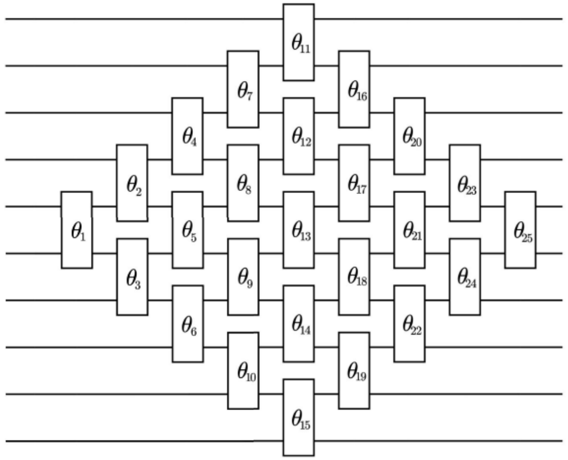

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Quantum Approximate Optimization of Non-Planar Graph Problems on a Planar Superconducting Processor

Google AI Quantum and Collaborators*
(Dated: April 10, 2020)

We demonstrate the application of the Google Sycamore superconducting qubit quantum processor to discrete optimization problems with the quantum approximate optimization algorithm (QAOA). Like past QAOA experiments, we study performance for problems defined on the connectivity graph of our hardware; however, we also apply the QAOA to the Sherrington-Kirkpatrick model and 3-regular MaxCut, both high dimensional graph problems requiring significant compilation. Experimental scans of the QAOA energy landscape show good agreement with theory across even the largest instances studied (23 qubits) and we are able to perform variational optimization successfully. For problems defined on the planar graph of our hardware we obtain an approximation ratio that is independent of problem size and observe, for the first time, that performance increases with circuit depth. For problems requiring compilation, performance decreases with problem size but still provides an advantage over random guessing for circuits involving several thousand gates. This behavior highlights the challenge of using near-term quantum computers to optimize problems on graphs differing from hardware connectivity. As these graphs are more representative of real world instances, our results advocate for more emphasis on such problems in the developing tradition of using the QAOA as a holistic benchmark of quantum processors.

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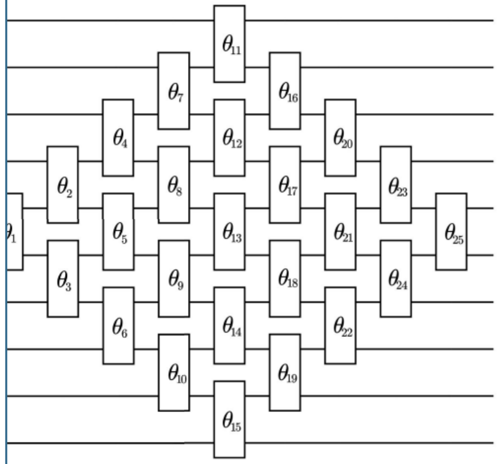

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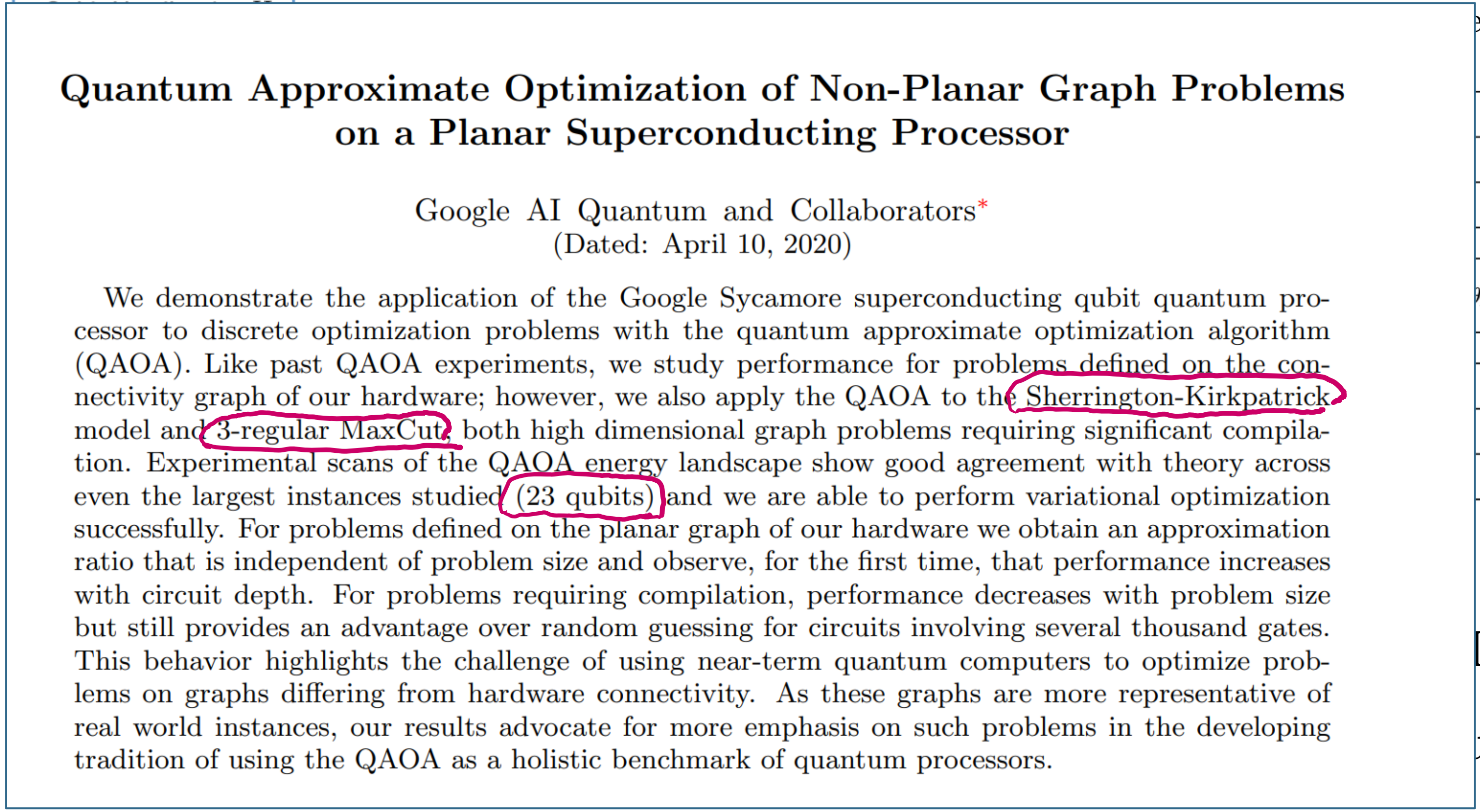

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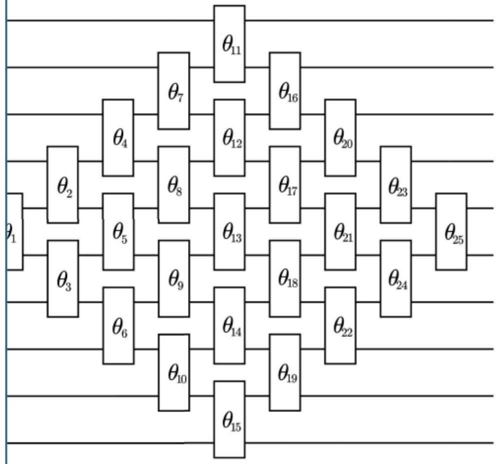

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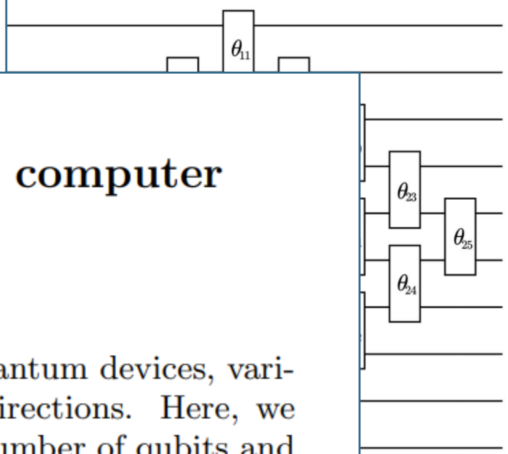

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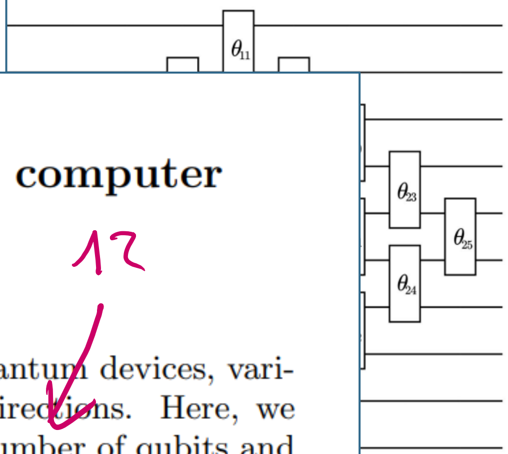

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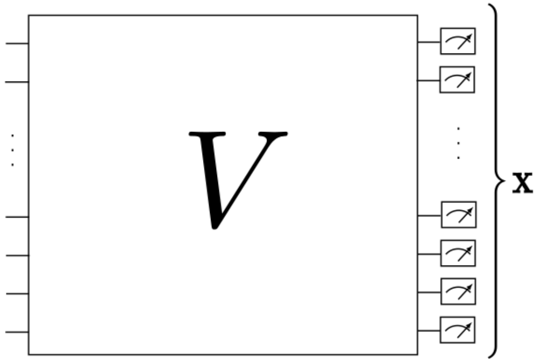

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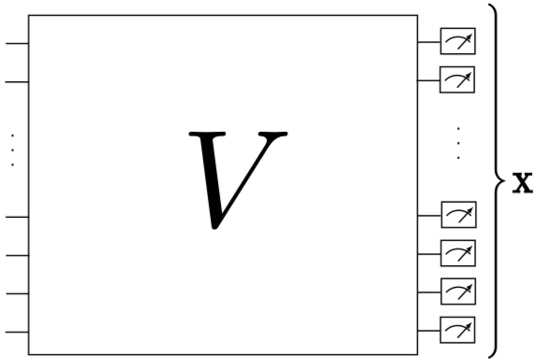

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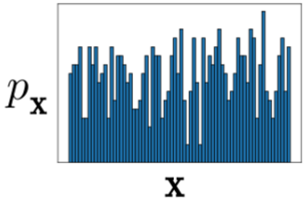

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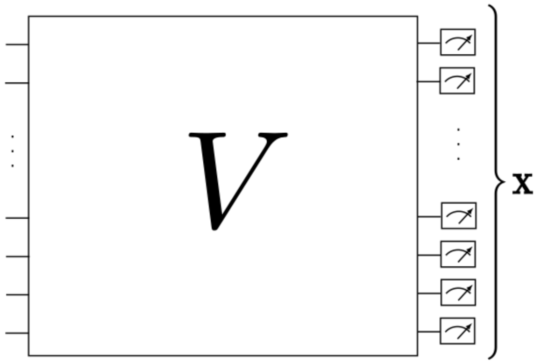

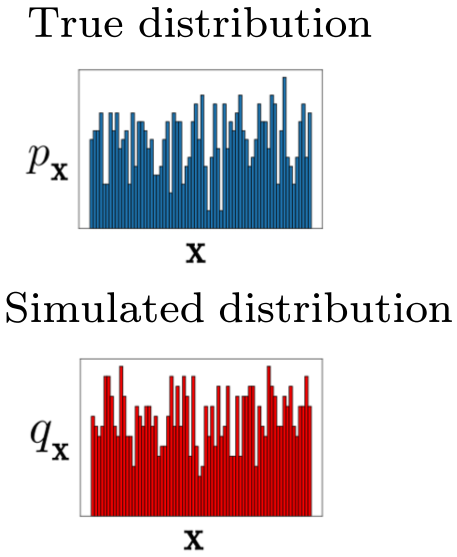

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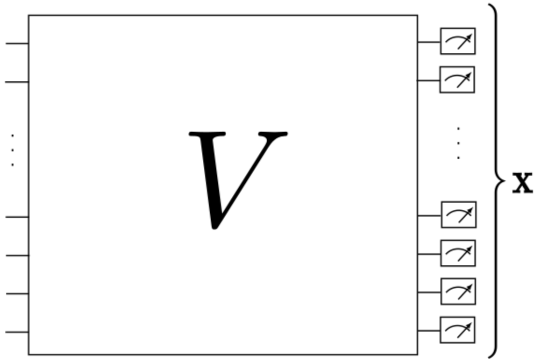

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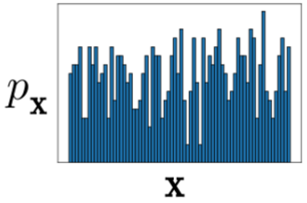

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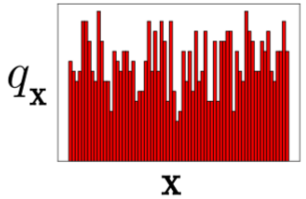

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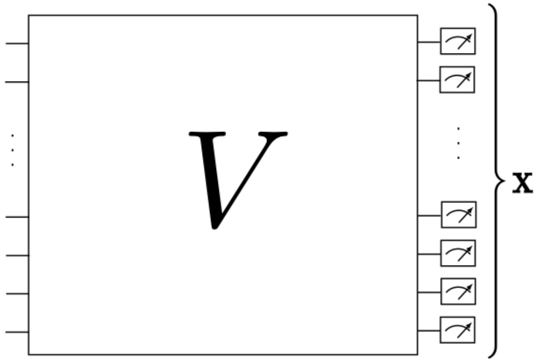

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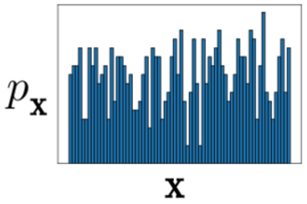

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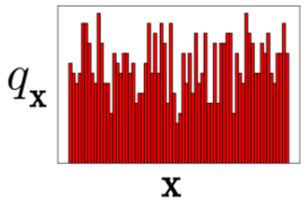

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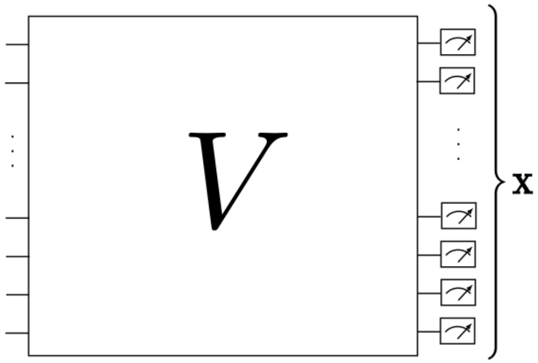

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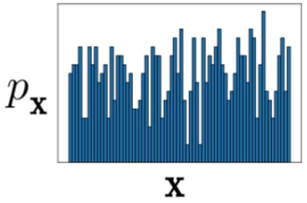

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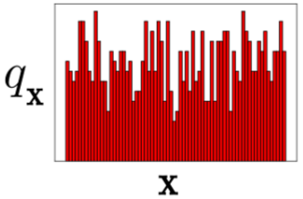

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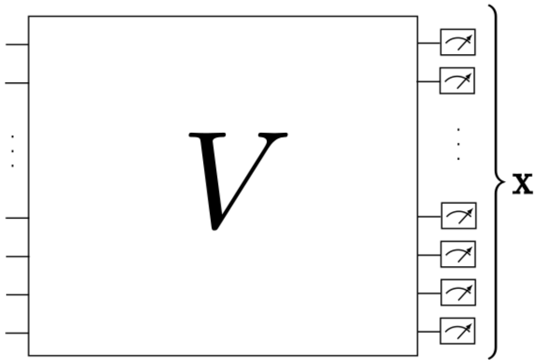

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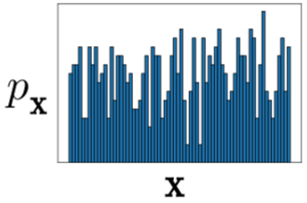

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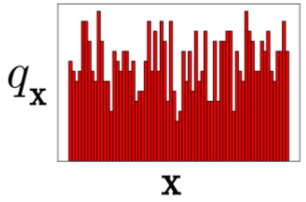

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- **Cons**: not practical, noise still affects such proposals

Quantum computational advantage/supremacy (II)

Computer science: **polynomial-time computation == efficient**

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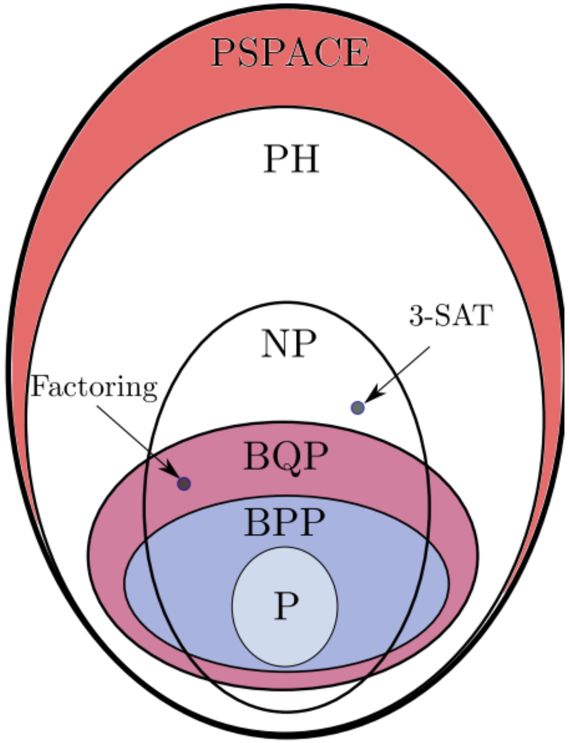

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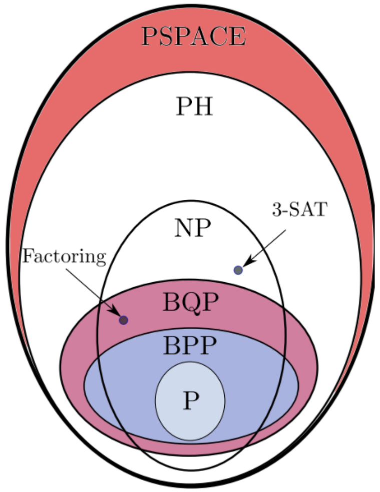

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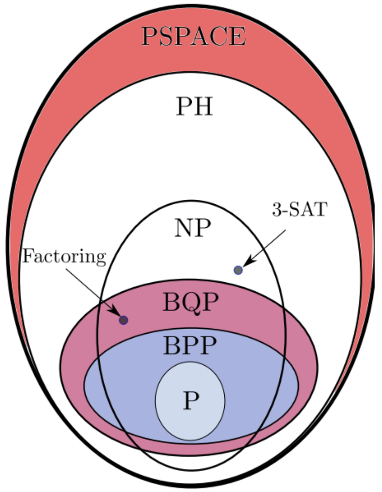
Quantum computational advantage/supremacy (II)

Computer science: **polynomial-time computation == efficient**



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$\Longrightarrow$

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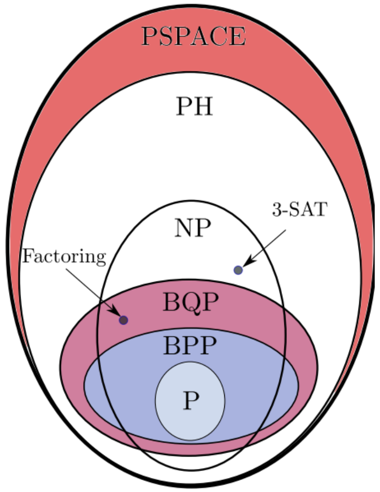

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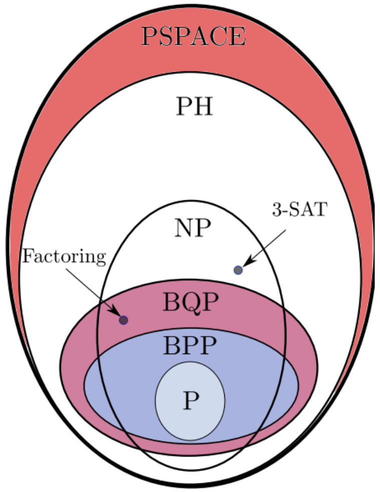

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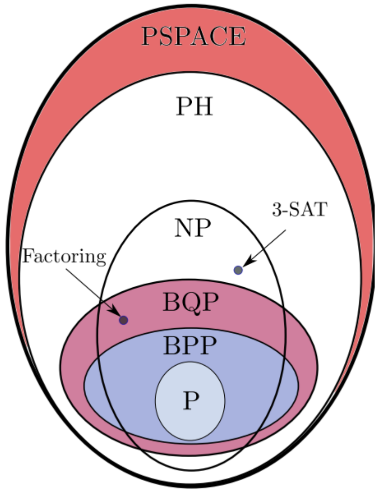

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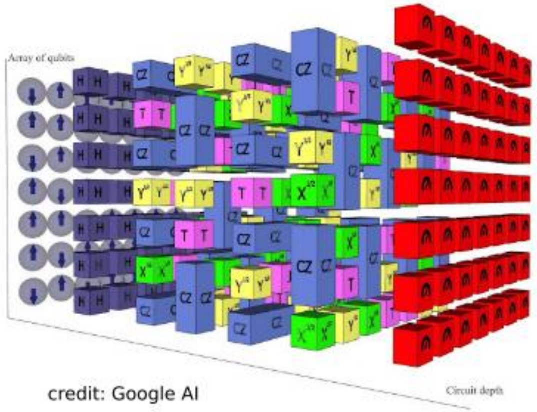
A: **Boson Sampling** [Aaronson-Arkhipov 2010], IQP [Bremner-Montanaro-Shepard 2016],

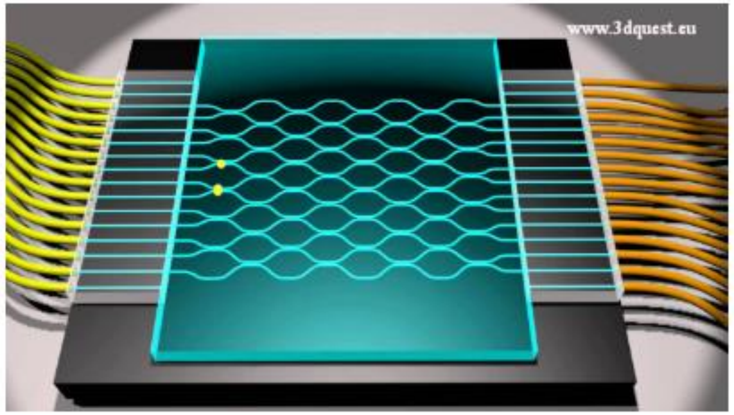
Random Circuit Sampling (RCS) [Boixo et al. 2018] [Bouland et al. 2018] [Movassagh 2019]

Quantum computational advantage/supremacy (III)

Main experimental platforms: **Random Circuit Sampling** & **Boson Sampling**



credit: Google AI



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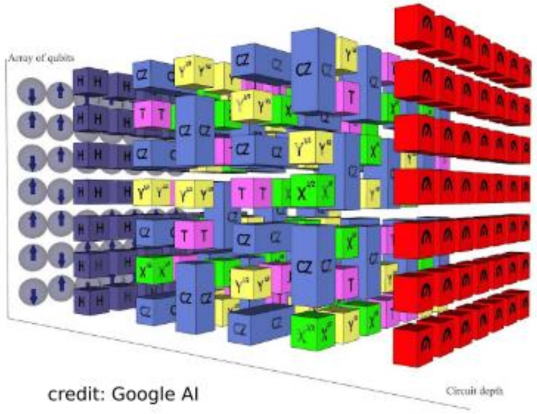

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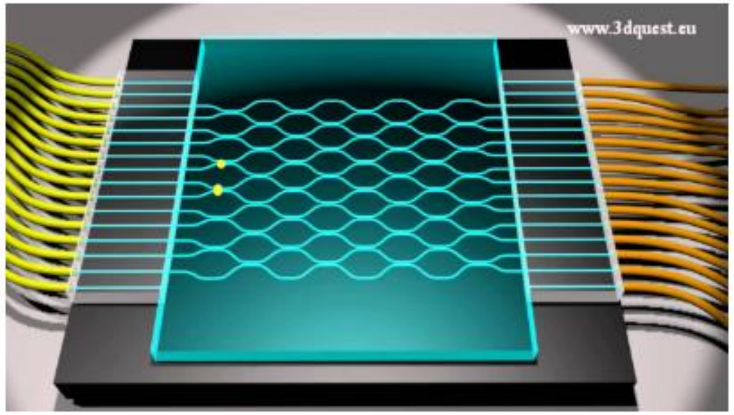

- Google/ UCSB experiment in 53 qubit Sycamore chip, depth ~20 [Arute *et al.* 2019]
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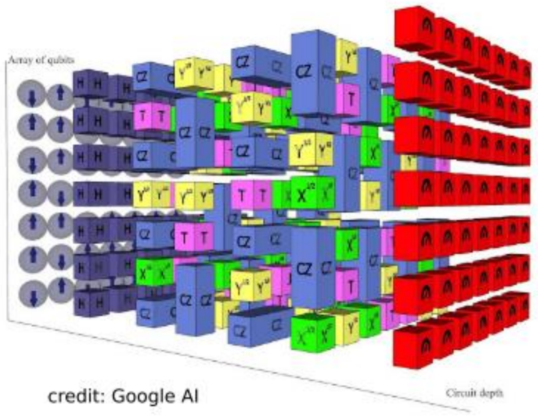

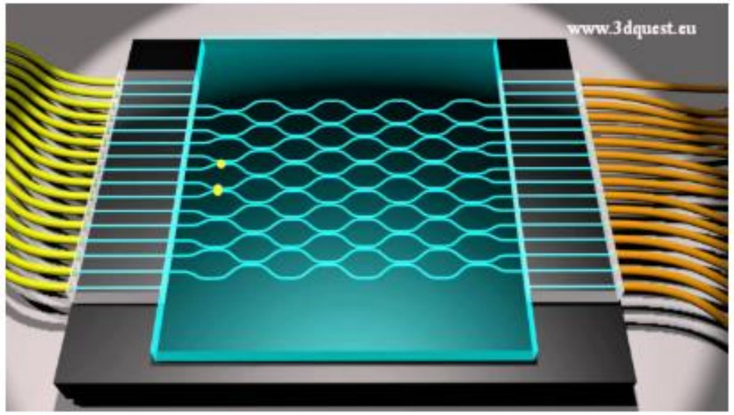

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Issues: certification [Hengleiter *et al.* 2019], spoofing by efficient classical simulations [Napp *et al.* 2019] [Renema *et al.* 2018]

Fermion Sampling with magic input states

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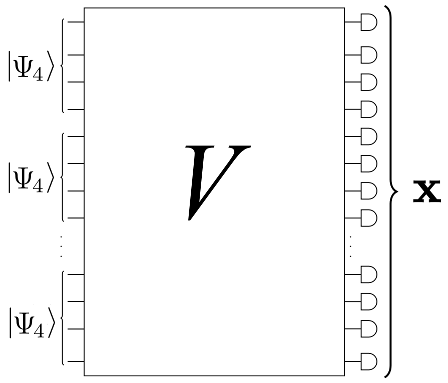
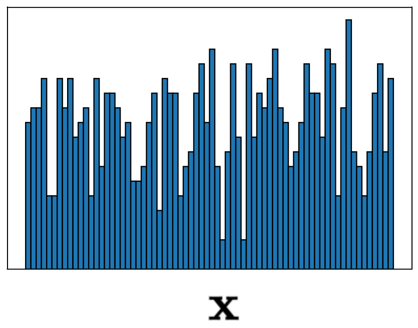
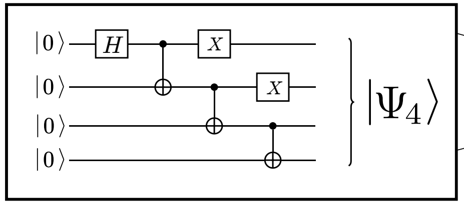


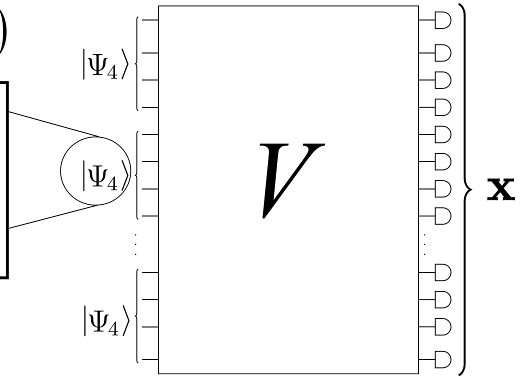


Fermion Sampling with magic input states

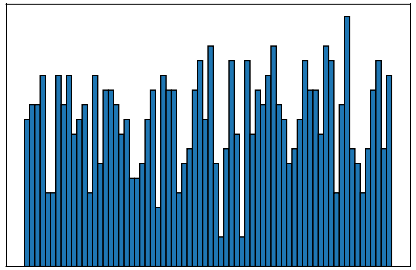
$$|\psi_4\rangle = \frac{1}{\sqrt{2}}\left(|0011\rangle + |1100\rangle\right)$$



$$|\psi_{in}\rangle = |\psi_4\rangle^{\otimes N}$$



$$p_{\mathbf{x}}\left(V, \psi_{in}\right)$$



x

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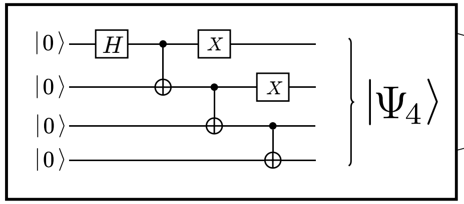

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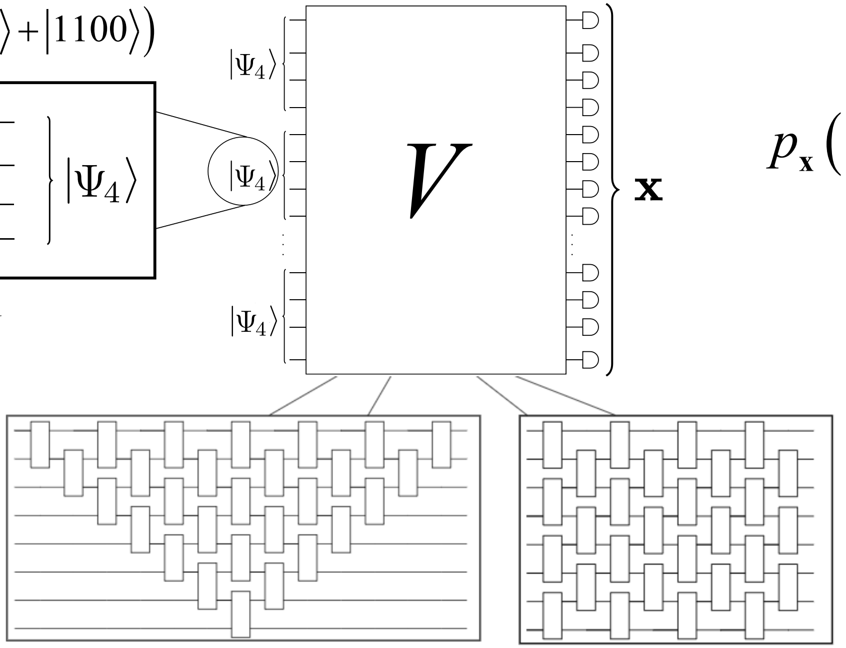


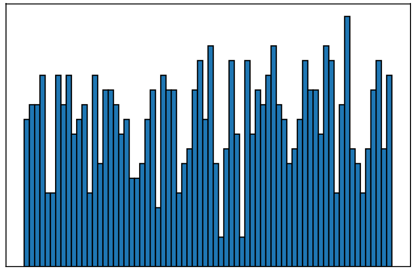
Passive FLO

Active FLO

$$f_i^\dagger \rightarrow \sum_{j=1}^{d} U_{ji} f_i^\dagger$$

$$m_i \rightarrow \sum_{j=1}^{2d} O_{ji} m_i$$

$$p_{\mathbf{x}}\left(V, \psi_{in}\right)$$



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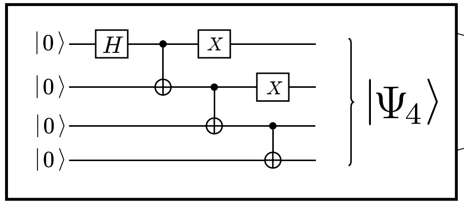

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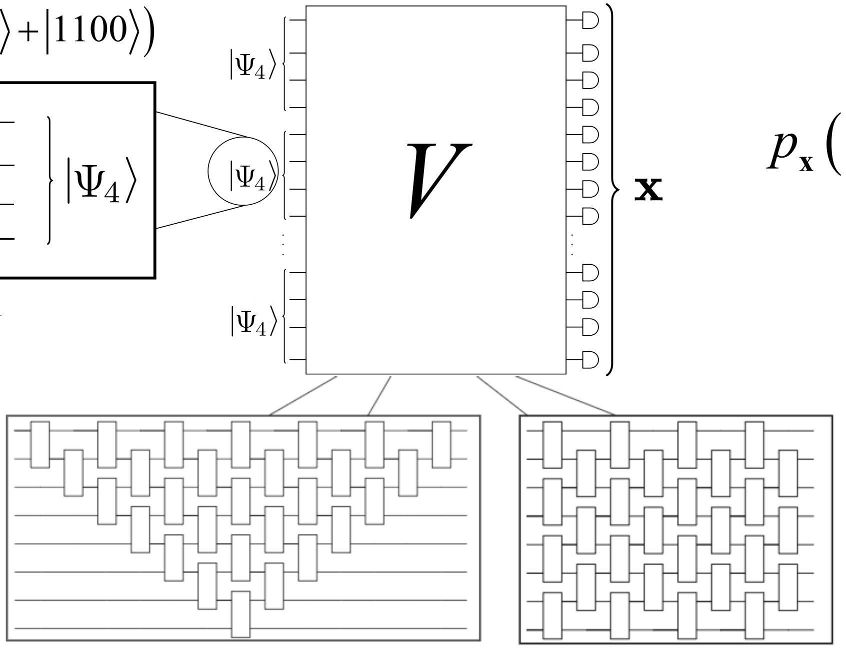

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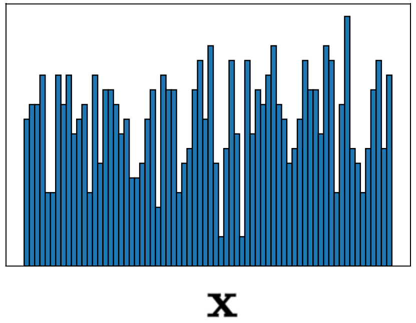

Main results:

- Anticoncentration of $p_{\mathbf{x}}\left(V, \psi_{in}\right)$
- Average-case $\#P$-hardness of $p_{\mathbf{x}}\left(V, \psi_{in}\right)$
- Efficient certification of $V \in FLO$

Fermionic linear optics

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Fermionic system of d modes: $\mathcal{H} = \oplus_{k=0}^{d} \Lambda^k \left(\mathbb{C}^d\right)$ k -number of fermions

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Representation of $U(d)$

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Active FLO:

$$m_i \to \sum_{j=1}^{2d} O_{ji} m_i \ , \ O \in SO(2d)$$

$$V = \exp\left(\frac{1}{4} \sum_{k,l=1}^{2d} [\log(O)]_{kl} m_k m_l\right)$$

(projective) representation of $SO(2d)$

Jordan-Wigner transformation

Jordan-Wigner transformation

d fermionic modes

d qubits

Jordan-Wigner transformation

$$U_{JW}$$

d fermionic modes $\longrightarrow$ d qubits

Jordan-Wigner transformation

U_{JW}

d **fermionic modes** $\longrightarrow$ d **qubits**

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Majorana operators $\begin{array}{l} m_{2i-1} \\ m_{2i} \end{array}$ $\longrightarrow$ Pauli operators $\begin{array}{l} Z_1 Z_2 \cdots Z_{i-1} X_i \\ Z_1 Z_2 \cdots Z_{i-1} Y_i \end{array}$

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Particle-numer measurements → Computational basis measurements

Jordan-Wigner transformation

$$U_{JW}$$

d **fermionic modes**	$\longrightarrow$	d **qubits**
$\lvert \mathbf{n}_F \rangle = \prod_{i=1}^{d} (f_i^\dagger)^{n_i} \lvert 0_F \rangle$	$\longrightarrow$	$\lvert \mathbf{n} \rangle = \otimes_{i=1}^{d} \lvert n_i \rangle$
Majorana operators m_{2i-1}, m_{2i}	$\longrightarrow$	Pauli operators $Z_1 Z_2 \cdots Z_{i-1} X_i$, $Z_1 Z_2 \cdots Z_{i-1} Y_i$
Particle-numer measurements	$\longrightarrow$	Computational basis measurements
Local particle-preserving quadratic hamiltonians	$\longrightarrow$	Hamiltonians generated by $Z_i\,,\, Z_{i+1}, X_i Y_{i+1} - Y_i X_{i+1},\, X_i X_{i+1} + Y_i Y_{i+1}$

Implementation in supercoducting qubits

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Elements of $U(d)$ and $SO(2d)$ can be decomposed into **mode-local transformations** on a line

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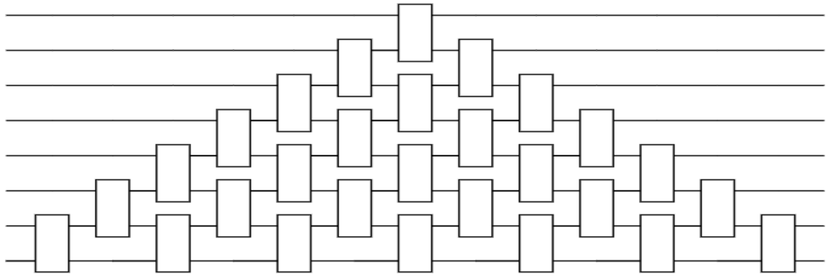
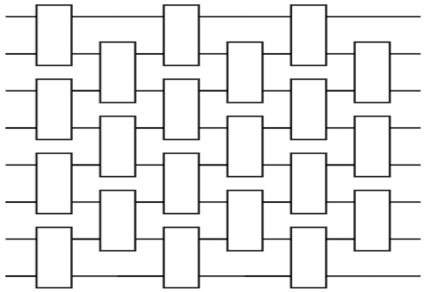


[Reck–Zelinger 1994]



[Clements *et al.* 2016]

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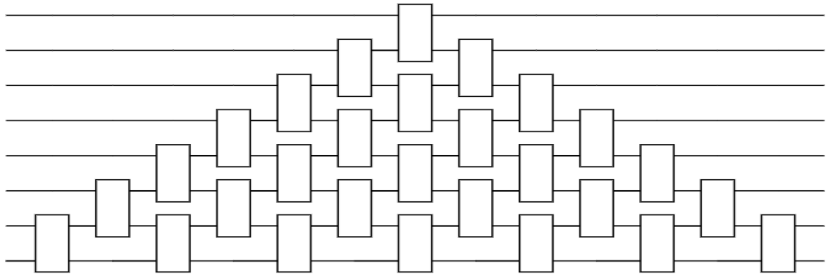

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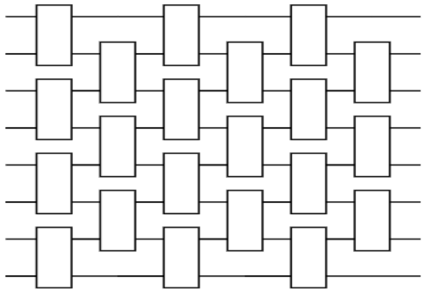

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Arbitrary FLO circuit can be realized by circuit of depth $\sim d$ in **1D architecture**

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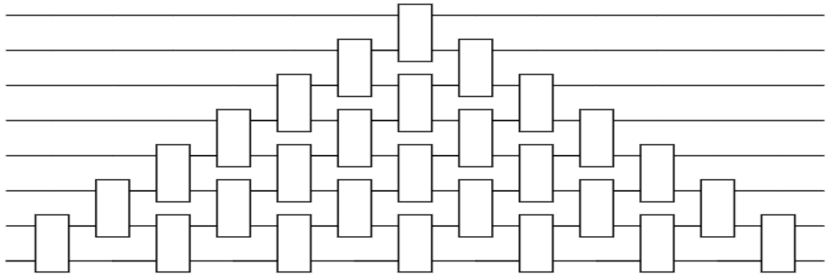

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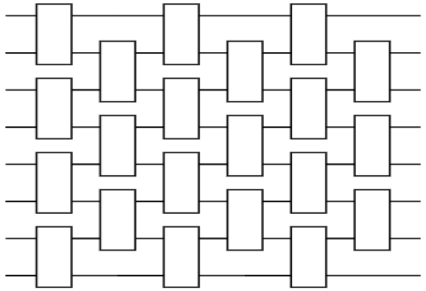

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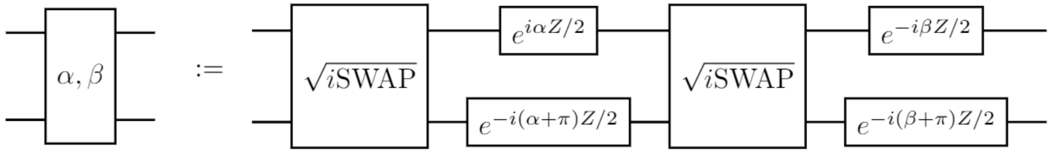
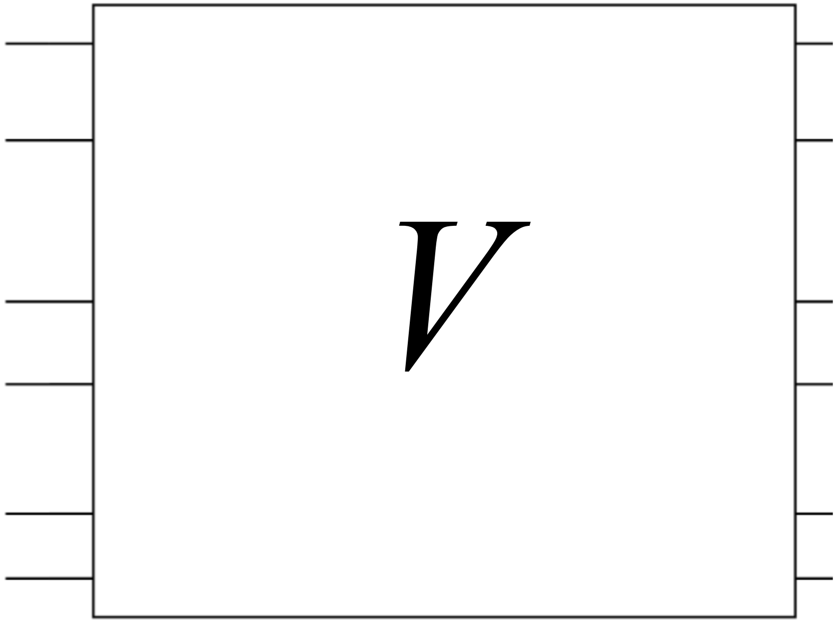
Arbitrary FLO circuit can be realized by circuit of depth $\sim d$ in **1D architecture**

Necessary gates: **native to** superconducting architectures [Arute *et al.* 2020]



$$\sqrt{i\mathrm{SWAP}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{-i}{\sqrt{2}} & 0 \\ 0 & \frac{-i}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Efficient tomography of FLO circuits

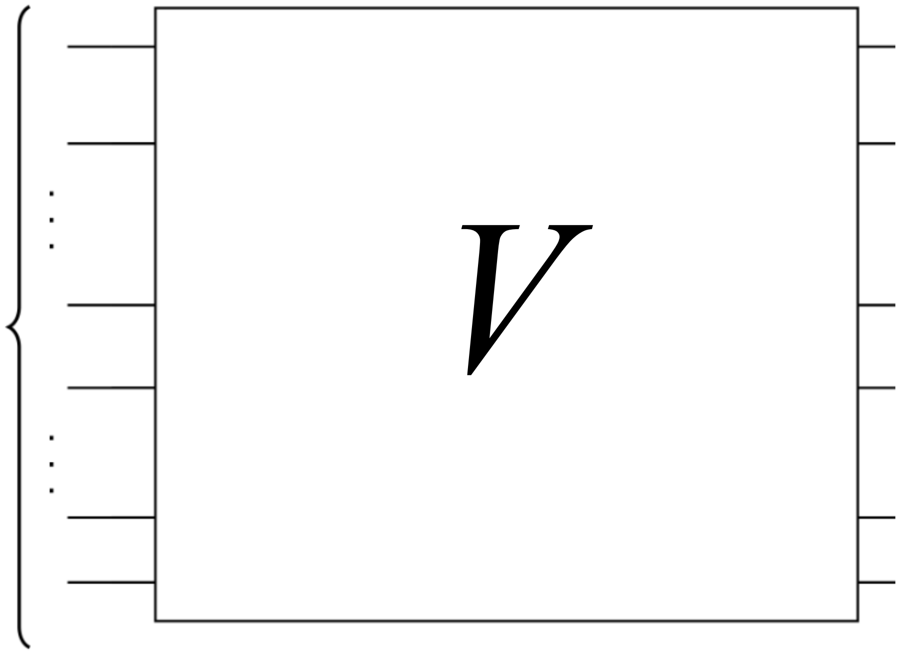


Efficient tomography of FLO circuits

Prepare

$$|+_X^p\rangle = |0\rangle^{\otimes(p-1)} \otimes |+_X\rangle \otimes |0\rangle^{\otimes(d-p)}$$
$$|+_Y^p\rangle = |0\rangle^{\otimes(p-1)} \otimes |+_Y\rangle \otimes |0\rangle^{\otimes(d-p)}$$

$$p = 1, 2, \cdots, d$$



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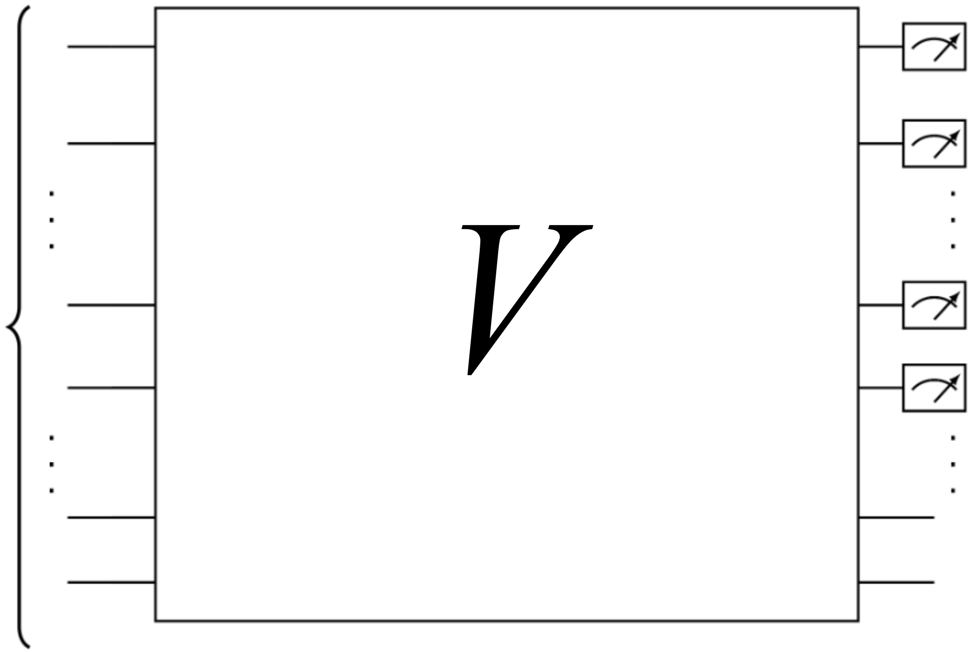

Measure

Z_1	Z_1
Z_2	Z_2
$\vdots$	$\vdots$
Z_{q-1}	Z_{q-1}
X_q	Y_q
m_{2q-1}	m_{2q}

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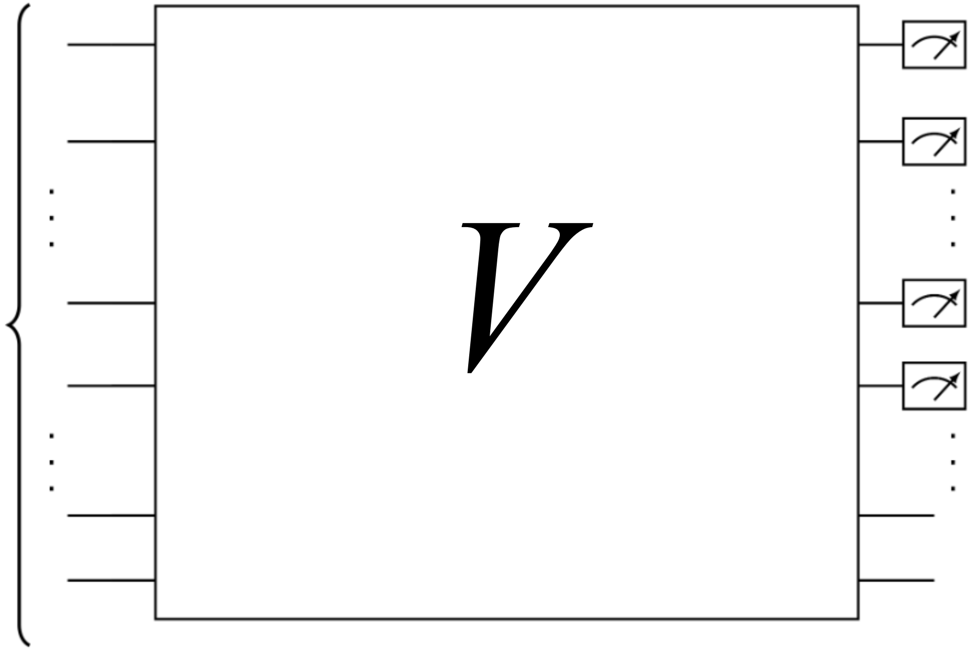

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Collect satatistics

Efficient tomography of FLO circuits

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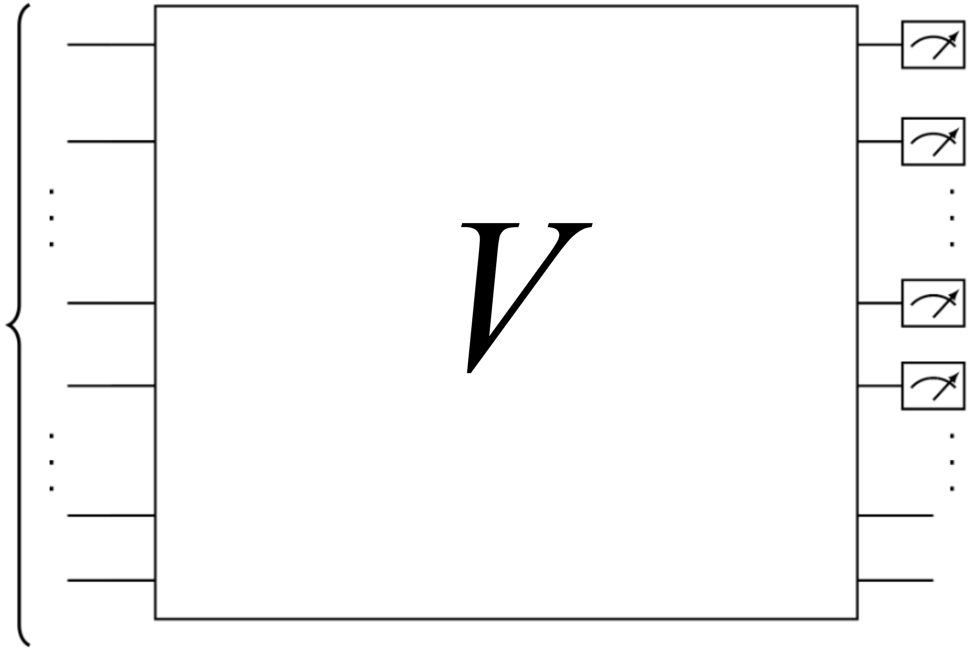


Measure

Z_1	Z_1
Z_2	Z_2
$\vdots$	$\vdots$
Z_{q-1}	Z_{q-1}
X_q	Y_q

m_{2q-1} m_{2q}

$q = 1, 2, \cdots, d$

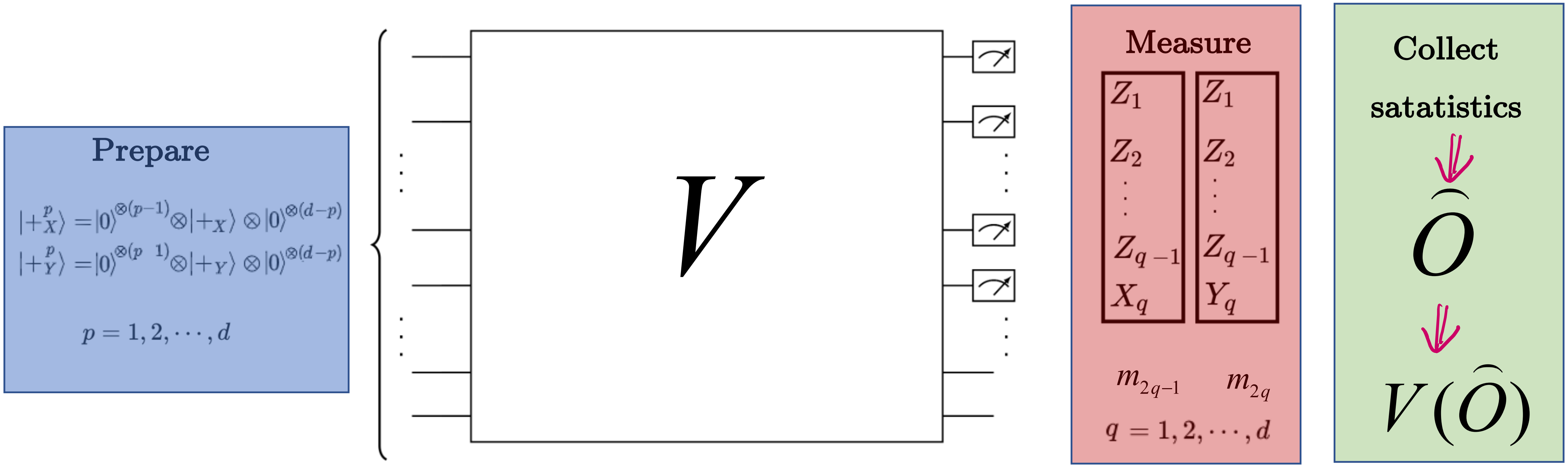
Collect

satatistics

$\Downarrow$

$\widehat{O}$

Efficient tomography of FLO circuits



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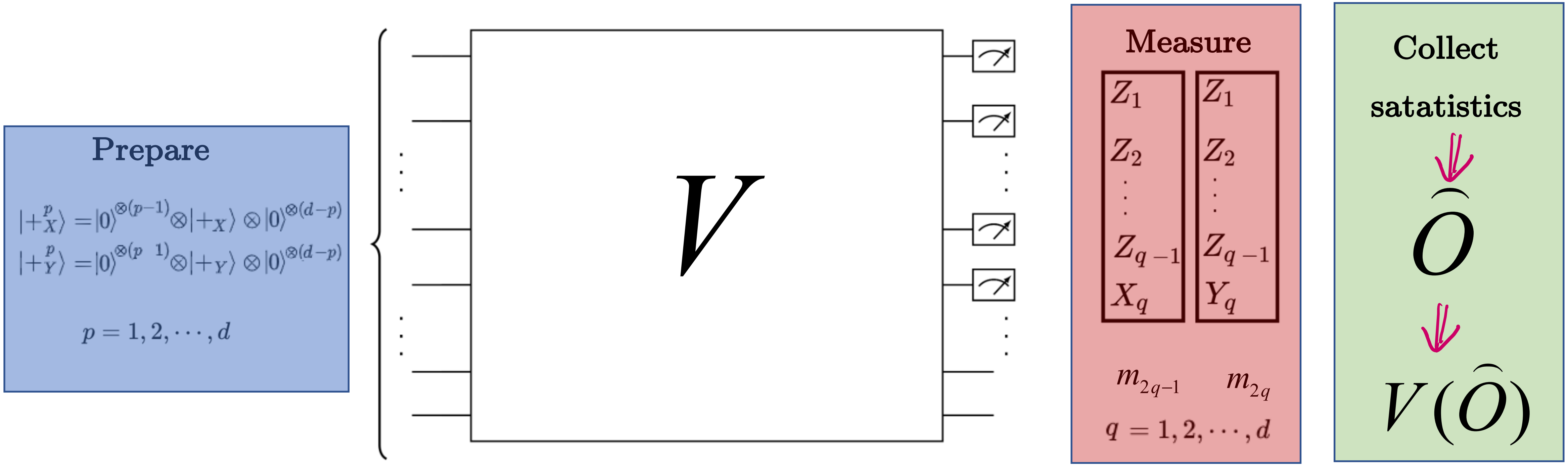
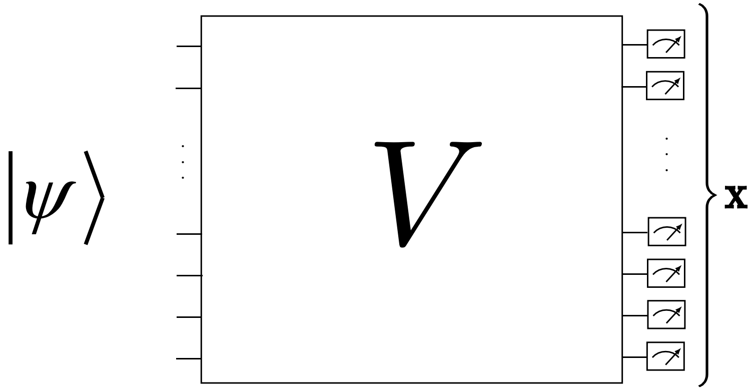
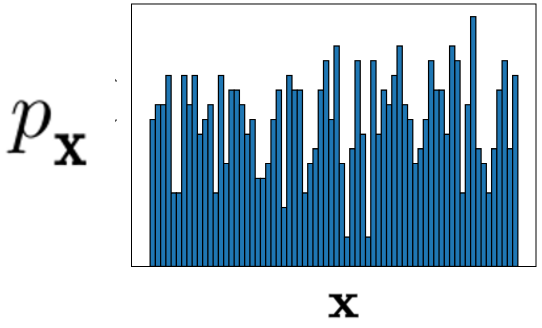


Result: If $V \in FLO$, then the above scheme gives an estimate $V(\hat{O})$ such that $\|V - V(\hat{O})\|_\diamond \leq \varepsilon$ using $O\left(\frac{d^3}{\varepsilon^2}\right)$ measurement rounds.

Hardness of Fermion Sampling





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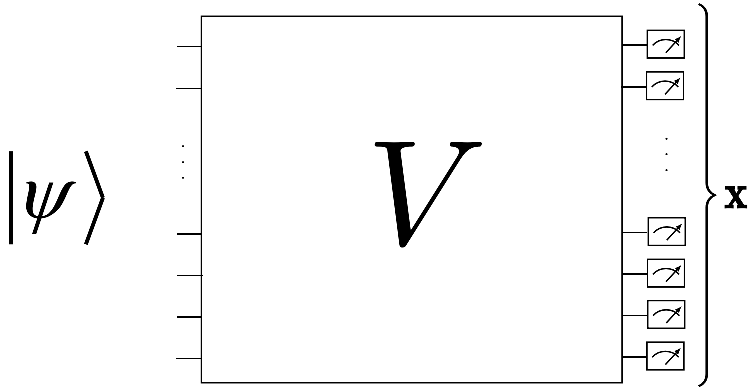

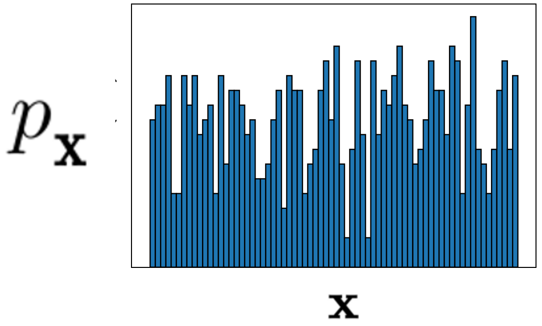

If $|\psi\rangle$ is free (fermionic Gaussian or Slater determinant), then **sampling is classically easy** [Valiant 2000] [Terhal-DiVincenzo 2001] [Jozsa-Miyake 2008]

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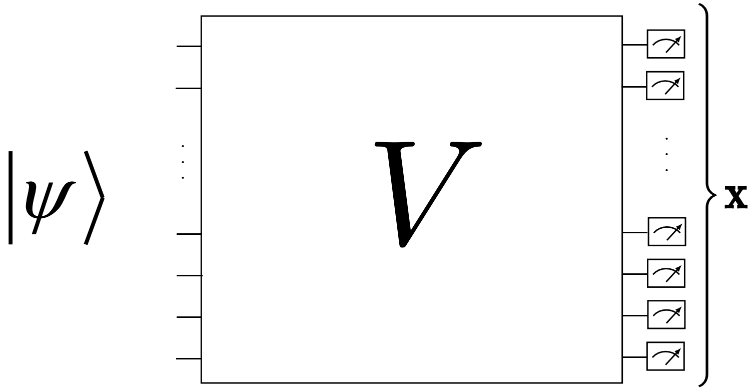

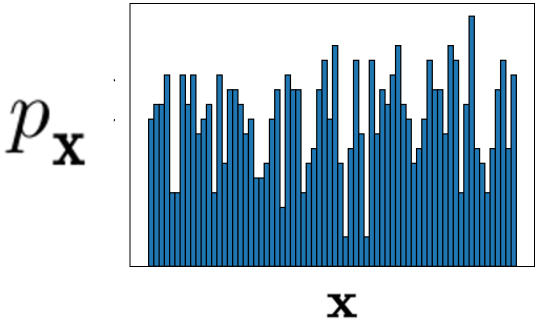

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Striking difference between Fermion Sampling and Boson Sampling!

$$p_{\mathbf{x}}^{bos} \propto |Per(U_{\mathbf{x}})|^2 \qquad p_{\mathbf{x}}^{fer} \propto |Det(U_{\mathbf{x}})|^2$$

$$Per(A) = \sum_{\sigma \in S_n} \prod_{i=1}^{n} A_{\sigma(i),i} \qquad Det(A) = \sum_{\sigma \in S_n} \mathrm{sgn}(\sigma) \prod_{i=1}^{n} A_{\sigma(i),i}$$

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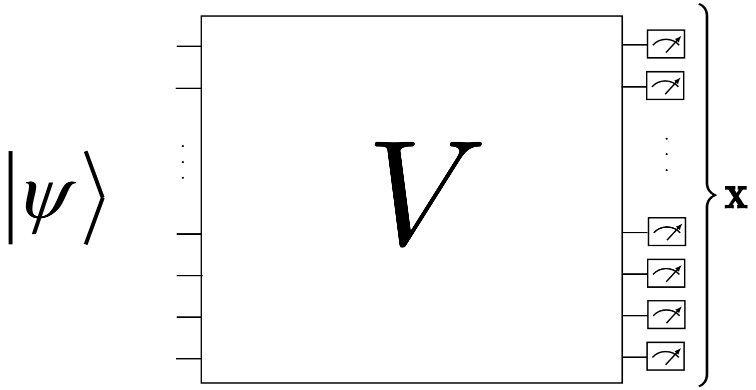

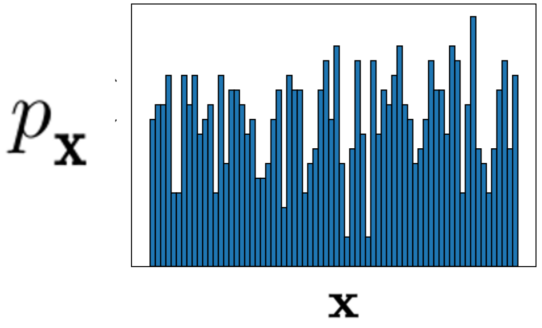

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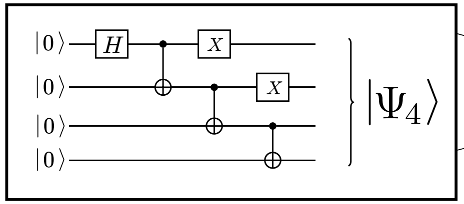
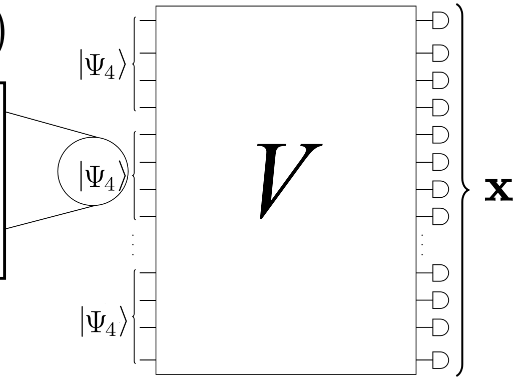
Avi Wigderson

Determinant vs Permanent dichotomy in complexity theory (#P -hardness of Per !)

Hardness of Fermion Sampling (II)

$$|\psi_4\rangle = \frac{1}{\sqrt{2}}\left(|0011\rangle + |1100\rangle\right)$$





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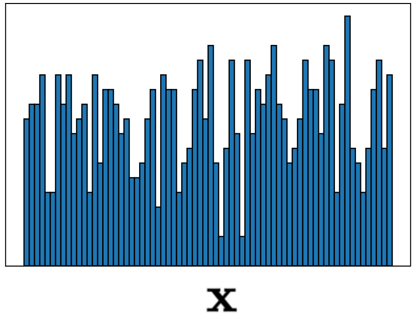
$$p_{\mathbf{x}}\left(V, \psi_{in}\right)$$



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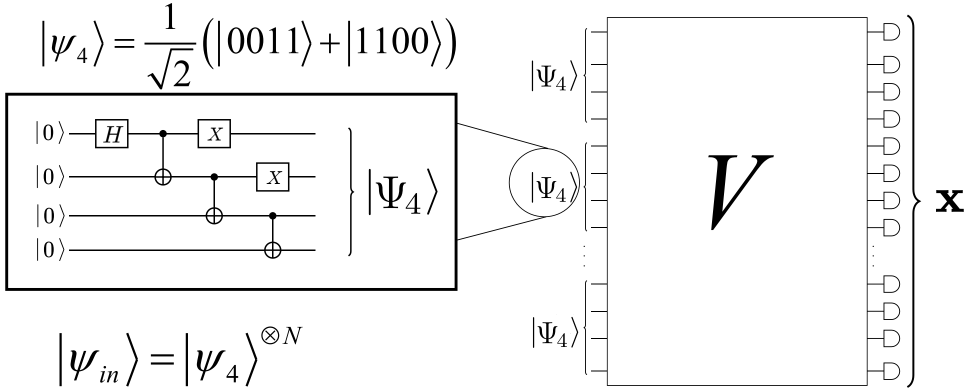

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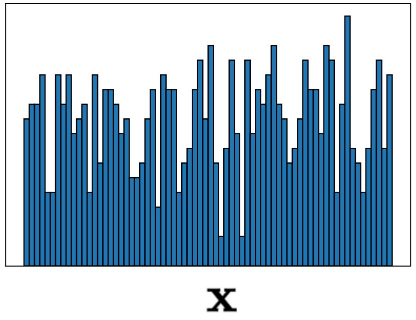

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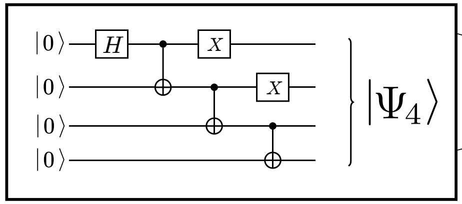

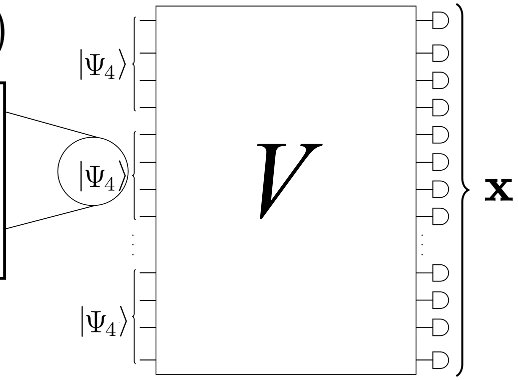

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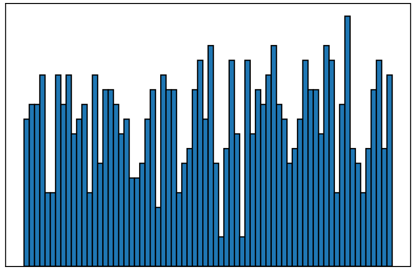

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Dimitri Ivanov

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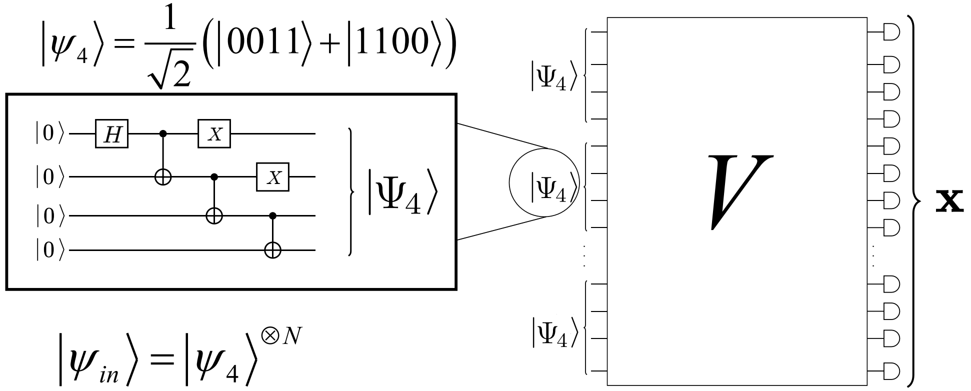

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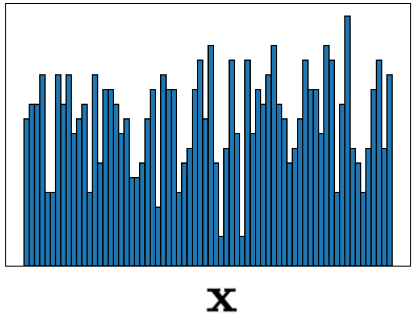

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Dimitri Ivanov

Mixed discriminants are #P-hard to compute.

Alternativelly, $|\psi_4\rangle$ promote atcive FLO to universality [Bravyi 2006] [Hebenstreit *et al.* 2019]

Hardness of Fermion Sampling (III)

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Anticoncentration

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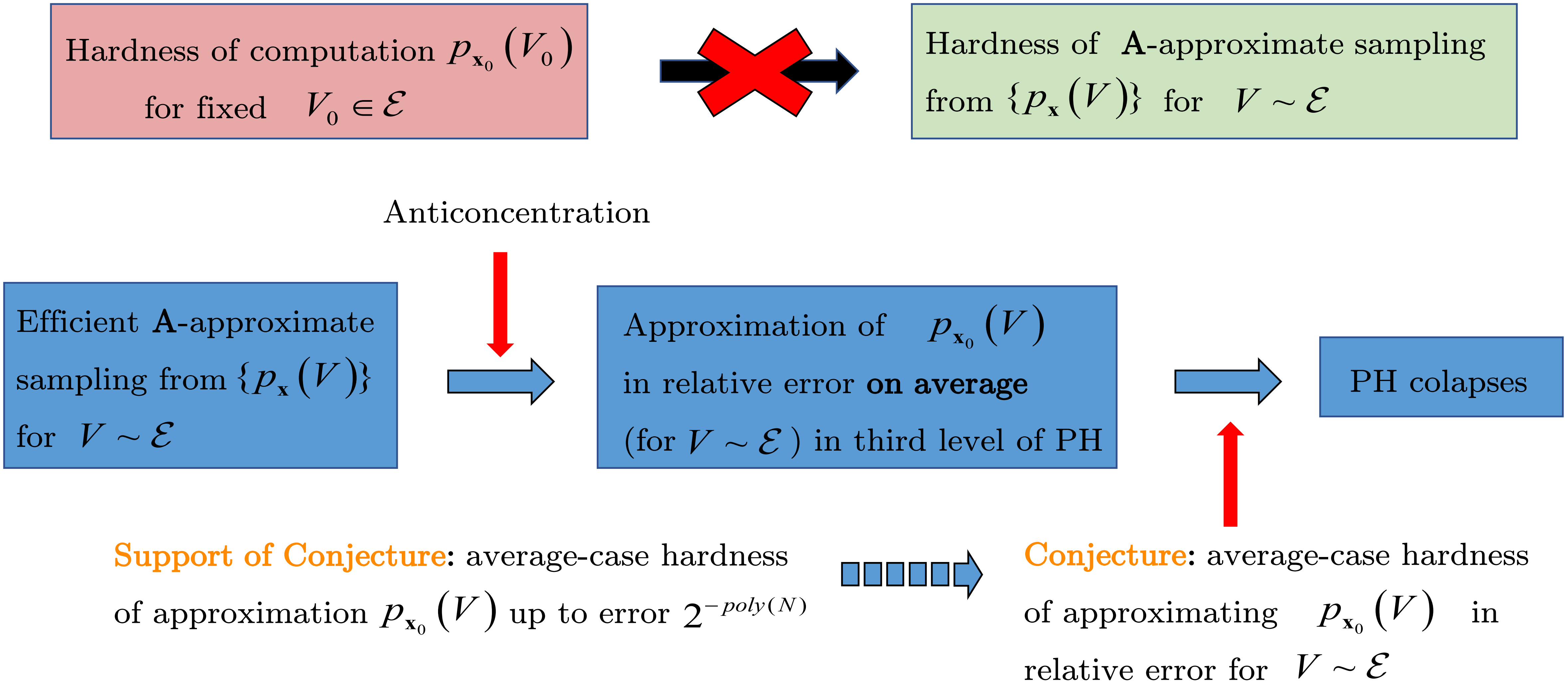

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Holds for **Random Quantum Circuits** via approximate 2-design property

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Holds for **RQC** [Movassagh 2019], [Bouland *et al.* 2018]

Hardness of Fermion Sampling (IV)

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Anticoncentration for FLO circuits

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Result: Anticoncentration for FLO circuits

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Numerics suggests that for Gaussian μ probabilities $p_{\mathrm{x}}(V,\psi)$ **do not anticoncentrate.**

Anticoncentration for Fermion Sampling (II)

Proof sketch:

- Payley-Zygmund $\quad \Pr\left(X > \alpha \mathbb{E} X \right) \geq (1-\alpha)^2 \frac{(\mathbb{E} X)^2}{\mathbb{E} X^2} \quad , \quad \alpha \in [0,1]$

- We set $X_V = p_{x_0}(V, \Psi_{in}) = \mathrm{tr}\left(|x_0\rangle\langle x_0| \Pi(V) \Psi_{in} \Pi(V^{-1}) \right)$, $\Pi : G \longrightarrow U(\mathcal{H})$ suitable irrep of G

- $\mathbb{E}_{V\sim\mu} X_V = \frac{1}{|\mathcal{H}|}$

- $\mathbb{E}_{V\sim\mu} X_V^2 = \int_G d\mu(V) \, \mathrm{tr}\left(\Pi(V)^{\otimes 2} |x_0\rangle\langle x_0|^{\otimes 2} \Pi(V^{-1})^{\otimes 2} \Psi_{in}^{\otimes 2} \right) = \frac{1}{|\tilde{\mathcal{H}}|} \mathrm{tr}\left(\mathbb{P}_{\tilde{\mathcal{H}}} \Psi_{in}^{\otimes 2} \right)$

- Inserting to P-Z: $\Pr_{V\sim\mu}\left(p_{x_0}(V, \Psi_{in}) \geq \alpha \frac{1}{|\mathcal{H}|} \right) \geq (1-\alpha)^2 \frac{|\tilde{\mathcal{H}}|^2}{|\mathcal{H}|^2} \frac{1}{\mathrm{tr}(\mathbb{P}_{\mathcal{H}} \Psi_{in}^{\otimes 2})} = O\left(\frac{1}{N^\alpha}\right)$

  $\sim \frac{1}{N^\alpha}$ CONSTANT

Average-case hardness

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- **Goal:** construct a low-degree rational interpolation between a #P-hard FLO circuit and generic circuits
- Use polynomial interpolation technique to recover the value of the worst-case probability from those of generic circuits
- To achieve the goal, we use the **Cayley-path** deformation [Movassagh 2019]

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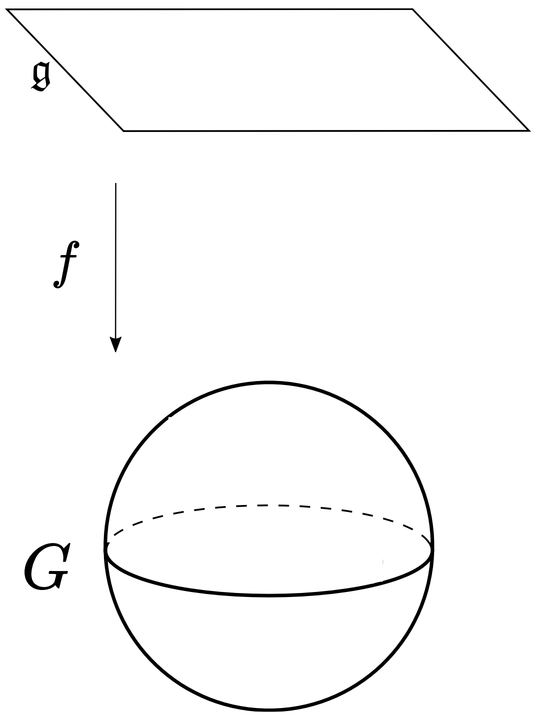

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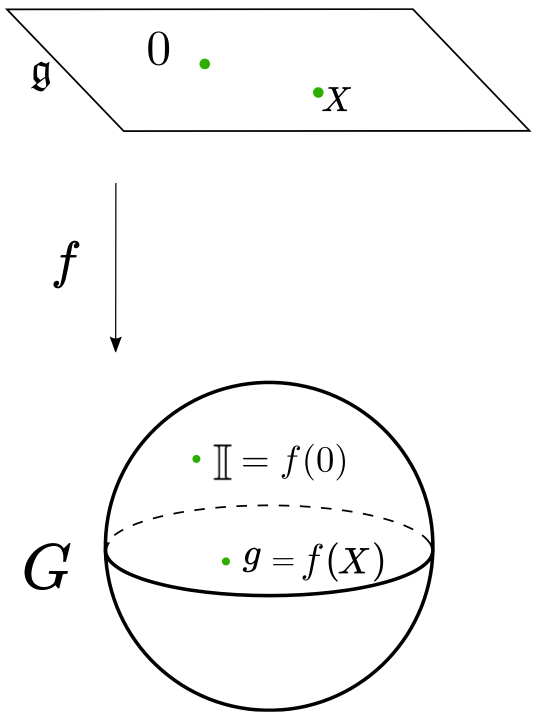

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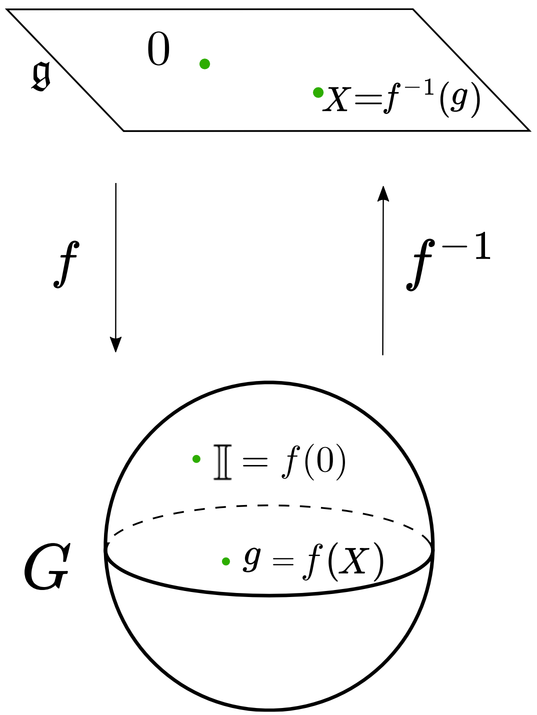

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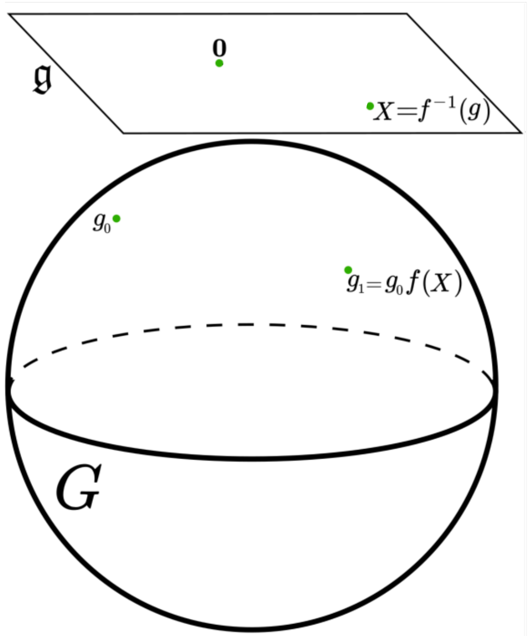

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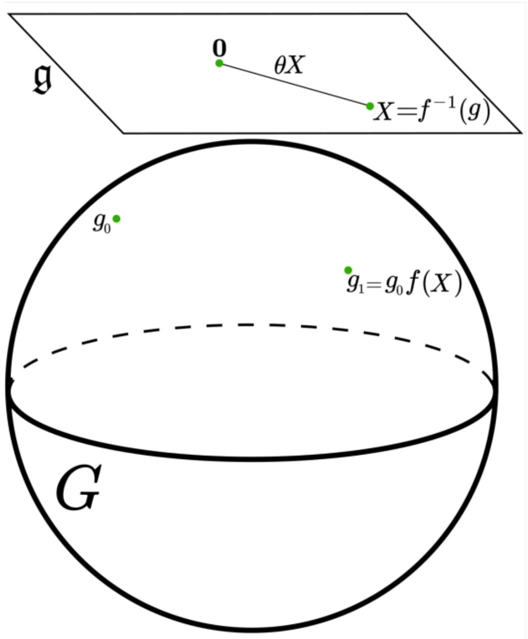

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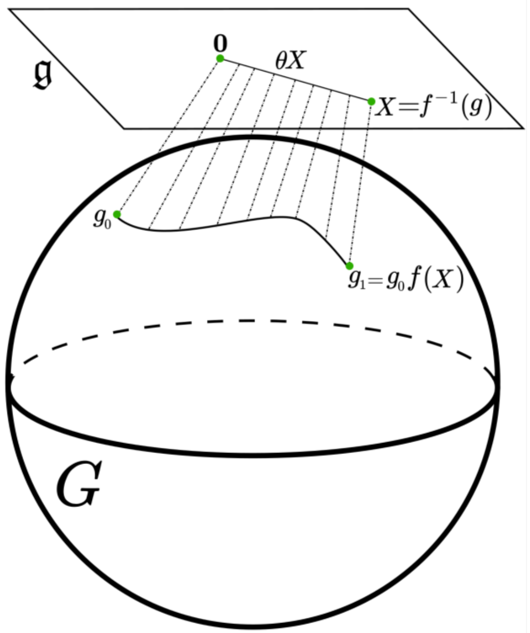

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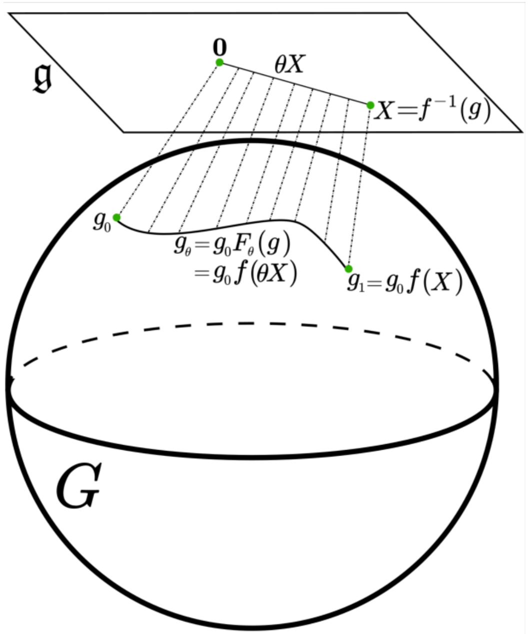
- Use polynomial interpolation technique to recover the value of the worst-case probability from those of generic circuits
- To achieve the goal, we use the **Cayley-path** deformation [Movassagh 2019]



Cayley map:

$$f(X) = (\mathbb{I} - X)(\mathbb{I} + X)^{-1}$$

Inverse Cayley map:

$$f^{-1}(g) = (\mathbb{I} - g)(\mathbb{I} + g)^{-1}$$

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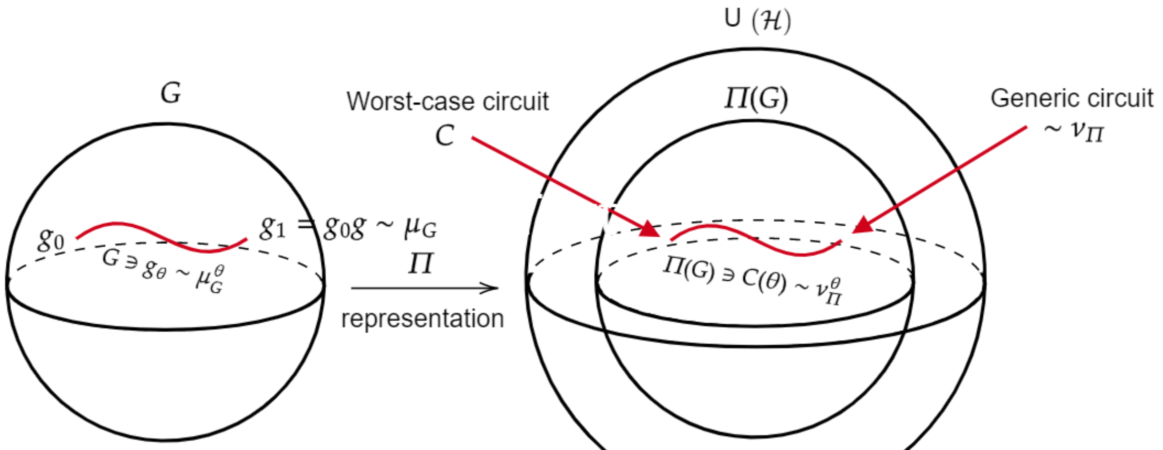

$$g_\theta = g_0 \frac{(1-\theta)I + (1+\theta)g}{(1+\theta)I + (1-\theta)g}$$

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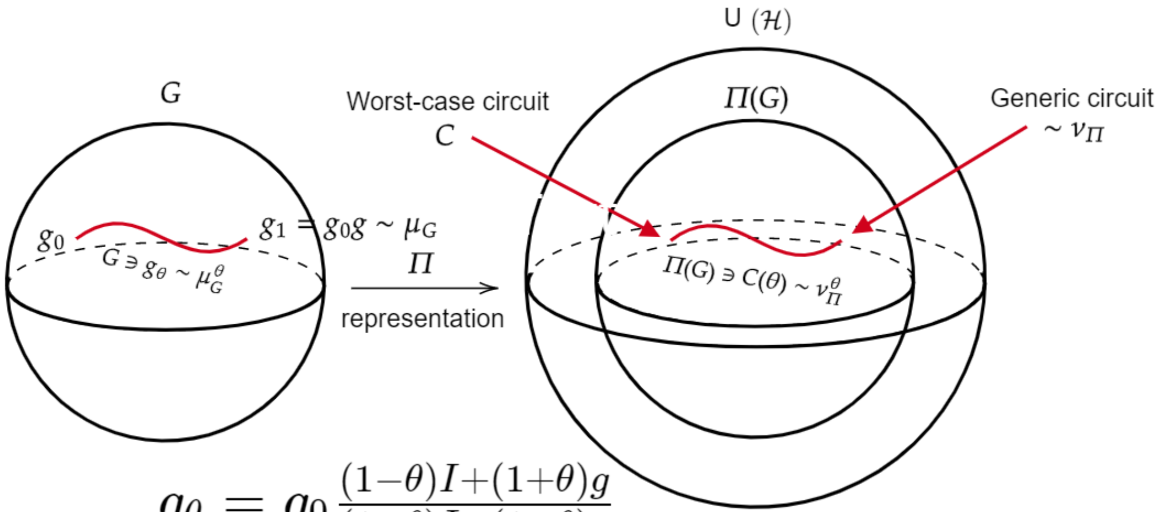

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Difference to previous work: instead of deforming individual gates, we deform at the level of the symmetry group, which is represented as a global circuit.

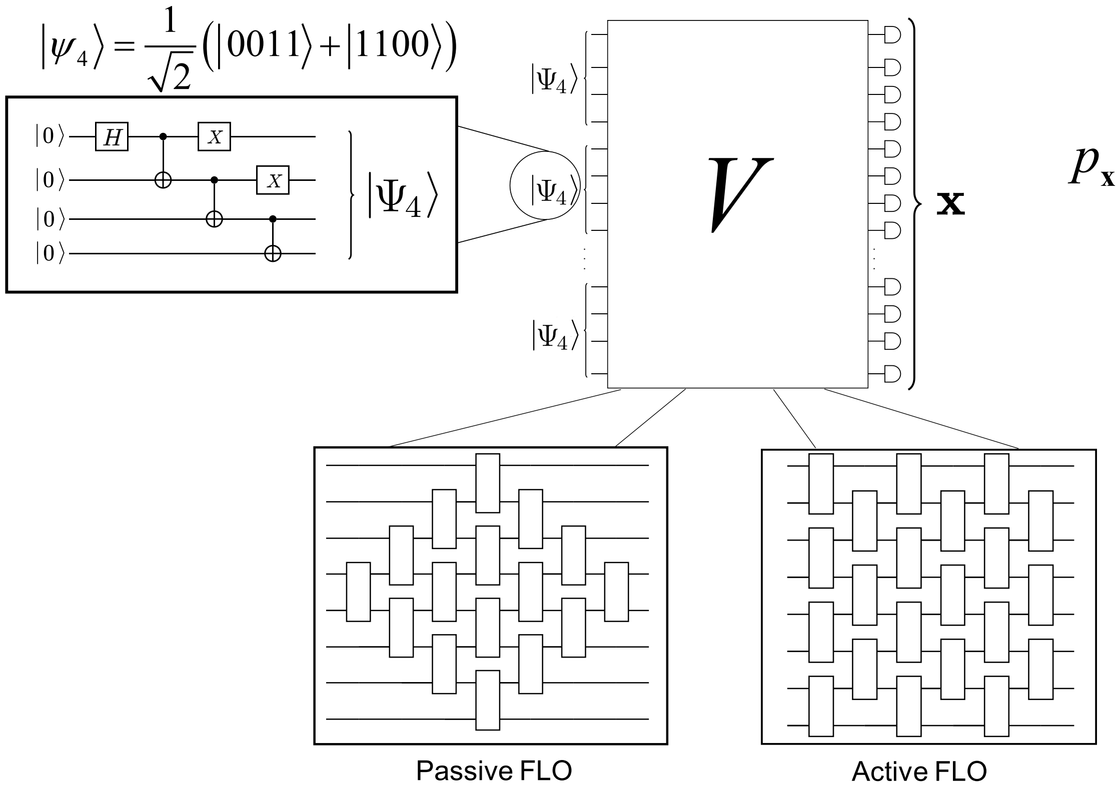
Average-case hardness (II)

Result: It is #P-hard to compute values of $p_{\mathbf{x}_0}(V, \Psi_{in})$ with probability greater than $\frac{3}{4} + \frac{1}{\mathrm{poly} N}$ over the choice of $V \sim \mu$

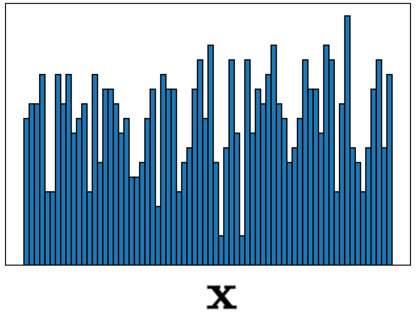
Result: It is #P-hard to approximate probability $p_{\mathbf{x}_0}(V, \Psi_{in})$ to within accuracy $\epsilon = \exp(-\Theta(N^6))$ with probability greater than $1 - o(N^{-2})$ over the choice of $V \sim \mu$

- Movassagh's result: $\epsilon = \exp(-\Theta(N^{4.5}))$ for the Google's layout
- Supremacy conjecture: constant relative error with constant probability over the choice of $V \sim \mu$

Conclusions



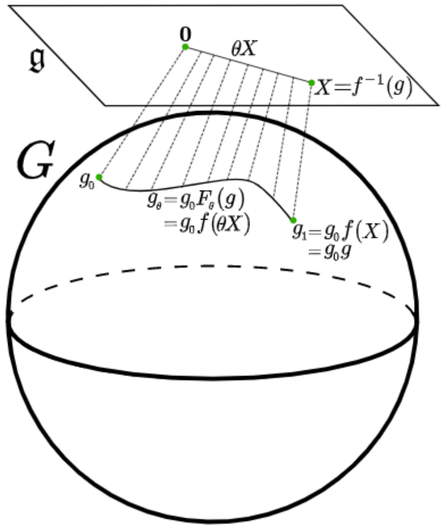
$p_{\mathbf{x}}(V, \psi_{in})$



Fermion Sampling with magic input states

- Experimentaly feasible
- Strong hardness guarantees
  - Anticoncentration of $p_{\mathbf{x}}(V, \psi_{in})$
  - Average case hardness of $p_{\mathbf{x}}(V, \psi_{in})$
- FLO unitaries can be efficiently certified

Outlook and open problems



$$f(X) = (\mathbb{I} - X)(\mathbb{I} + X)^{-1}$$

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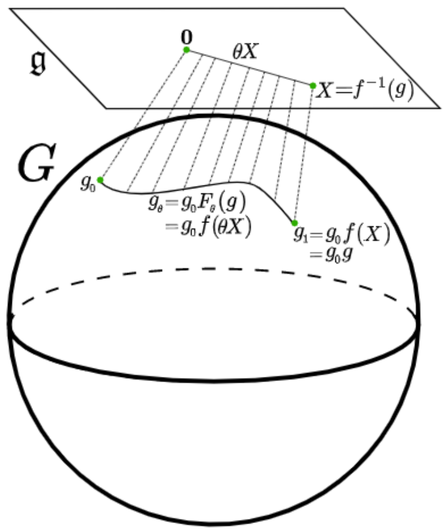
- Classical simulation of Fermion Sampling/ Matchgate circuits
- Verification and certification of Fermion Sampling
- Interesting applications originating from this quantum advantage paradigm?
- Application to other scenarios (Boson Sampling, Gaussian Boson Sampling)

Outlook and open problems



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Thank you!

Discussion

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