Boson bunching is not maximized by indistinguishable particles

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In this talk

- Overview of my work
  - Context of this thesis: BosonSampling
  - Boson bunching and indistinguishability
  - Validation of BosonSamplers
  - BosonSampling.jl
- In depth topic: Boson bunching and indistinguishability


arXiv > quant-ph > arXiv:2203.01306

Quantum Physics

[Submitted on 2 Mar 2022]

Boson bunching is not maximized by indistinguishable particles

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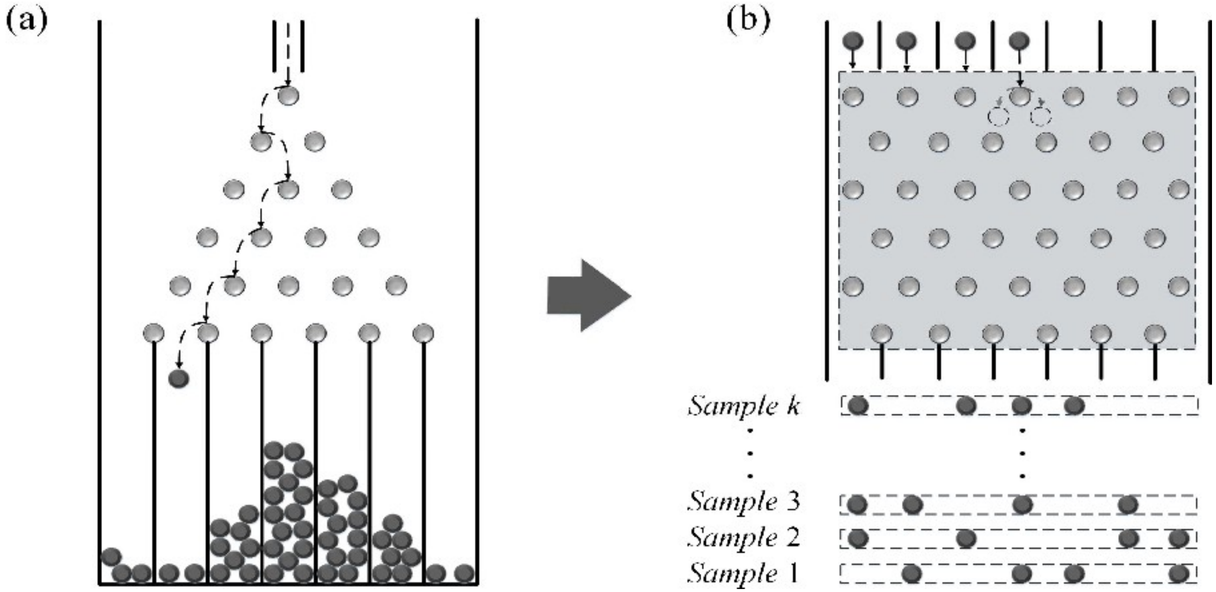
Section 0: BosonSampling

- Single photons sent in a linear interferometer
- Galton's board quantum equivalent
- Prototypical non-universal quantum computer



Probabilities

- Evolution of the quantum state

$$|\Psi\rangle_{in} = \prod_{j=1}^{m} \frac{1}{\sqrt{r_j!}} \left(\hat{a}^{\dagger}_{j,\Phi_j}\right)^{r_j} |0\rangle$$

$$\hat{a}^{\dagger}_{j,\Phi_j} \rightarrow \hat{U}\hat{a}^{\dagger}_{j,\Phi_j}\hat{U}^{\dagger} = \sum_{k=1}^{m} U_{jk}\hat{b}^{\dagger}_{k,\Phi_j}$$

- Event probabilities are given by matrix permanents

$$|\mathrm{perm}(M)|^2$$

Matrix permanents

- Same as the determinant, without the parity signs:

$$\mathrm{perm}(A) = \sum_{\sigma \in S_n} A_{i\sigma_i}$$

$$\det(A) = \sum_{\sigma \in S_n} \mathrm{sign}(\sigma) A_{i\sigma_i}$$

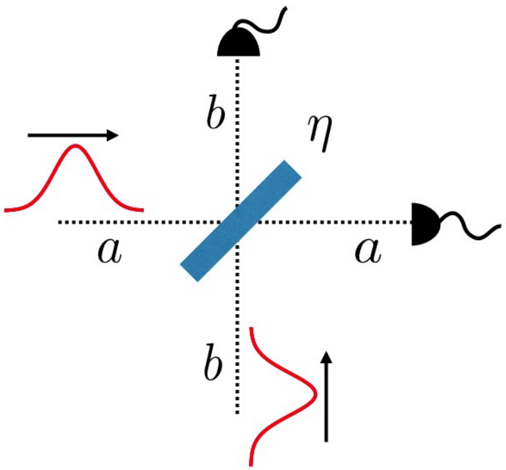
- But lacks many of the property symmetries, hence hard/few results
- For instance: computational complexity $\mathcal{O}(n^{2.373}),\ \mathcal{O}(n2^n)$

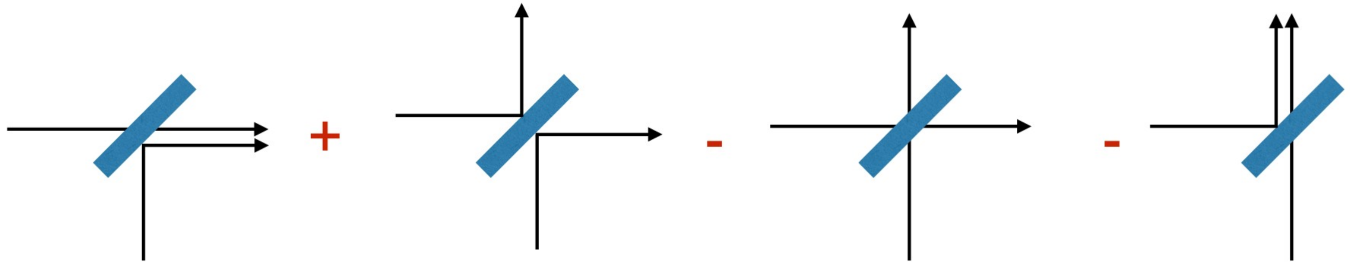
Section 1: Boson bunching, indistinguishability

- Reminder: Hong-Ou-Mandel effect, (in)distinguishability
- Conjecture: indistinguishable bosons bunch most
- Mathematical counterpart: permanents and Bapat-Sunder
- Counter-example: Drury
- Quantum optics interpretation
- Perspectives

Hong-Ou-Mandel

- Two particles sent in a balanced beam-splitter
  - Each has a ½ probability of going up or down
  - Particles don't speak to each other
  - Simplest fully quantum optics experiment
  - *Doesn't work if polarization mismatch*

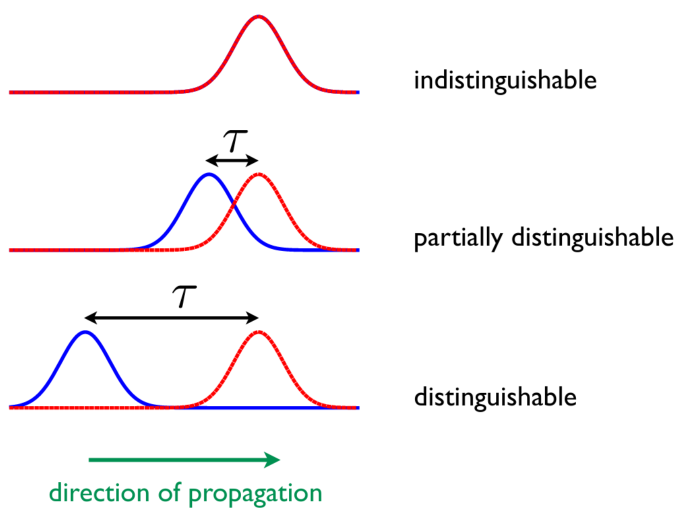


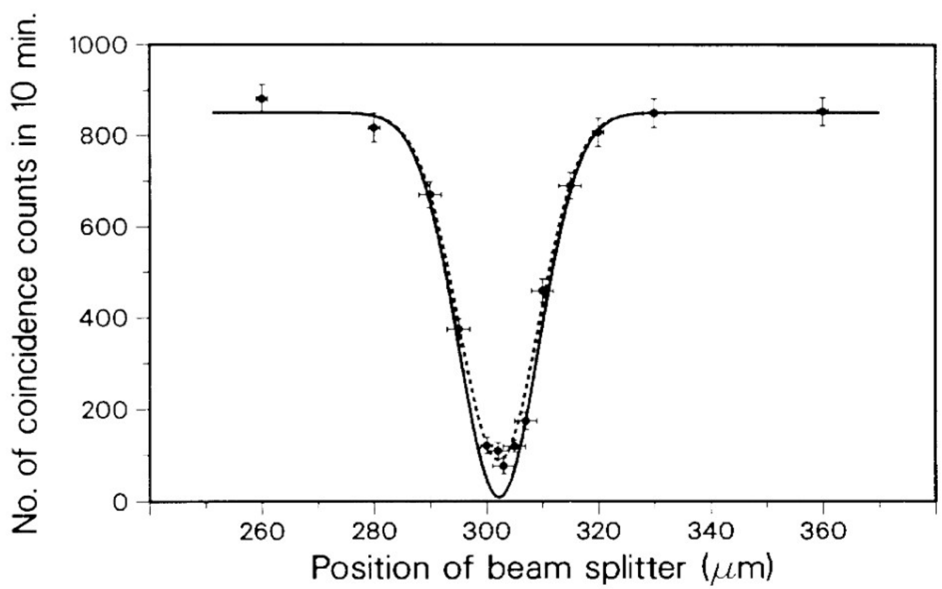


Left detector | Right detector

HOM: (in)distinguishability

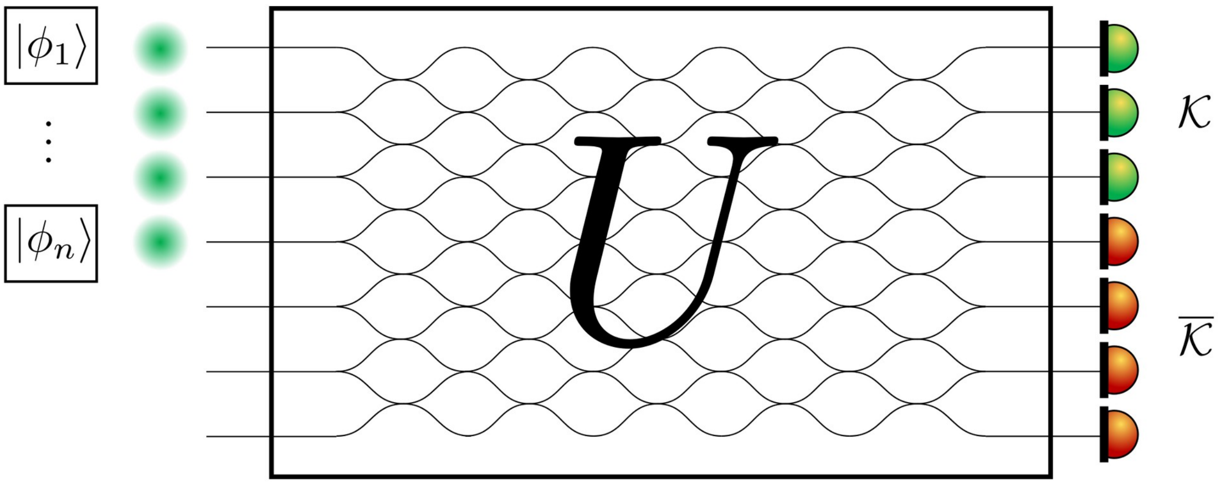
- Degrees of freedom:
  - Mode occupation j $|j\rangle$
  - Internal wave function (spin, arrival time,…) $|\phi_j\rangle$
- Strength of HOM effect decreases monotically





Generalized bunching

- A natural (*much stronger*) extension to the HOM effect:
  - Shchesnovich (2015): *Consider **any** input state of classically correlated photons. For **any** linear interferometer U and **any** nontrivial subset K of output modes, the probability that all photons are found in K is maximal if the photons are (perfectly) indistinguishable.*



Mathematical formulation

- Partial distinguishability: the Gram matrix $\mathcal{S}_{ij} = \langle \phi_i | \phi_j \rangle$

$$\mathcal{S}_{ij}^{\text{indist}} = 1 \qquad \mathcal{S}_{ij}^{\text{dist}} = \delta_{ij}$$

- Bunching probability:

$$H_{a,b} = \sum_{l \in \mathcal{K}} U_{l,a}^{*} U_{l,b}, \ \text{hpsd}$$

$$P_n(S) = \text{perm}(H \odot S^T)$$

$$P_n(\text{bos}) = \text{perm}(H)$$

Statement of the conjecture

- Physical conjecture

$$\mathrm{perm}(H \odot S^T) \overset{?}{\leq} \mathrm{perm}(H)$$

- Already known to mathematicians: Bapat-Sunder (1985)

$$\mathrm{perm}(A \odot B) \leq \mathrm{perm}(A) \prod_i b_{ii}, \ \ A, B \geq 0$$

Extra motivations

- Single mode bunching: the conjecture holds

$$P_n(S) = \prod_{i=j}^{n} |U_{1j}|^2 \operatorname{perm}(S) = P_n^{\mathrm{dist}} \operatorname{perm}(S)$$

- Fermion anti-bunching: bunching *minimized,* holds:

$$P_n^{\mathrm{ferm}}(S) = \det(H \odot S^T) \geq \det H$$

  from Oppenheim's inequality (A,B psd)

$$\det(A \odot B) \geq \det A \, \det B.$$

- Numerical evidence: random circuit (unitaries) trials do not find counter examples

A counter-example

- Drury (2017)

$$A = \begin{pmatrix}
1 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
0 & 1 & \frac{1}{\sqrt{2}}e^{\frac{4}{5}i\pi} & \frac{1}{\sqrt{2}}e^{\frac{2}{5}i\pi} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}e^{-\frac{2}{5}i\pi} & \frac{1}{\sqrt{2}}e^{-\frac{4}{5}i\pi} \\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}e^{-\frac{4}{5}i\pi} & 1 & \cos\left(\frac{1}{5}\pi\right)e^{-\frac{1}{5}i\pi} & \cos\left(\frac{2}{5}\pi\right)e^{-\frac{2}{5}i\pi} & \cos\left(\frac{2}{5}\pi\right)e^{\frac{2}{5}i\pi} & \cos\left(\frac{1}{5}\pi\right)e^{\frac{1}{5}i\pi} \\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}e^{-\frac{2}{5}i\pi} & \cos\left(\frac{1}{5}\pi\right)e^{\frac{1}{5}i\pi} & 1 & \cos\left(\frac{1}{5}\pi\right)e^{-\frac{1}{5}i\pi} & \cos\left(\frac{2}{5}\pi\right)e^{-\frac{2}{5}i\pi} & \cos\left(\frac{2}{5}\pi\right)e^{\frac{2}{5}i\pi} \\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \cos\left(\frac{2}{5}\pi\right)e^{\frac{2}{5}i\pi} & \cos\left(\frac{1}{5}\pi\right)e^{\frac{1}{5}i\pi} & 1 & \cos\left(\frac{1}{5}\pi\right)e^{-\frac{1}{5}i\pi} & \cos\left(\frac{2}{5}\pi\right)e^{-\frac{2}{5}i\pi} \\
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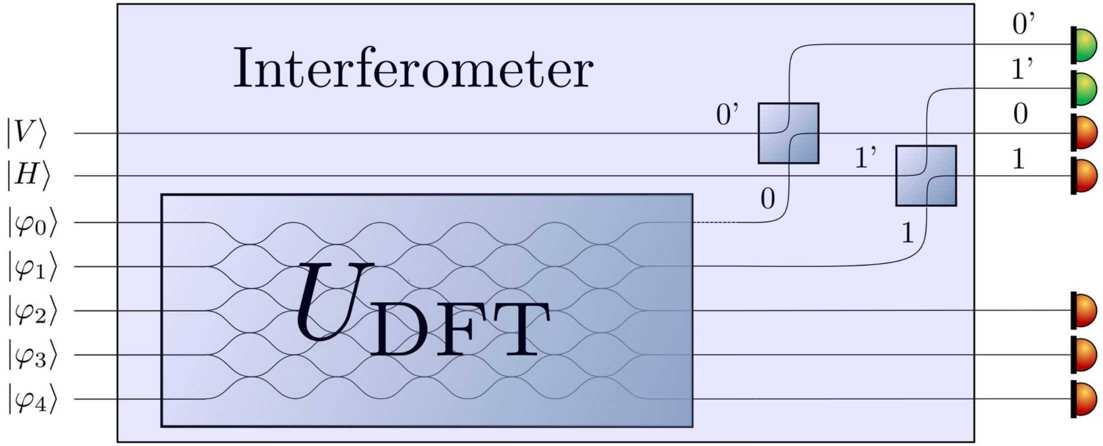
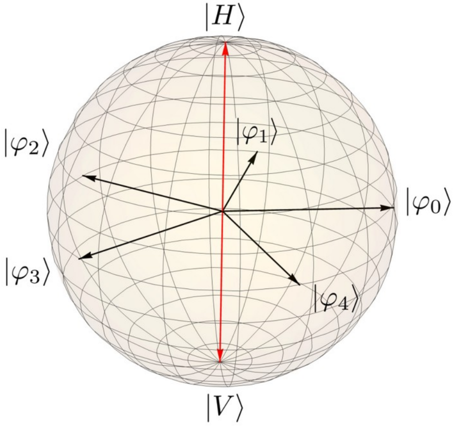
\end{pmatrix}$$

$$\frac{\mathrm{per}(A \circ A^T)}{\mathrm{per}(A)} = \frac{1237}{1152} > 1$$

Physical realization





- Counter-intuitive improved bunching with partially distinguishable photons:

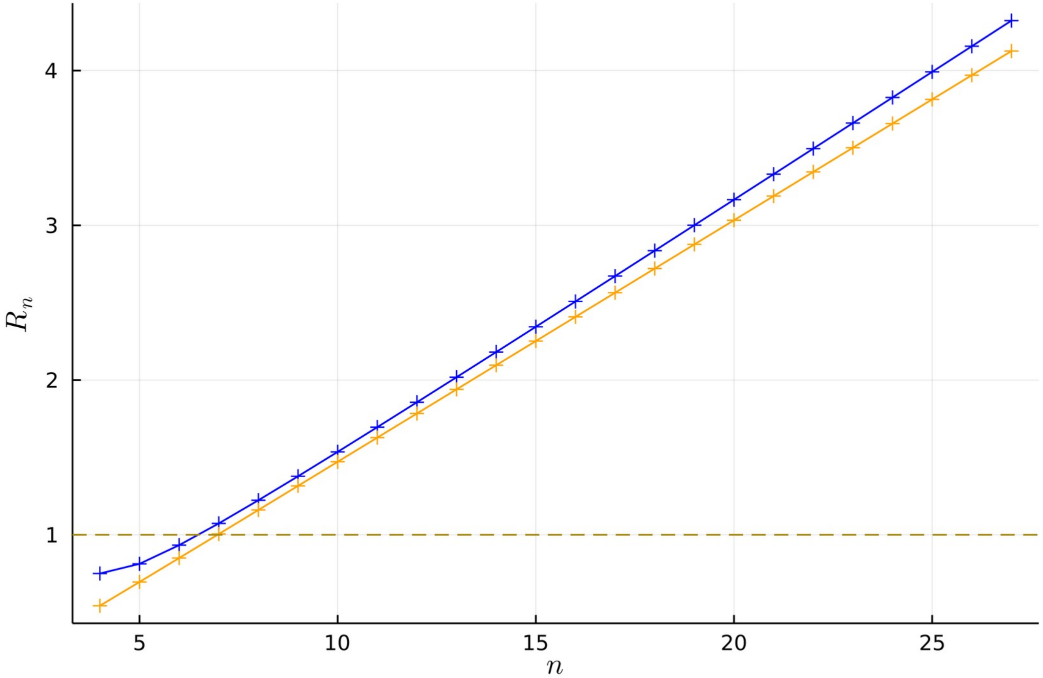
$$P_7^{(\star)} \approx 7.5 \times 10^{-3}$$

$$P_7^{(\mathrm{bos})} \approx 7.0 \times 10^{-3}$$

Natural extension

- Lower bound
- Asymptotic scaling

$$R_n = \frac{P_n^{(\star)}}{P_n^B} \geq \frac{n}{8} + \frac{1}{32}\frac{(n-2)^2}{n-1}$$



Perspectives

- Experimental realization: near term, contact with Pan’s group
- Stronger asymptotic scalings
- Lower dimension/real counter examples?
- Are violations always far from indistinguishability?

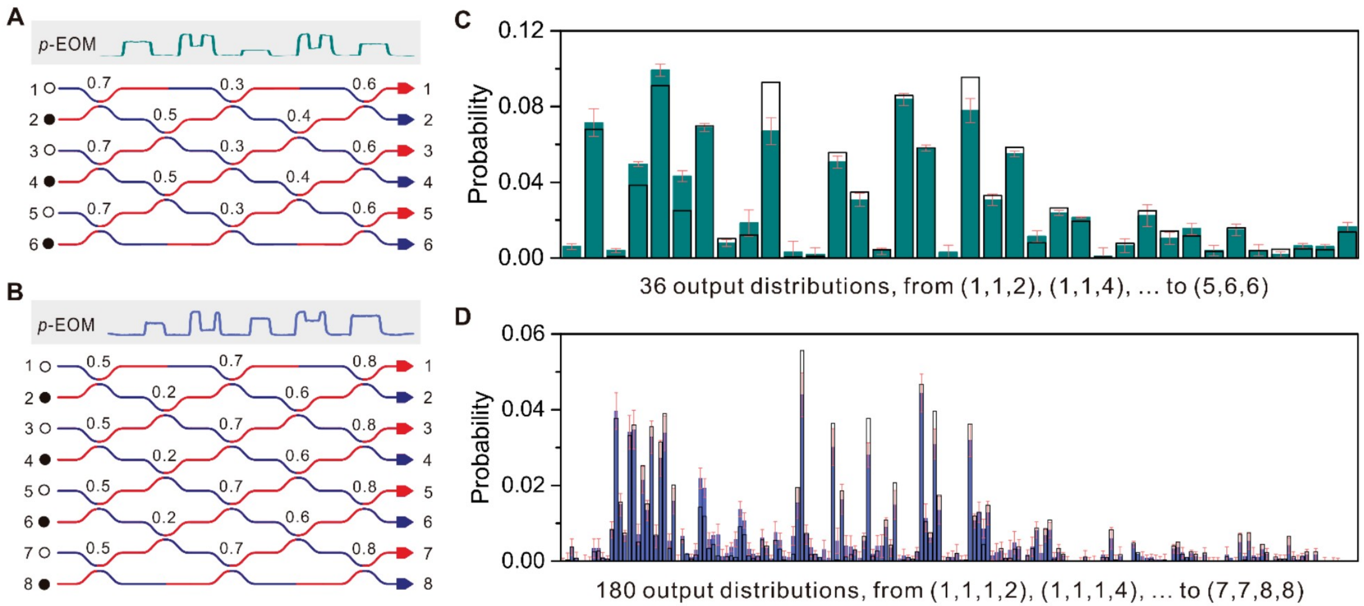
$$
\begin{aligned}
d(S) &= \mathrm{perm}(S)/n! \\
d(\mathrm{bos}) &= 1 \\
d(\mathrm{dist}) &= \frac{1}{7!} \approx 1.98 \times 10^{-4} \\
d(S^{(\star)}) &= \frac{45}{7!} \approx 8.93 \times 10^{-3} \ll 1
\end{aligned}
$$

Take away message

- The common rule of thumb that
  - *Ideal bosons bunch most (does **not** hold!)*
  - *Ideal fermions anti-bunch most (holds!)*
- Grey zone between bosonic and classical behaviour even more complex than expected

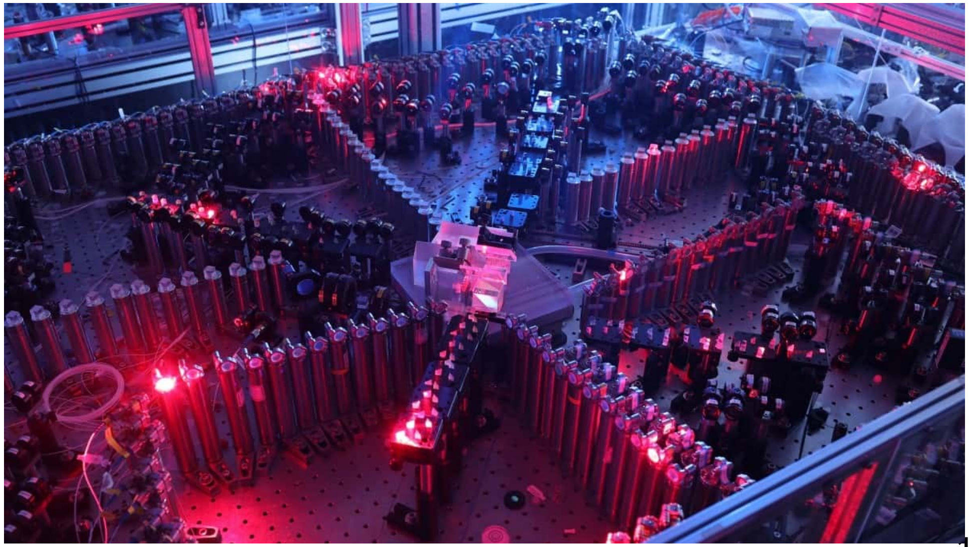
Section 2: Validation of BosonSamplers

- How do we know an experiment works?
  - Intrinsically hard to verify (vs NP)
  - Exponentially many probabilities, each a permanent
  - Many sources of noise, uncertainties



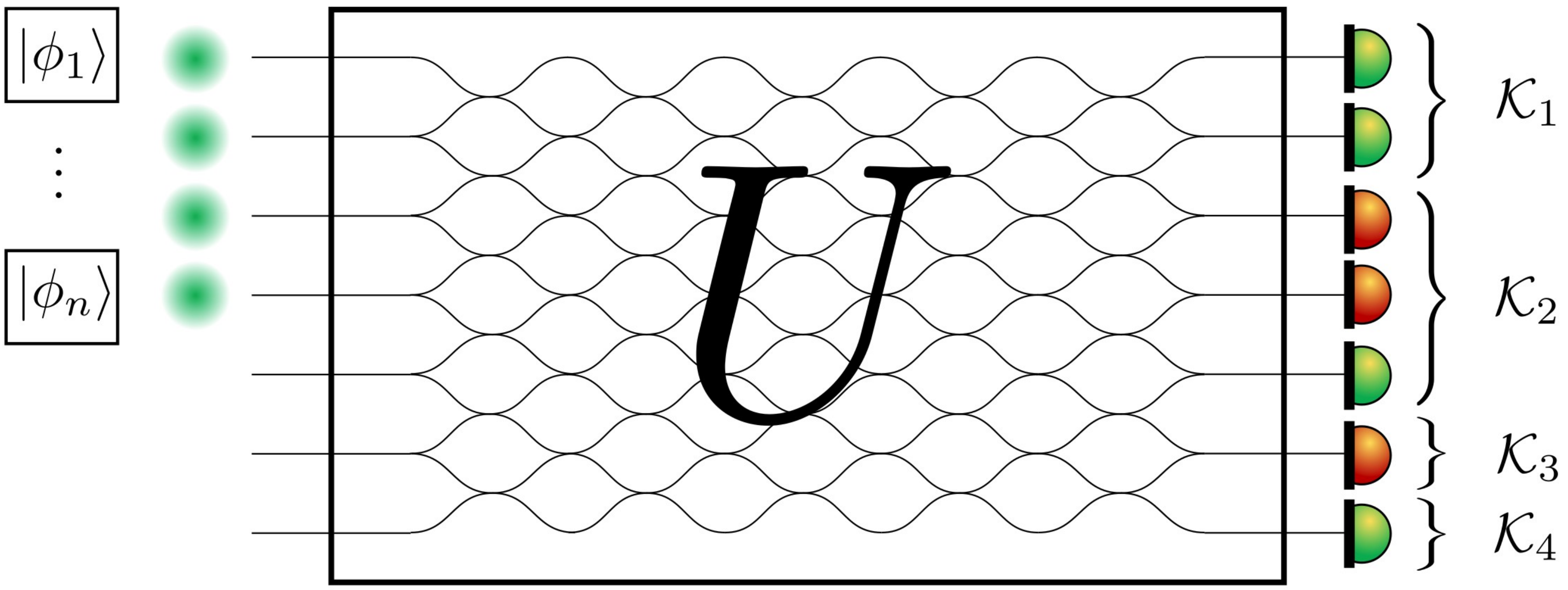
Goals

- Do we have a good input?
- Do we do something hard?
- Is it resilient to noise, uncertainties?
- Is it efficient (number of samples, computations)?
- Can our method be spoofed?



Our method: binning output modes

- Partition of output modes
- Photon counting probability for the bins



Mathematical results

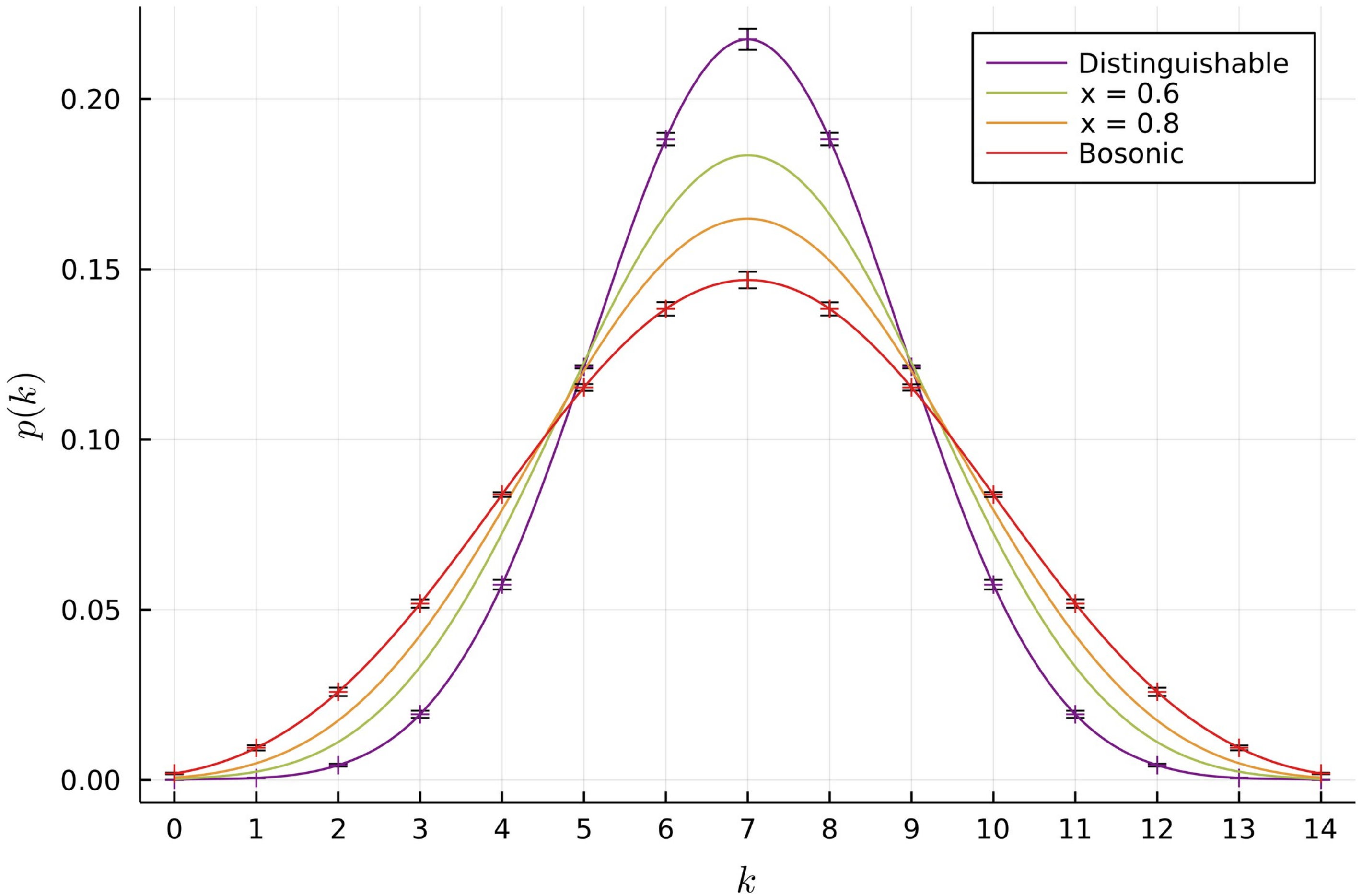
- Efficient way to approximate the entire probability distribution, even with noise
- Theorem: For a constant partition size K, there is a classical algorithm that computes an approximate distribution of probabilities such that

$$\sum_k |\tilde{P}(k) - P(k)| \leq \beta$$

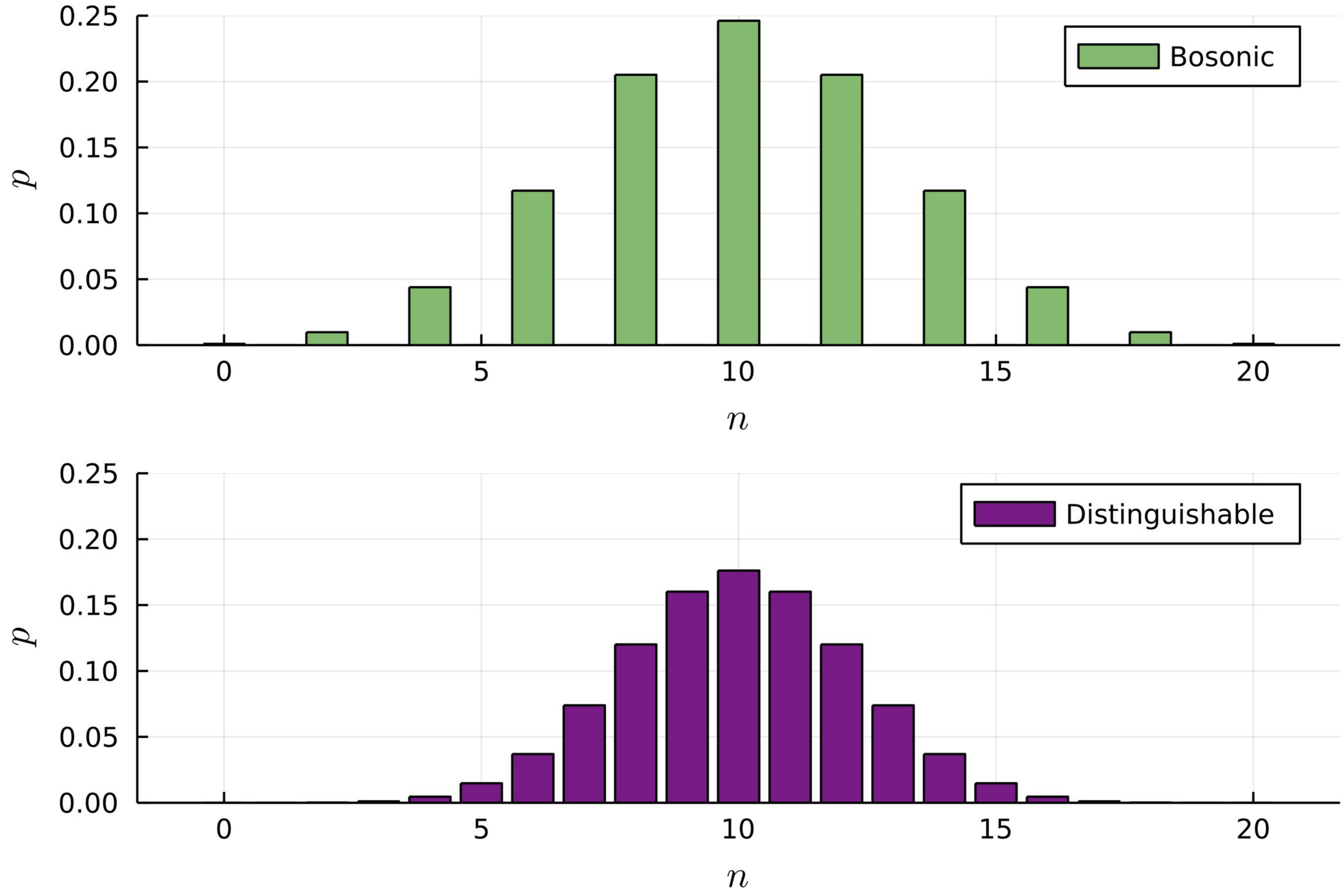
in time

$$O(n^{2K+2}\log(n)\beta^{-2})$$

Example: one bin

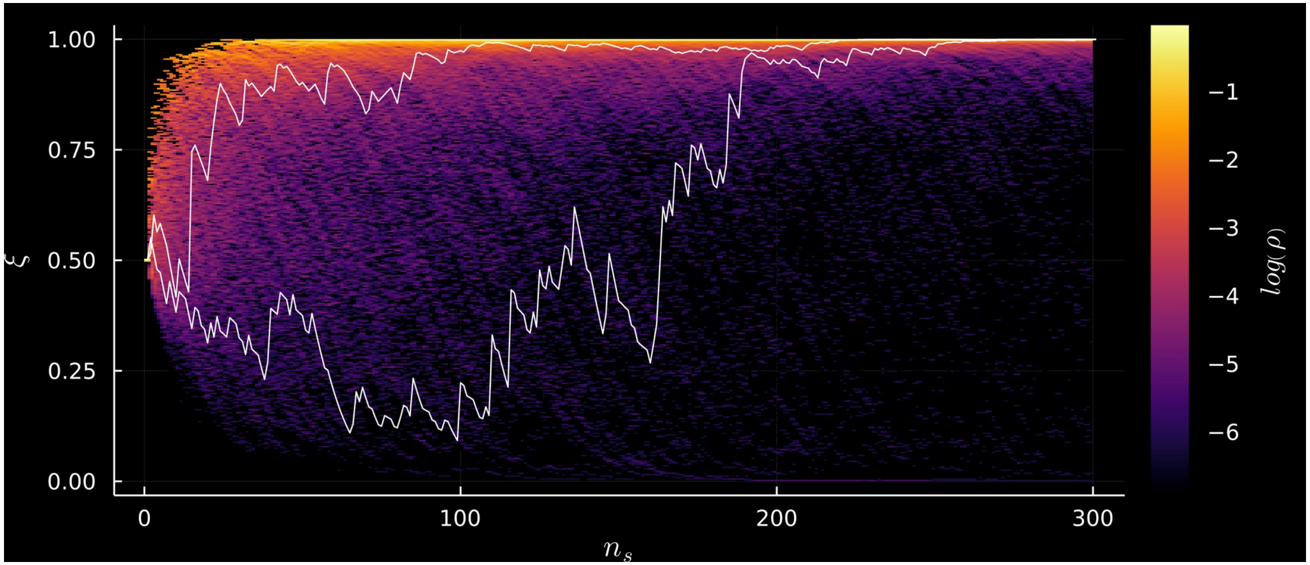


Example: HOM-like effect

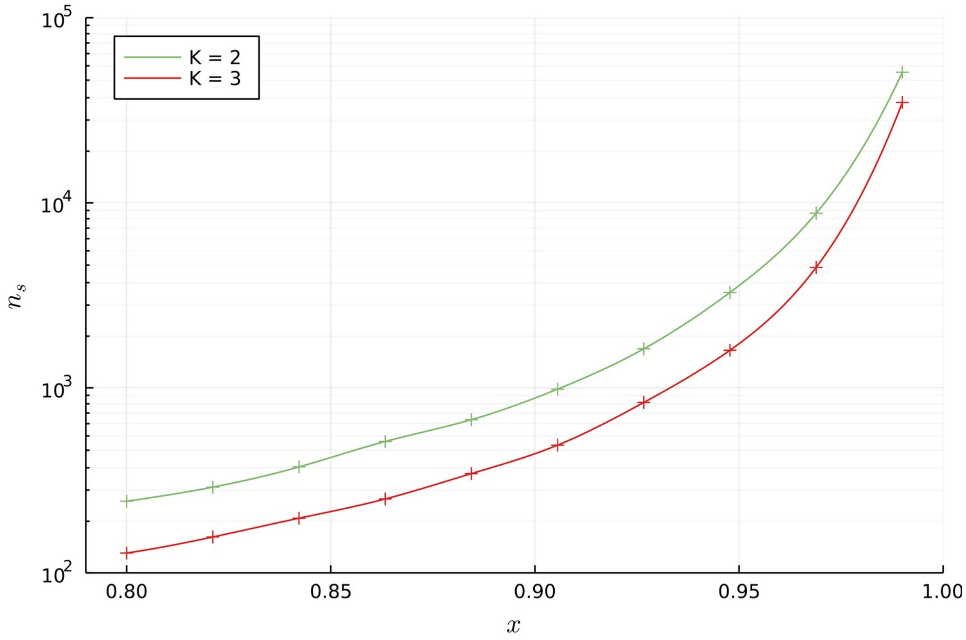


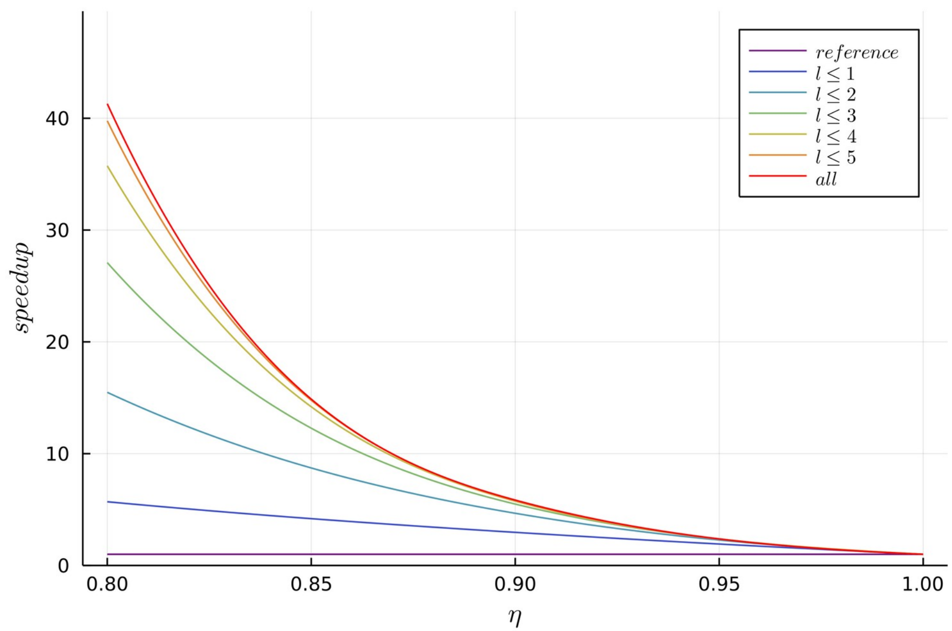
Practical implications

- Efficient way to distinguish between B and D inputs
- Realistic analysis of noise
- Resilient, universal scheme
- Can use information of lossy events



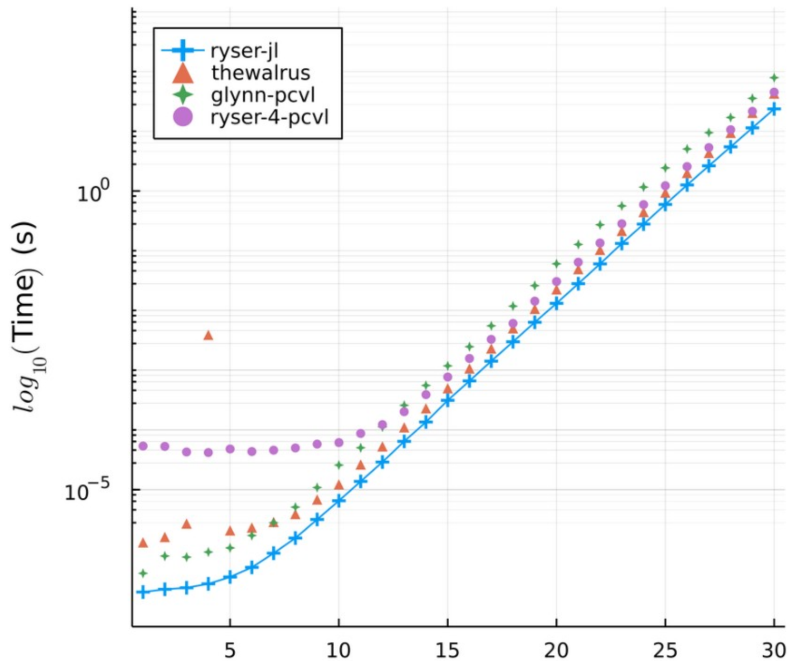
Number of samples needed





Section 3: BosonSampling package

- Permanent and BosonSampling packages for Julia (8 kloc src/)
- Made available publicly on GitHub
- First open source package regarding validation
- Easy to expand to new models while staying fast

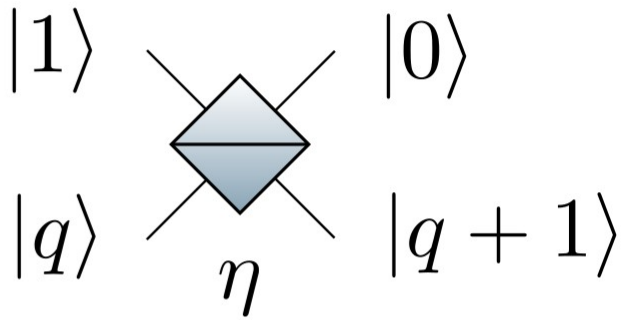


Images credit

- HOM illustrations: Agata M. Bra´nczyk, arXiv:1711.00080
- HOM experimental plot: Hong, C.K., Ou, Z.Y. and Mandel, L., 1987. Measurement of subpicosecond time intervals between two photons by interference. Physical review letters, 59(18), p.2044.
- HOM gif: Jachura, Chrapkiewicz, arXiv:1502.07917
- BosonSampling and Galton Board: Yong Liu; Junjie Wu; Xun Yi; 10.1109/ICNC.2015.7378023
- All other illustrations are from our preprint, arxiv:2203.01306
- Scalable boson sampling with a single-photon device, He et al.

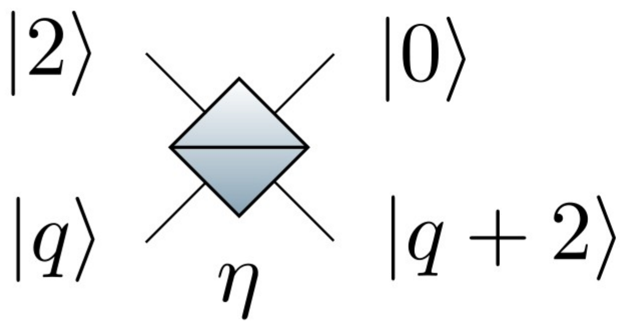
Appendix: violation mechanism

a)



$$P = (q+1)\eta(1-\eta)^q$$

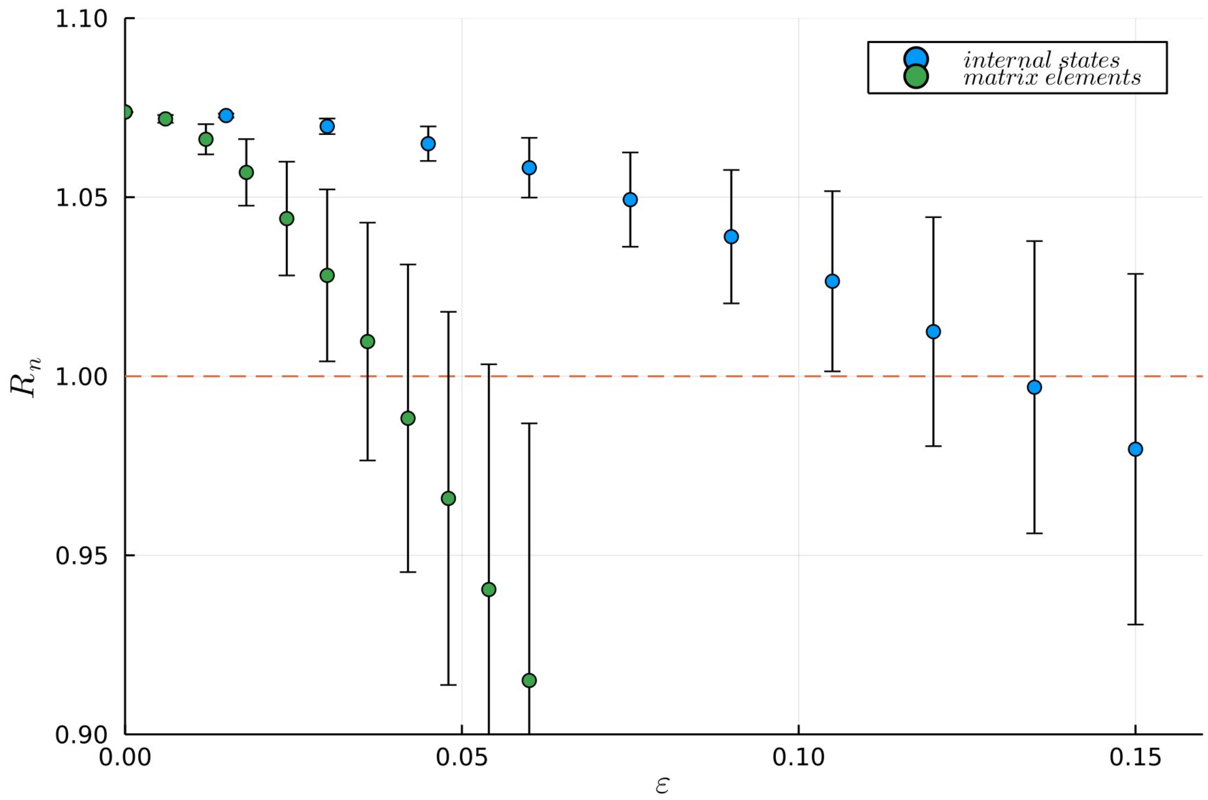
b)



$$P = \binom{q+2}{2}\eta^2(1-\eta)^q$$

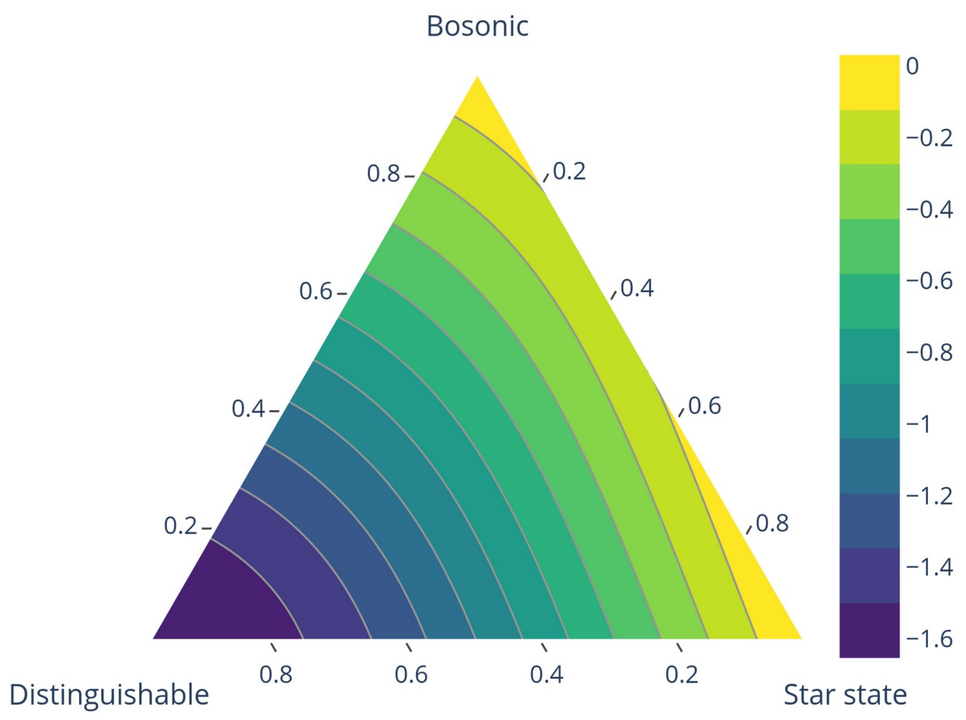
Appendix: resiliency

- The scheme is resilient to perturbations

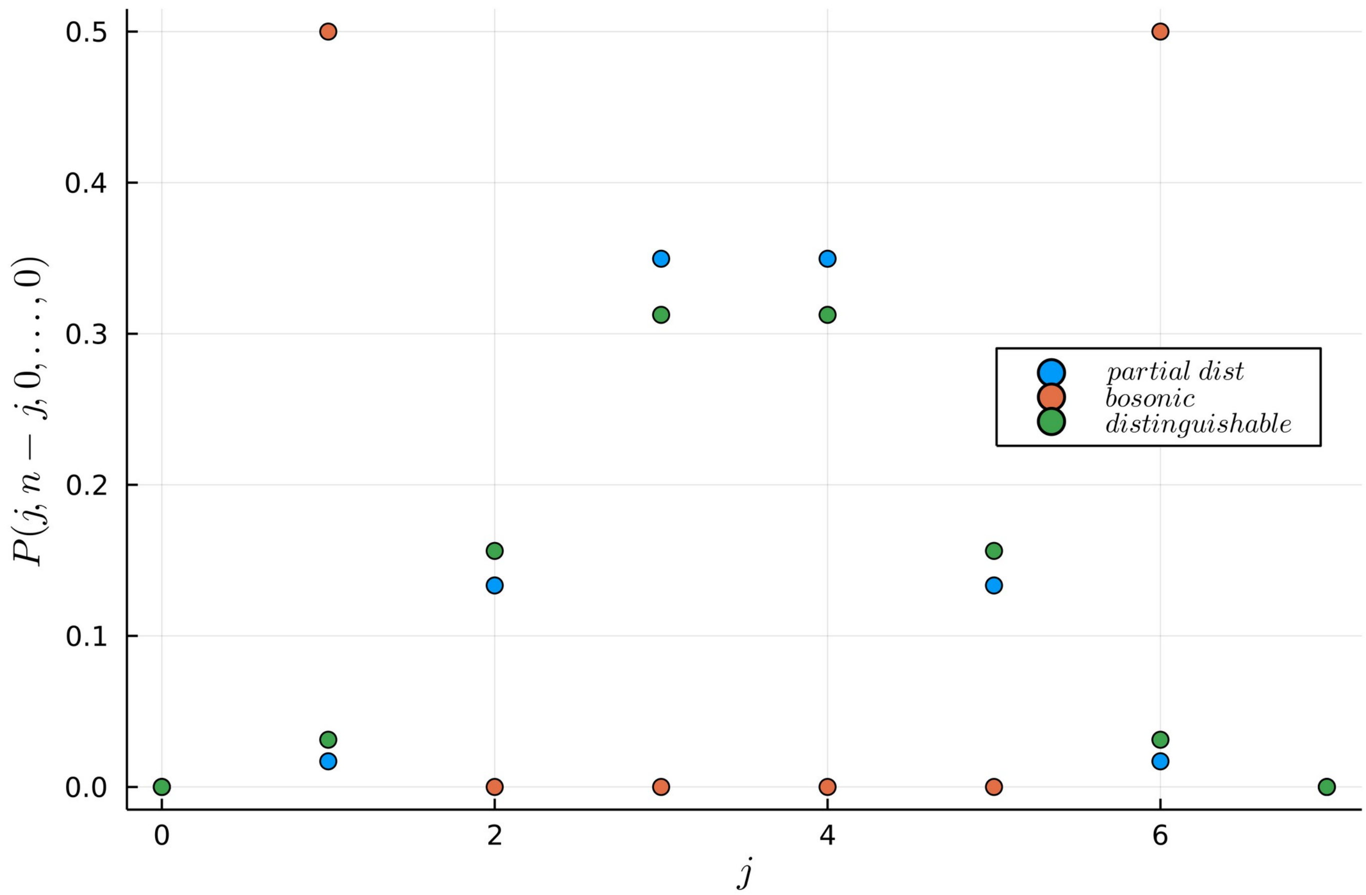


Appendix: stability around the bosonic case

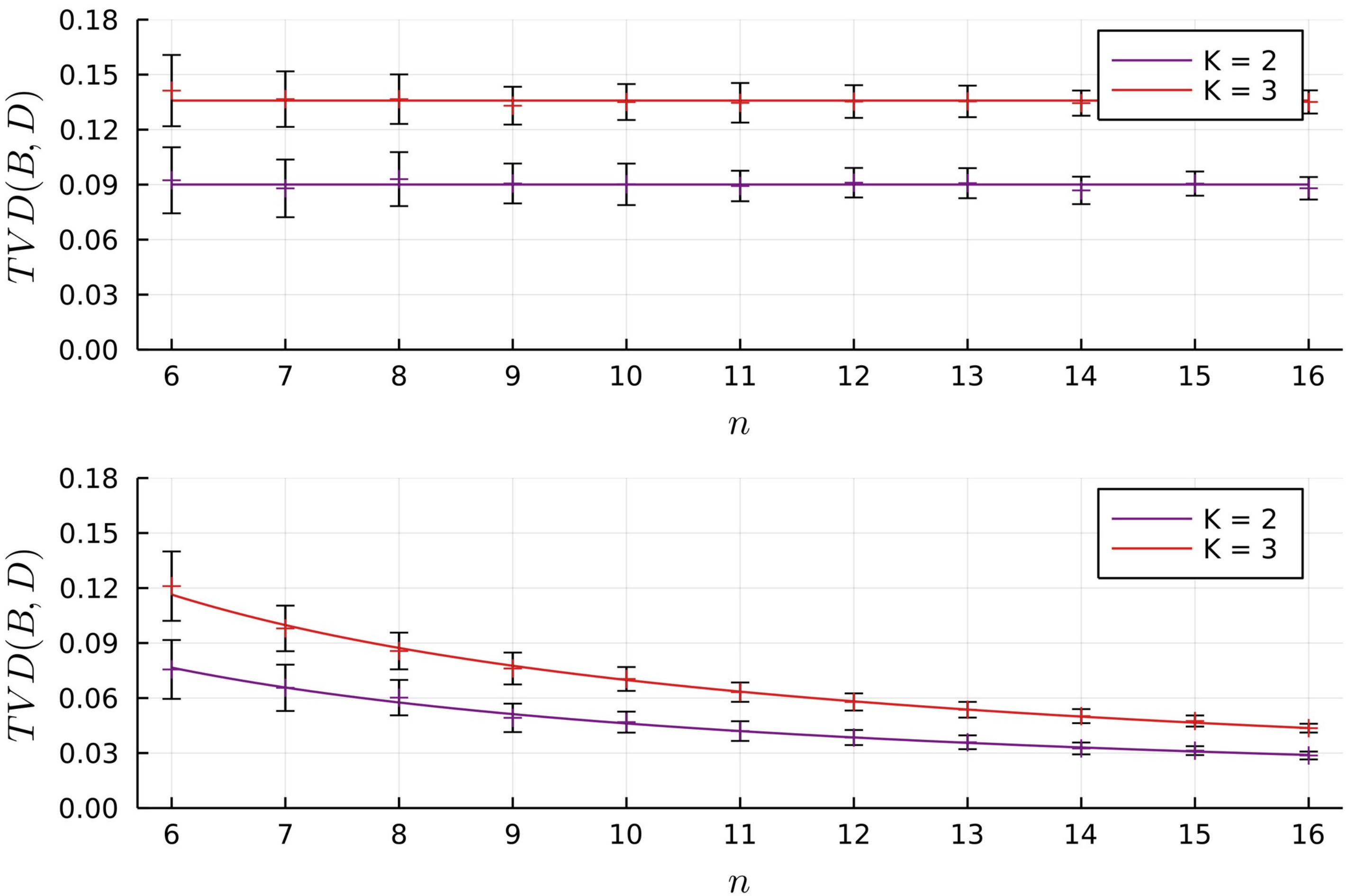
- Nil gradient around the bosonic Gram matrix



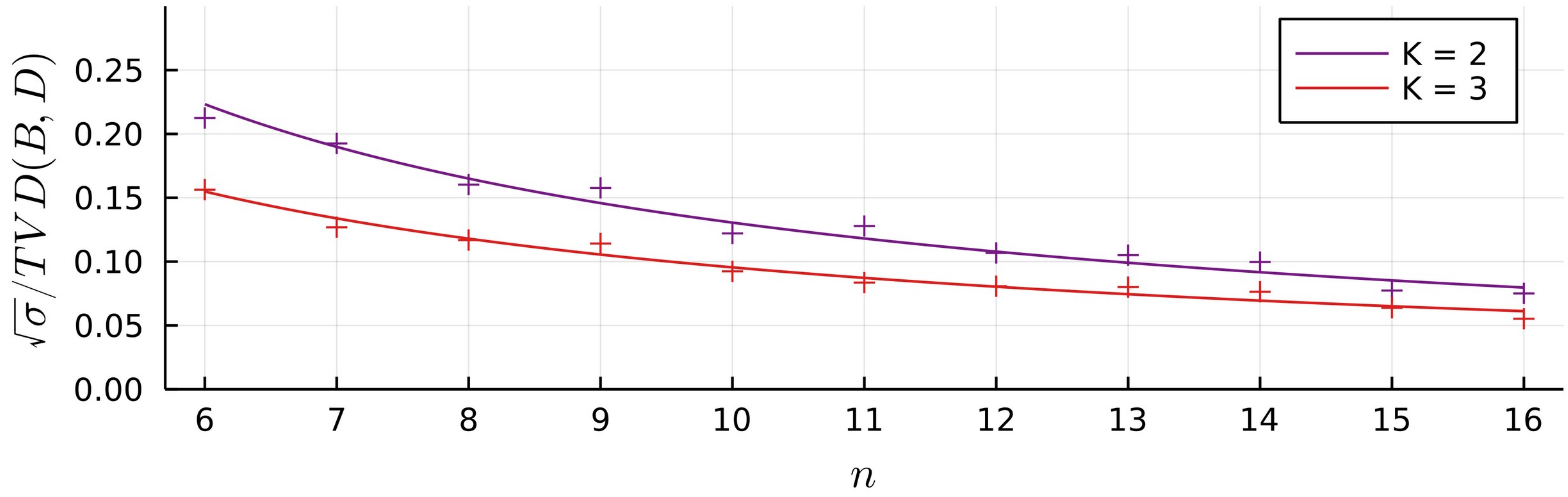
Appendix: photon number distribution

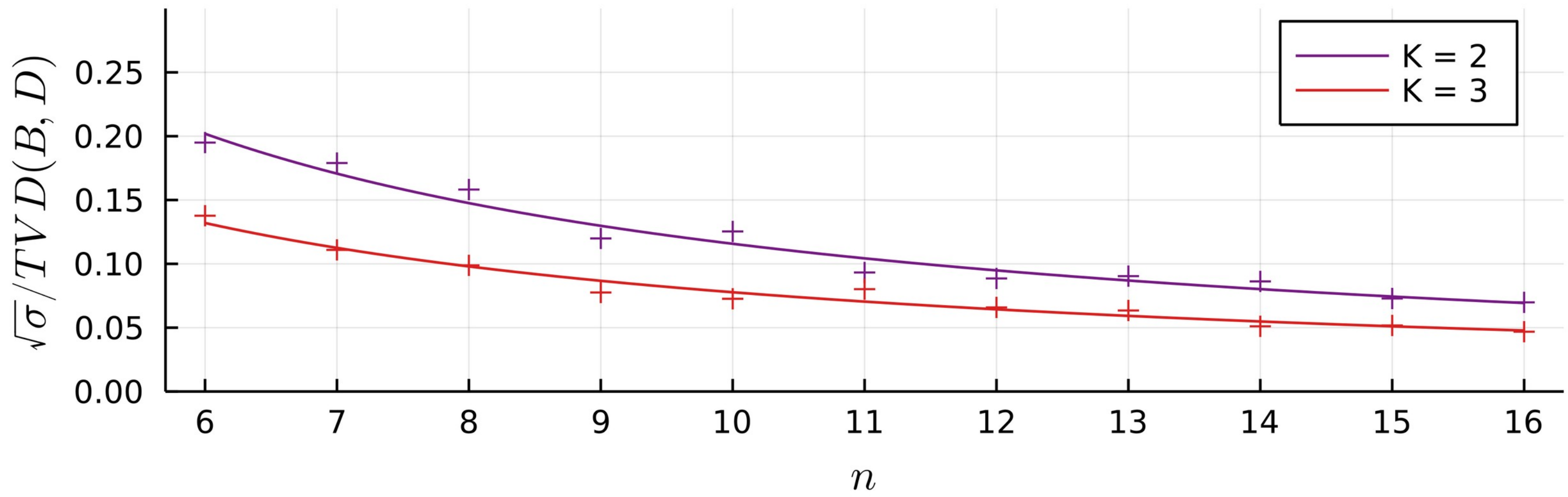


Appendix: validation

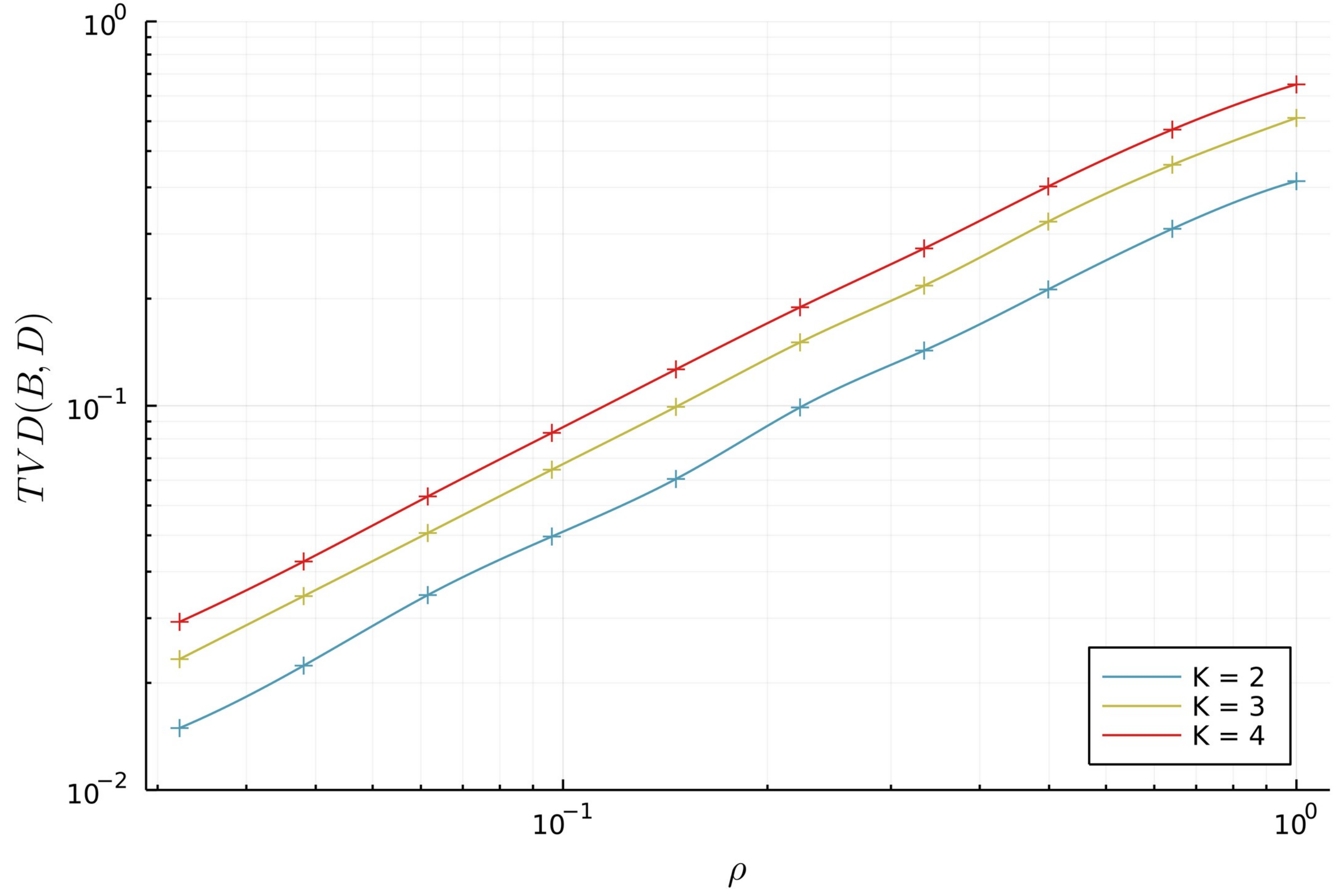


Appendix: validation

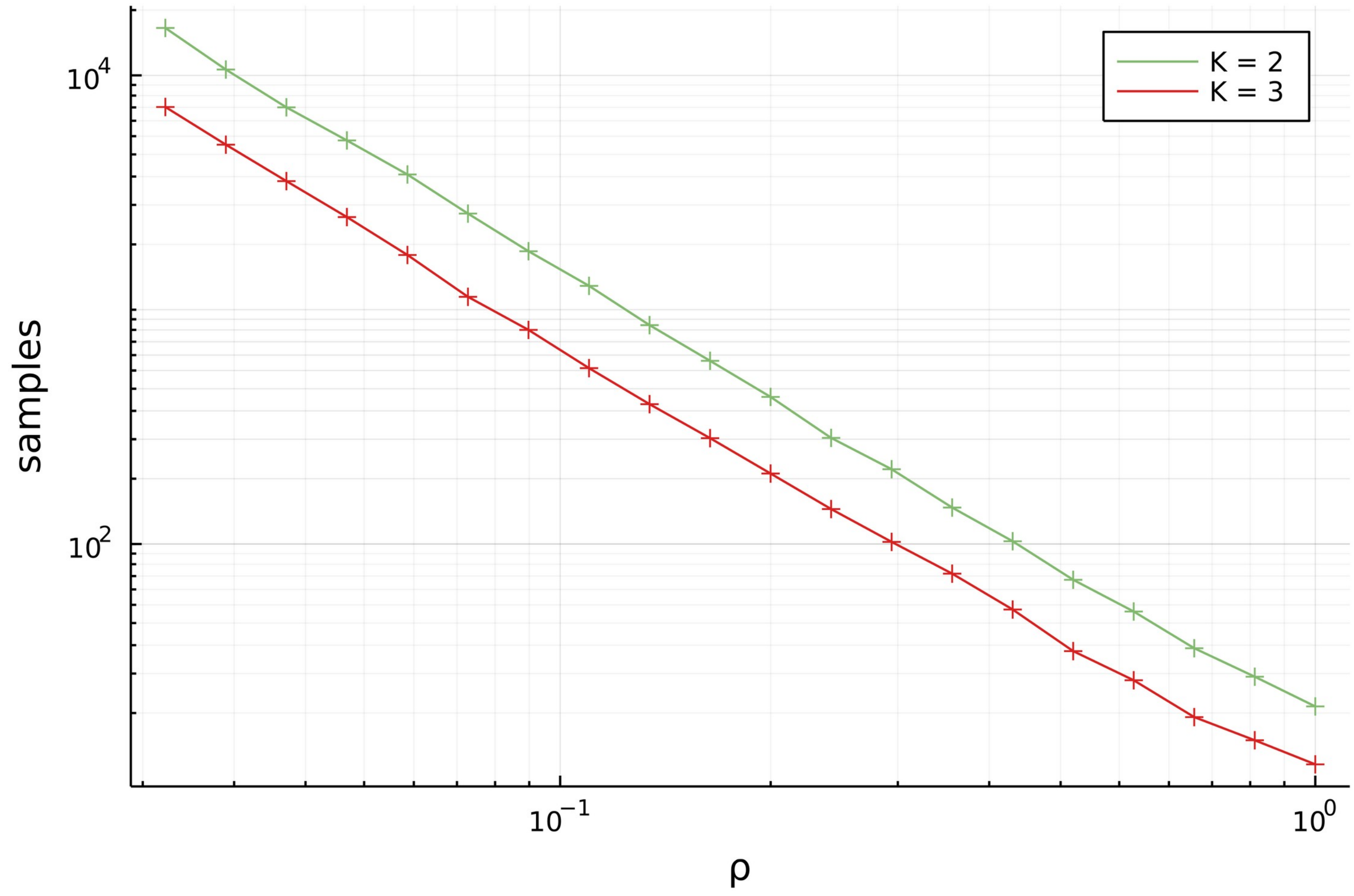




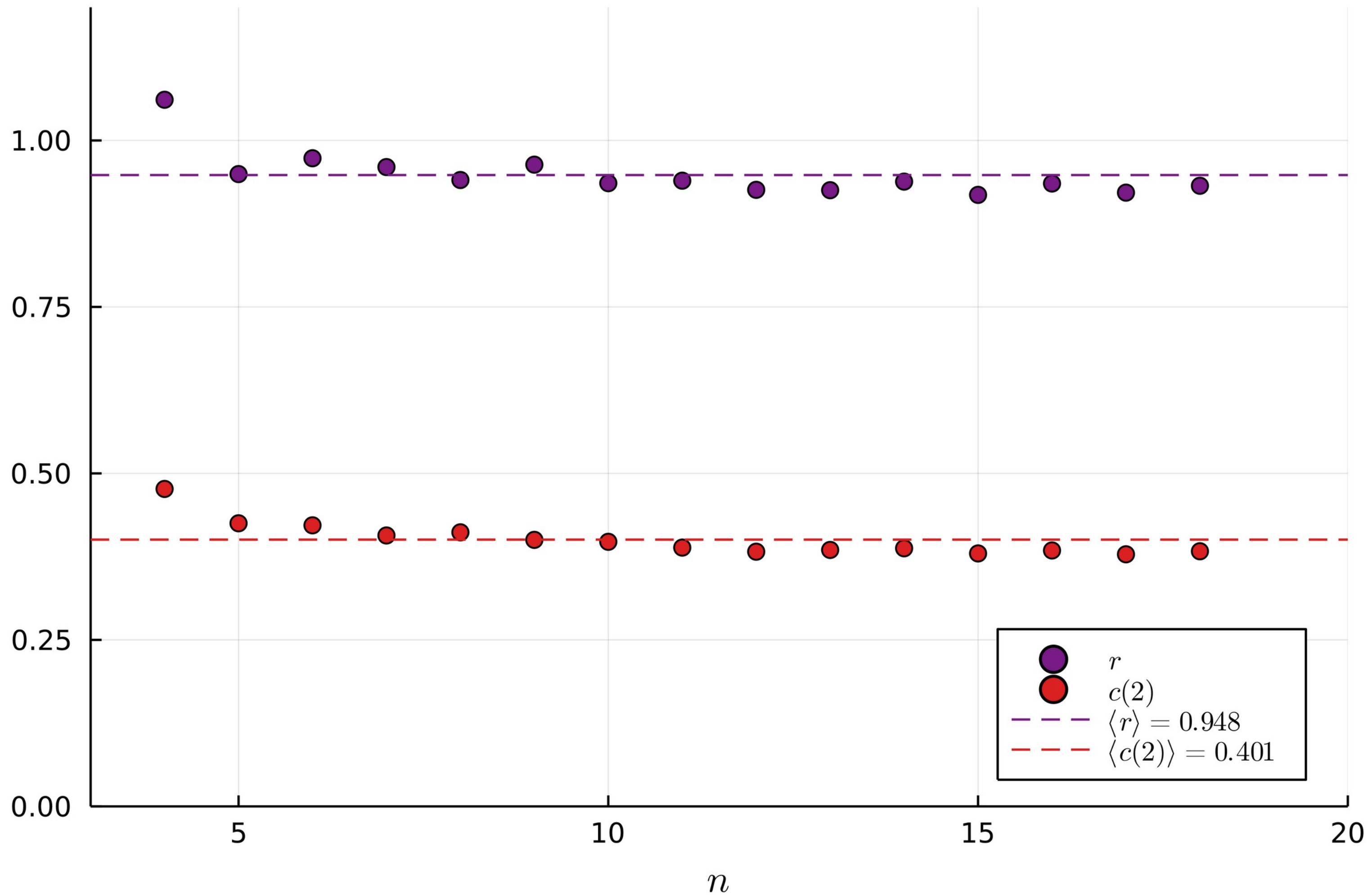
Appendix: validation



Appendix: validation



Appendix: validation



Appendix: validation