Quantum computing in the NISQ era

Towards quantum advantage in quantum digital simulation

José D. Guimarães | 2022

Universidade do Minho

CF-UM-UP

INL
INTERNATIONAL IBERIAN
NANOTECHNOLOGY
LABORATORY

Outline

Part I

Quantum TEDOPA (Q-TEDOPA).

Part II

Quantum error mitigation.

Part I

Quantum TEDOPA

José D. Guimarães, Mikhail I. Vasilevskiy, Luís S. Barbosa | 2022

Classical simulation of open quantum systems

Non-perturbative dynamics

- Reduced density operator methods: Hierarchical Equations of Motion (HEOM)

$$t_{exe} \sim O(2^N)$$

Tanimura, Yoshitaka, *The Journal of chemical physics* 153.2 (2020): 020901.

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- Matrix Product State (MPS) methods: Time Evolving Density operator with Orthogonal Polynomials (TEDOPA)

Tamascelli, Dario, et al., *Physical review letters* 123.9 (2019): 090402.

$$t_{exe} \sim O(D^3), \quad D \sim 2^t$$

D = bond dimension

Eisert, Jens, Marcus Cramer, and Martin B. Plenio, *Reviews of modern physics* 82.1 (2010): 277.

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Higher dimensional systems, e.g. NxN lattice,

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Schollwöck, Ulrich. *Annals of physics* 326.1 (2011): 96-192.

Our work

Quantum TEDOPA (Q-TEDOPA)

- Scales polynomially with *N* and *t* (no restriction on dimensionality).

Efficient method to simulate non-perturbative dynamics of an open quantum system using a quantum computer

José D. Guimarães*
Centro de Física das Universidades do Minho e do Porto, Laboratório de Física para Materiais e Tecnologias Emergentes (LaPMET), Universidade do Minho, Braga 4710-057, Portugal and International Iberian Nanotechnology Laboratory, Av. Mestre José Veiga s/n, Braga 4715-330, Portugal

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Classical non-perturbative simulations of open quantum systems dynamics face several scalability problems, namely, exponential scaling as a function of either the time length of the simulation or the size of the open system. In this work, we propose a quantum computational method which we term as Quantum Time Evolving Density operator with Orthogonal Polynomials Algorithm (Q-TEDOPA), based on the known TEDOPA technique, avoiding any exponential scaling in simulations of non-perturbative dynamics of open quantum systems on a quantum computer. By performing an exact transformation of the Hamiltonian, we obtain only local nearest-neighbour interactions, making this algorithm suitable to be implemented in the current Noisy-Intermediate Scale Quantum (NISQ) devices. We show how to implement the Q-TEDOPA using an IBM quantum processor by simulating the exciton transport between two light-harvesting molecules in the regime of moderate coupling strength to a non-Markovian harmonic oscillator environment. Applications of the Q-TEDOPA span problems which can not be solved by perturbation techniques belonging to different areas, such as dynamics of quantum biological systems and strongly correlated condensed matter systems.

Guimarães, José D., Mikhail I. Vasilevskiy, and Luís S. Barbosa. *arXiv preprint arXiv:2203.14653* (2022).

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- Mapping to a closed quantum system.

Very appealing for quantum computers!

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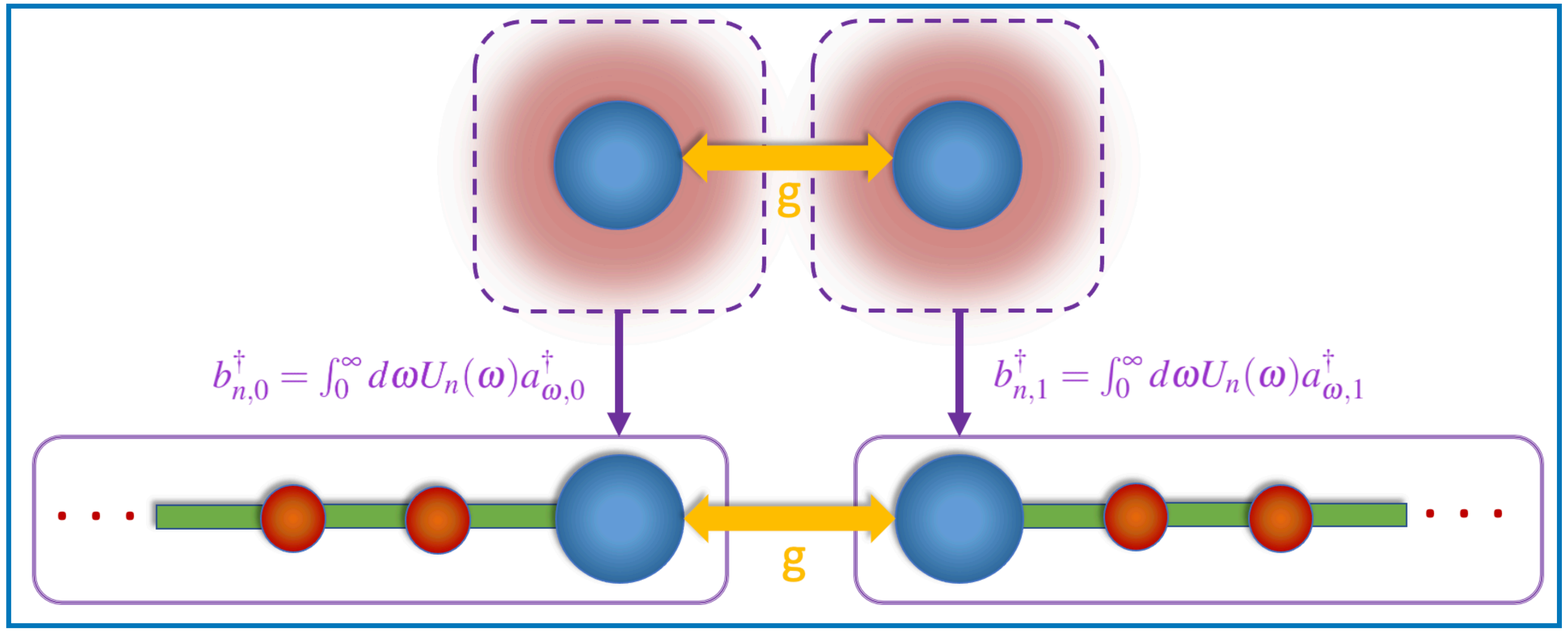
(T-)TEDOPA
How it works?

Full Hamiltonian:

$$H = H_S + H_E + H_{SE} \,,$$

$$H_E = \int_0^{+\infty} d\omega \omega a_\omega^\dagger a_\omega, \quad H_{SE} = A \otimes \int_0^{+\infty} d\omega \sqrt{J(\omega)} (a_\omega^\dagger + a_\omega)$$



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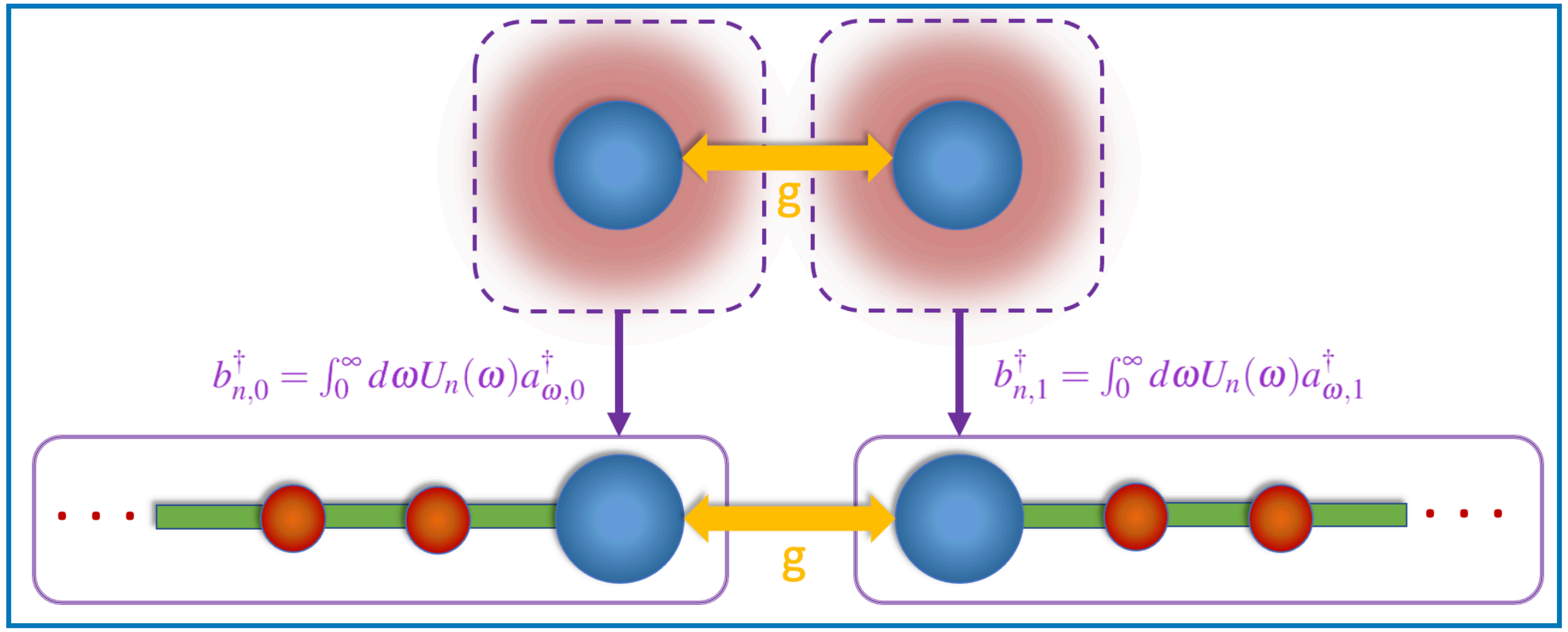
$$H_{SE}^C = t_0 A \otimes (b_0^\dagger + b_0)$$

$$H_E^C = \sum_{n=0}^{\infty} w_n b_n^\dagger b_n + \sum_{n=1}^{\infty} t_{n,n-1} b_n^\dagger b_{n-1} + h.c.$$



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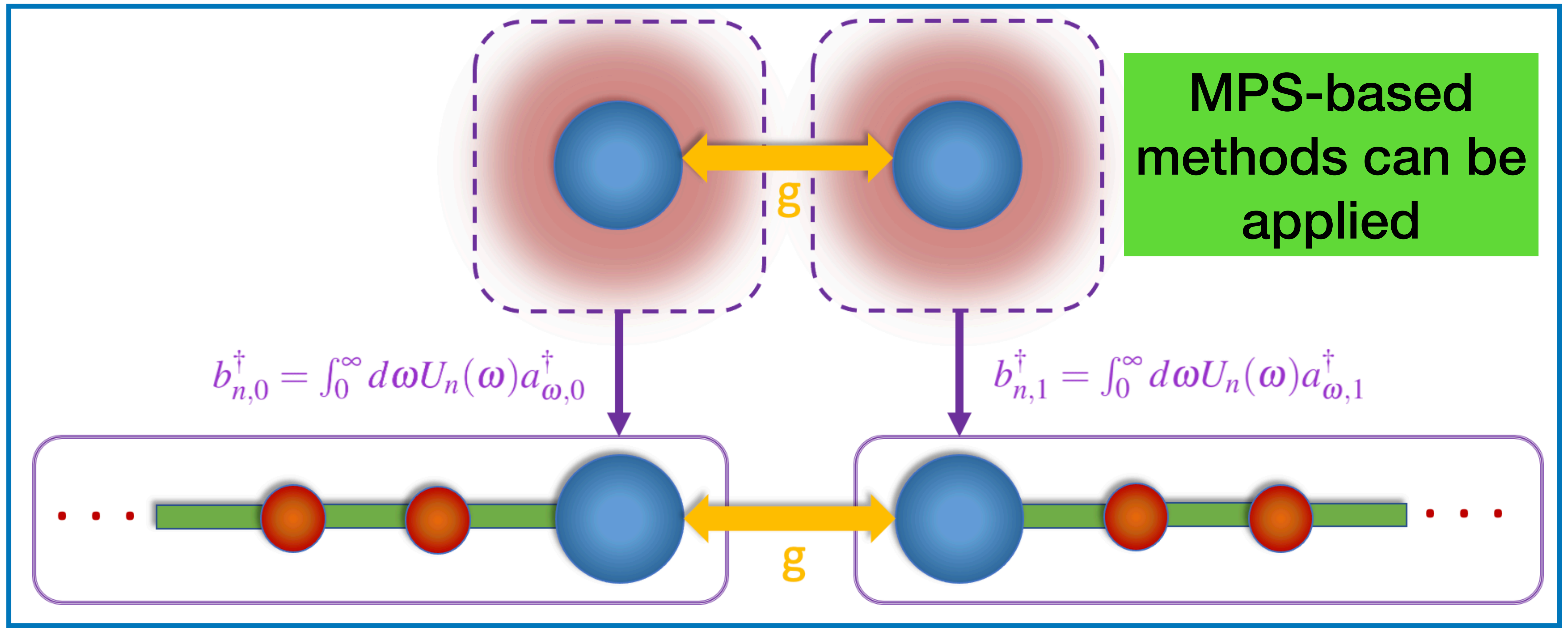

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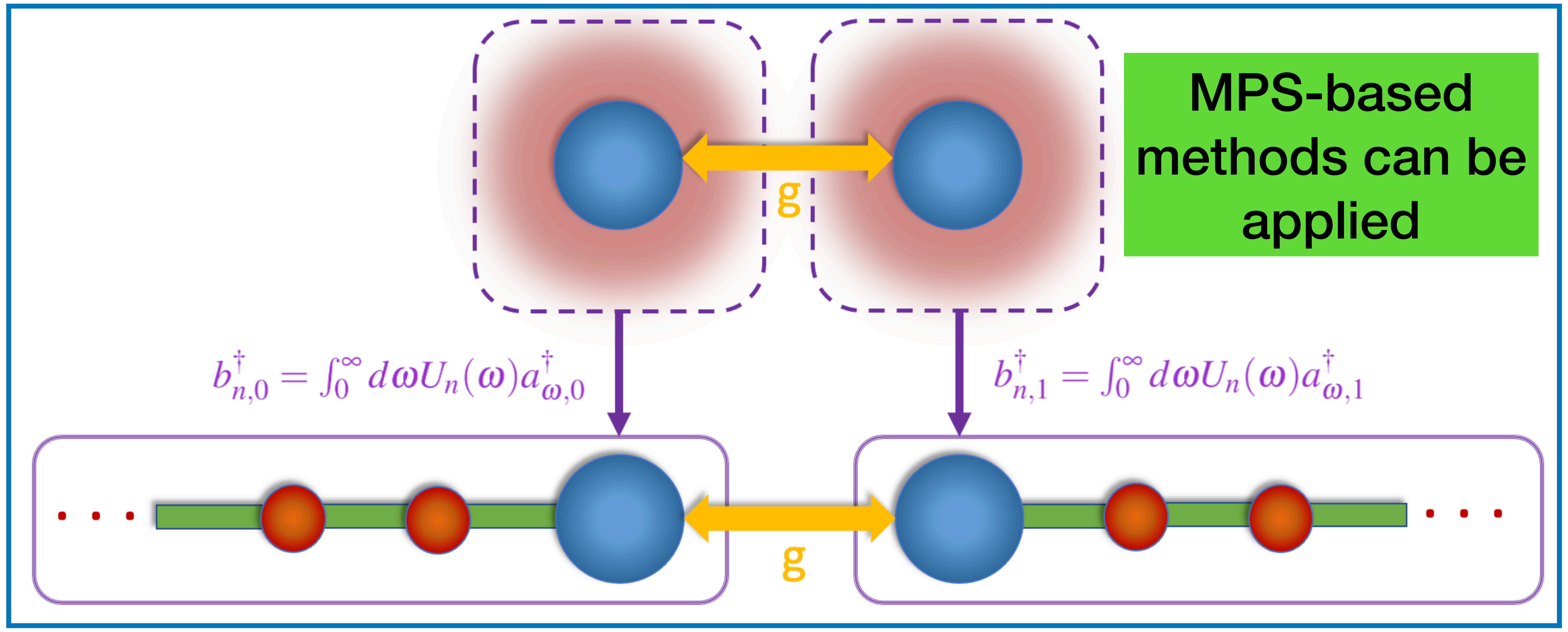

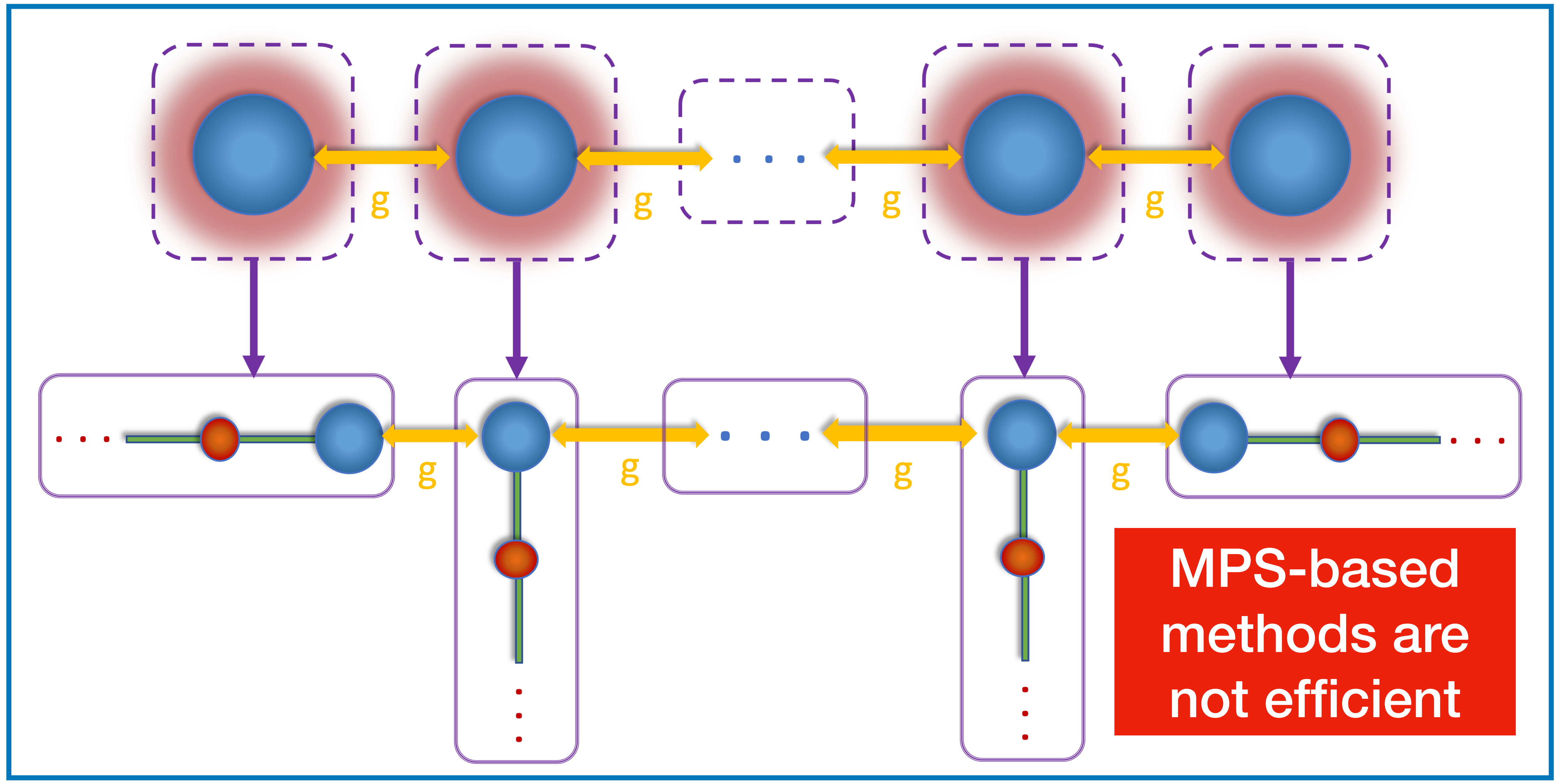

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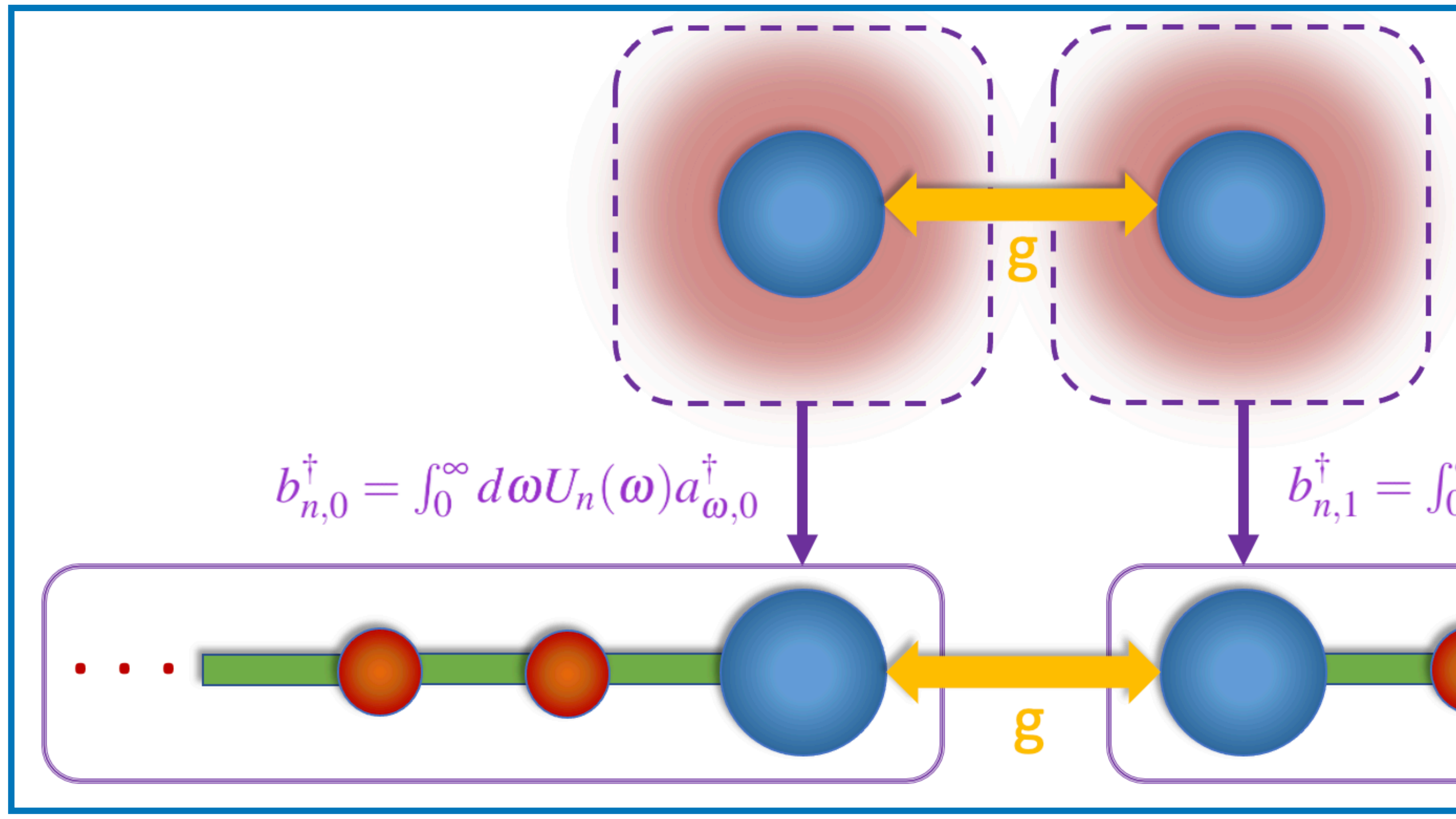
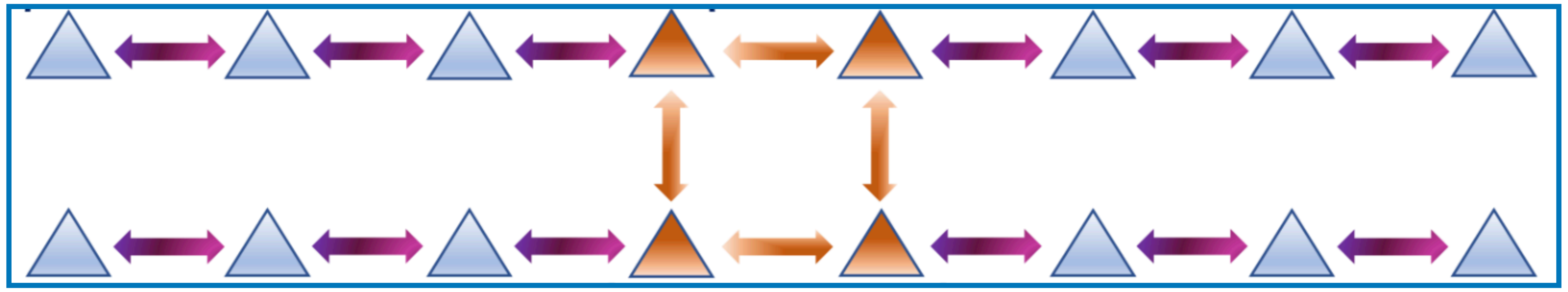


MPS-based methods can be applied



MPS-based methods are not efficient

Quantum TEDOPA

The solution

1. Map all operators to Pauli operators.

System

Qubit = two-level molecule

$$H_S = \sum_m E_m a_m^\dagger a_m + \sum_{m \neq n} g_{mn} a_m^\dagger a_n$$

$$= -\sum_m \frac{E_m}{2} \hat{Z}_m + \frac{1}{2} \sum_{m>n} g_{mn} (\hat{X}_m \hat{X}_n + \hat{Y}_m \hat{Y}_n)$$

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$$b = \sum_{m=1}^{2^n - 1} \sqrt{m} \, | m-1 \rangle \langle m |$$

Binary qubit encoding ($d=2^n$)

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Unary qubit encoding ($d=n$)

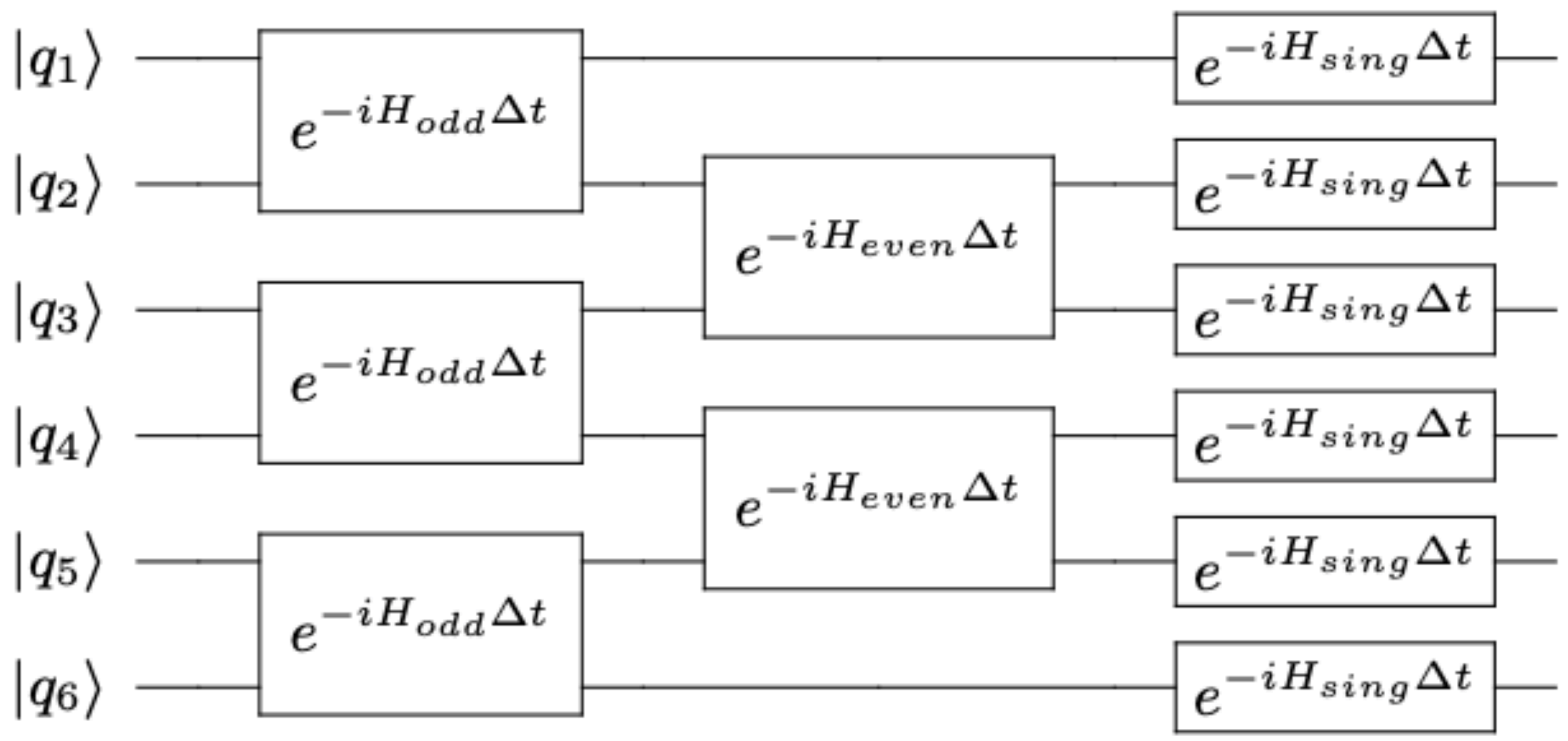
Quantum TEDOPA

The solution

1. Map all operators to Pauli operators.

2. Implement the evolution in a quantum circuit via some (hybrid) quantum algorithm.

Trotter-Suzuki product formula



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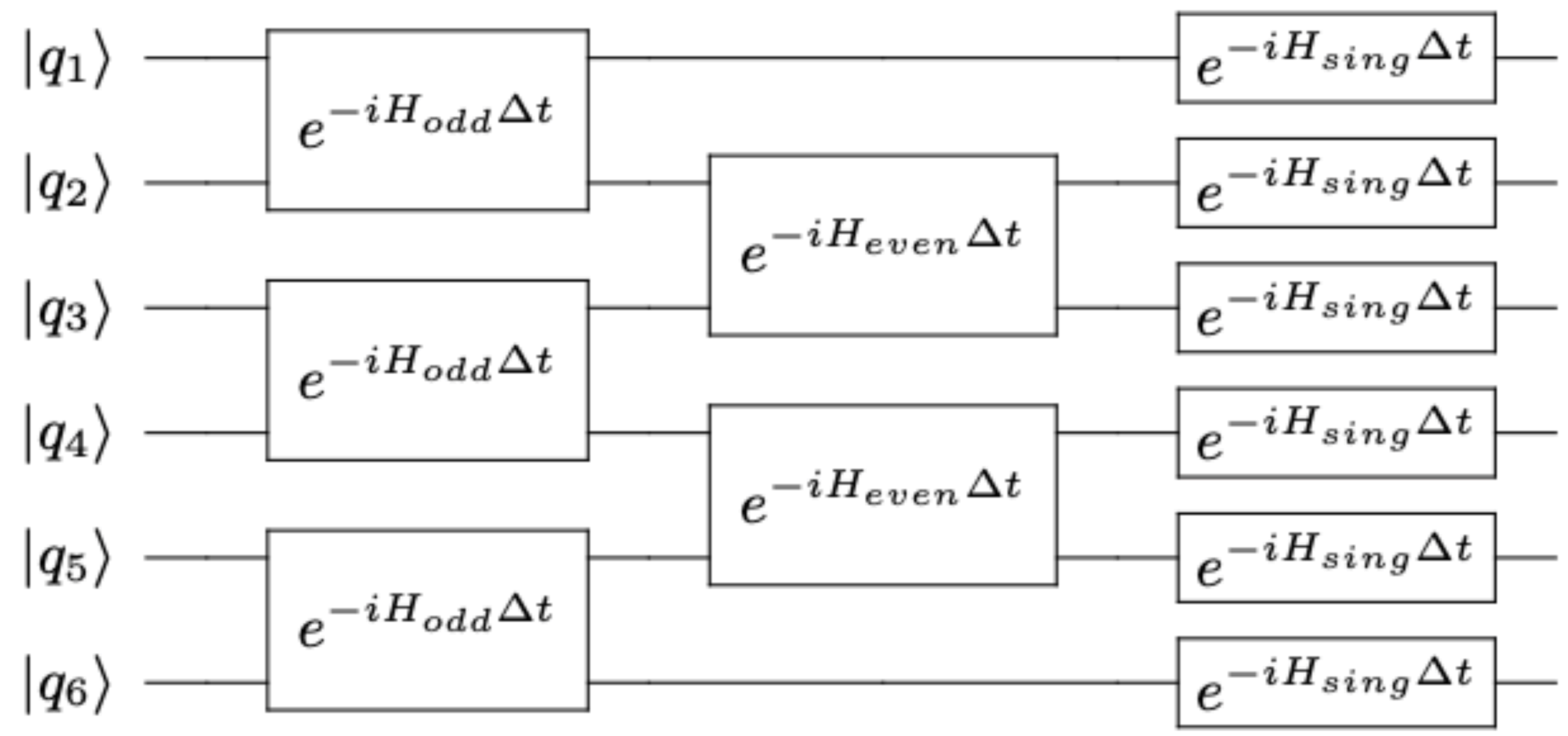

Hybrid quantum-classical algorithms:

- Variational Trotter compression.
- Variational Fast-Forwarding.
- Variational Quantum Simulator.

Y. Li and S. C. Benjamin, Physical Review X 7, 021050 (2017).

Packages for hybrid variational computing: Pennylane, Tequila, etc.

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Execution time, number of qubits, etc. ?

$$H_E = \sum_{n=0}^{\infty} w_n b_n^\dagger b_n + \sum_{n=1}^{\infty} t_{n,n-1} b_n^\dagger b_{n-1} + h.c.$$

Costier term

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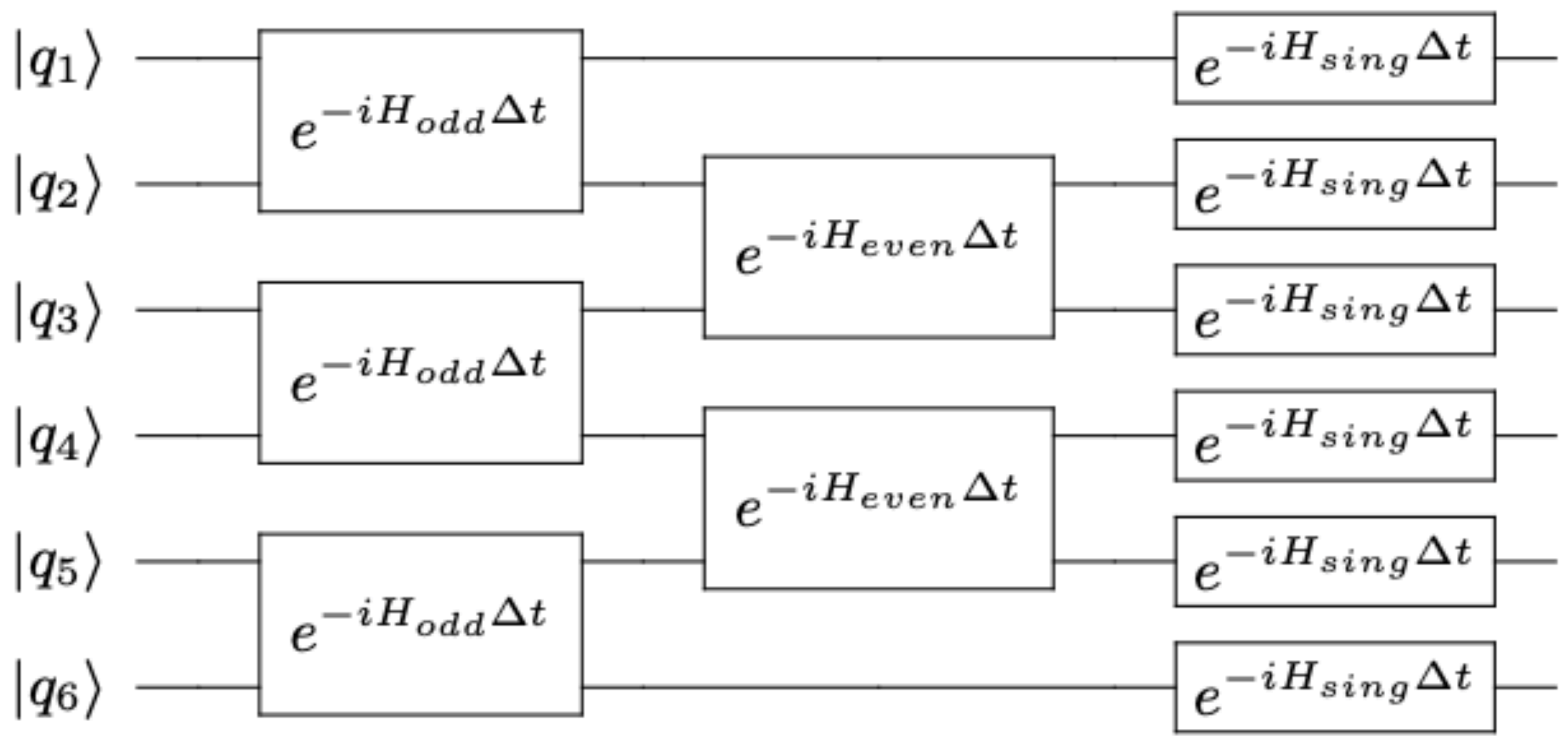

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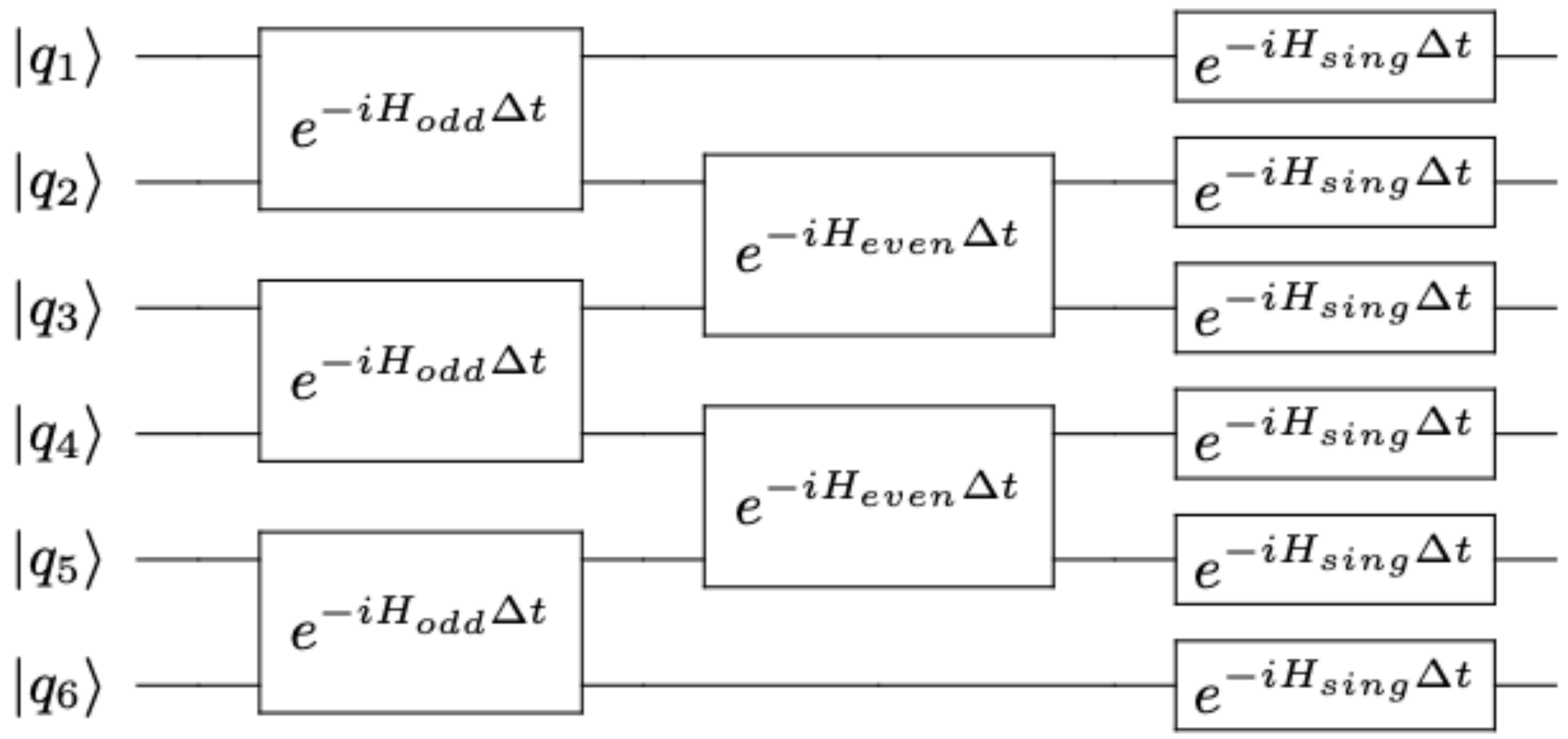

d = energy levels of the harmonic oscillator
l = number of oscillators in the chain
N = number of Trotter iterations

	Binary	**Unary**
Qubits	$log(d)l$	dl

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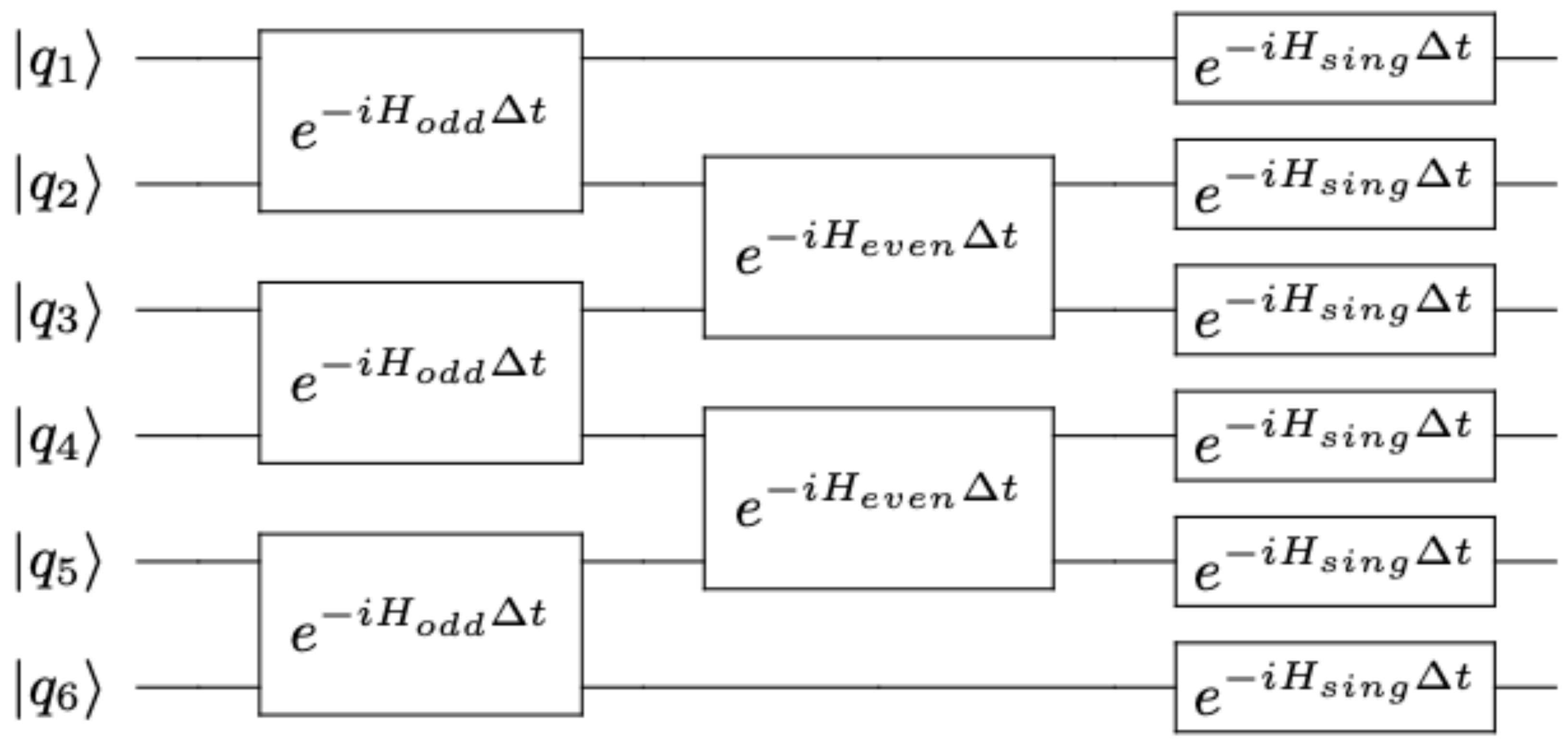

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Qubit connectivity	$2log(d)$	$2d$

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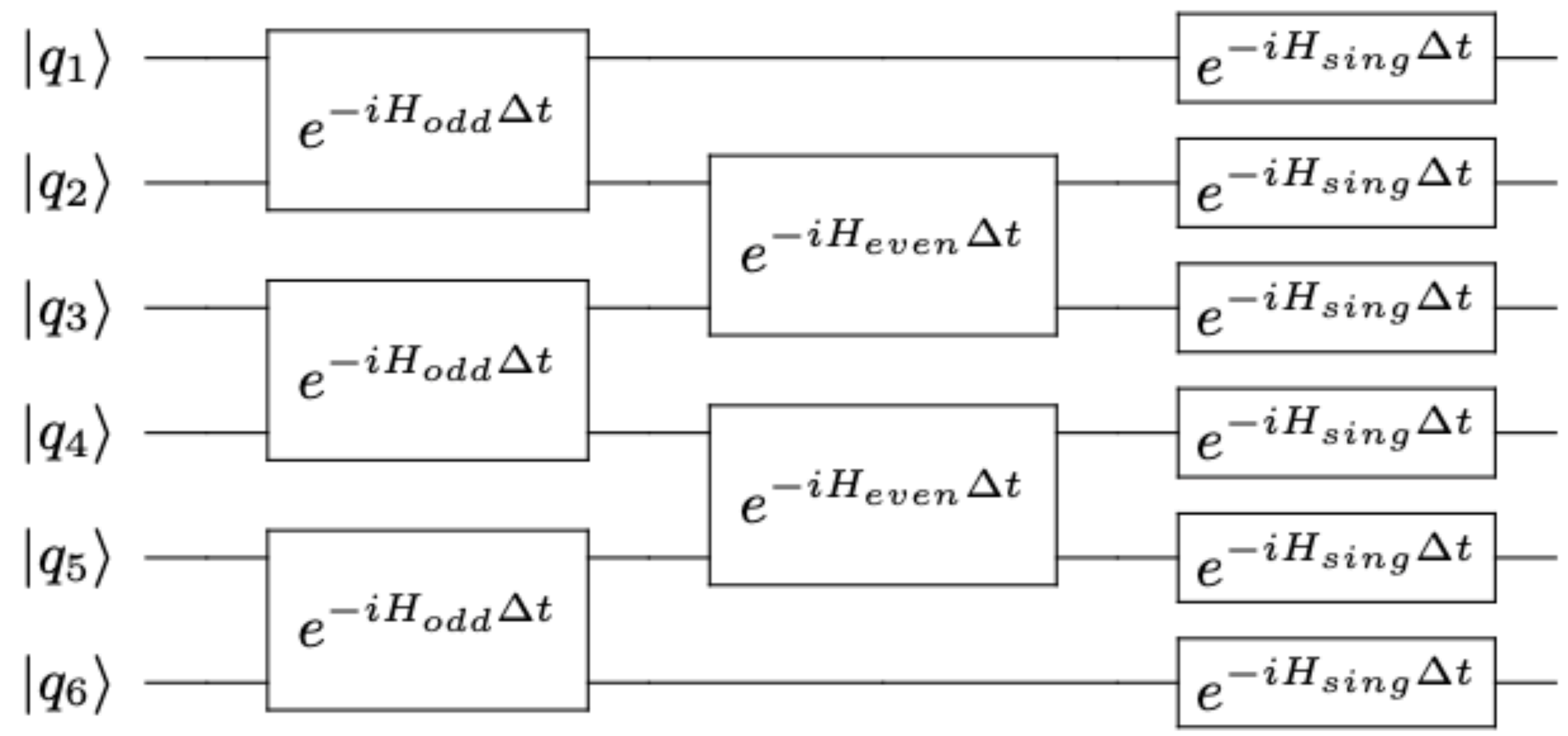
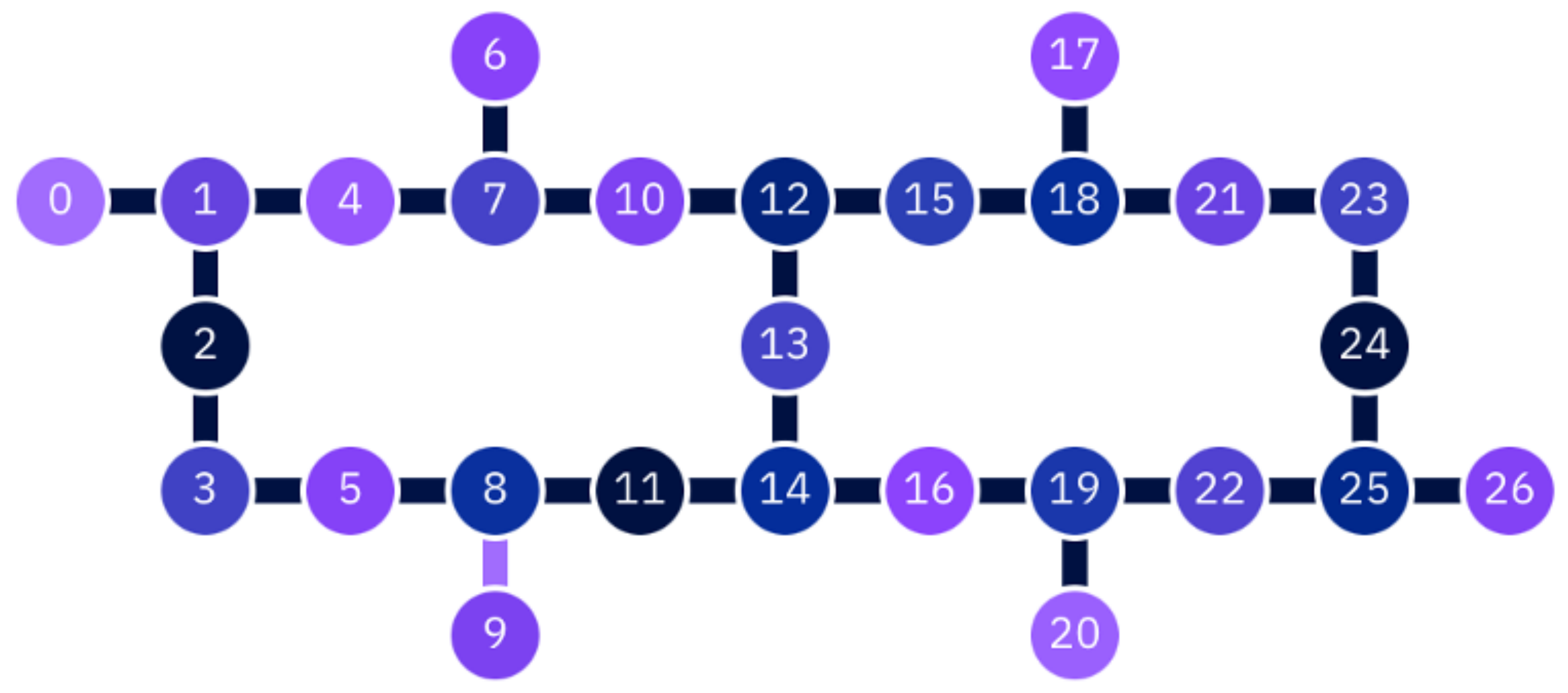
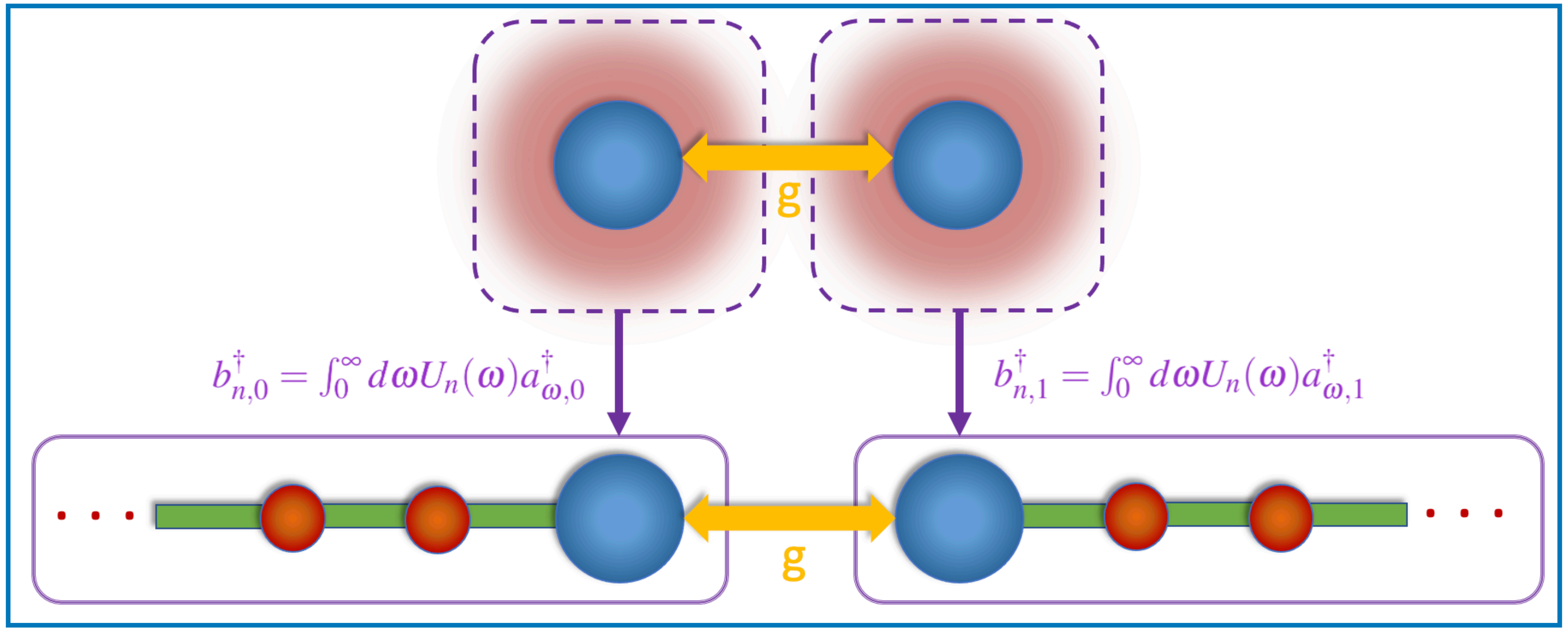




ibmq mumbai

Implementation of Q-TEDOPA

IBMQ superconducting processor



Implementation of Q-TEDOPA

IBMQ superconducting processor

$$J(\omega) = 2\pi\alpha\omega e^{-\omega/\omega_c}\theta(\omega)$$

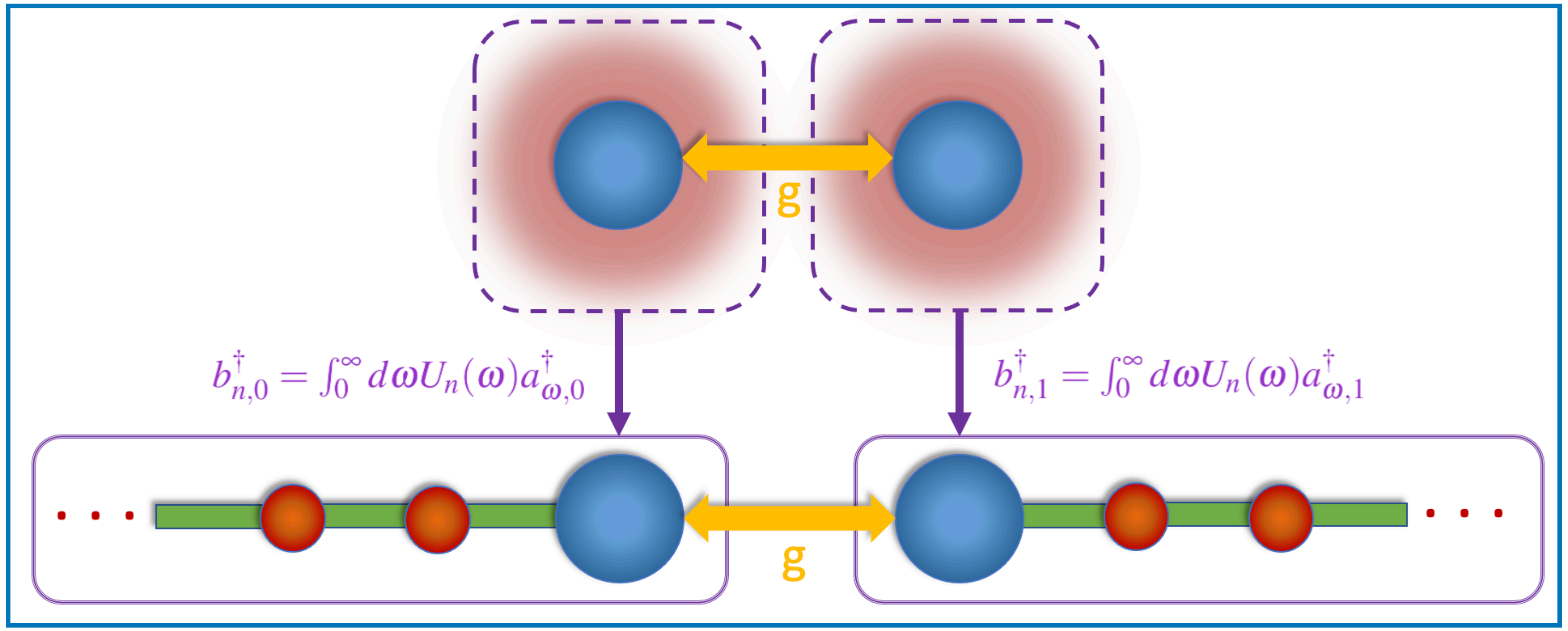
$$\alpha = 0.25, \quad \omega_c = 100cm^{-1}.$$

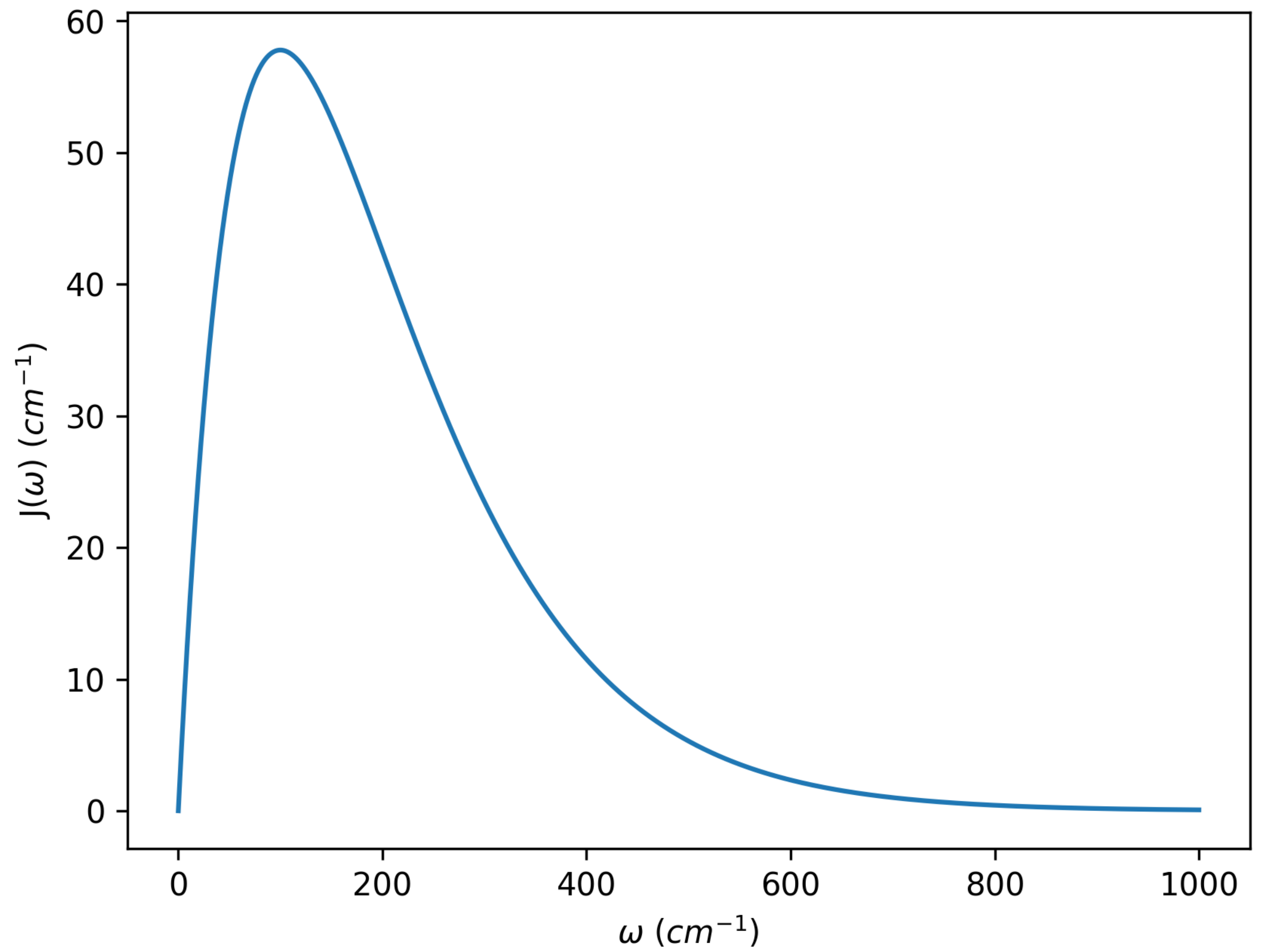
Moderate exciton-phonon coupling strength

$$g = 87.7cm^{-1}, \quad J(\Delta E) = g/2$$

Non-Markovian bath

$$J(\pm\Delta E) \not\approx const.$$





Implementation of Q-TEDOPA

IBMQ superconducting processor

- 2 molecules (qubits) + 5 oscillators (d=2) in each chain.
- Binary qubit encoding.
- 12 qubits, 10 iterations, ~400 CNOT gates.

$$J(\omega) = 2\pi\alpha\omega e^{-\omega/\omega_c}\theta(\omega)$$

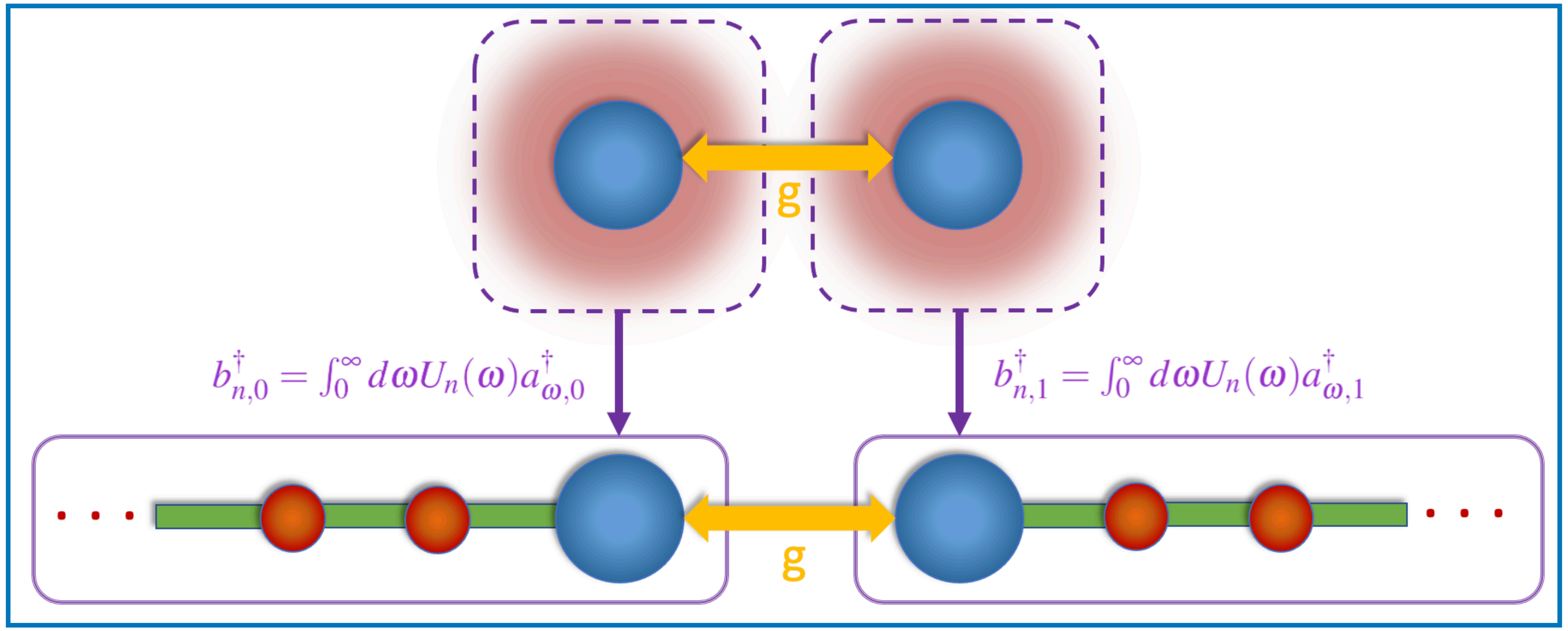
$$\alpha = 0.25, \quad \omega_c = 100cm^{-1}.$$

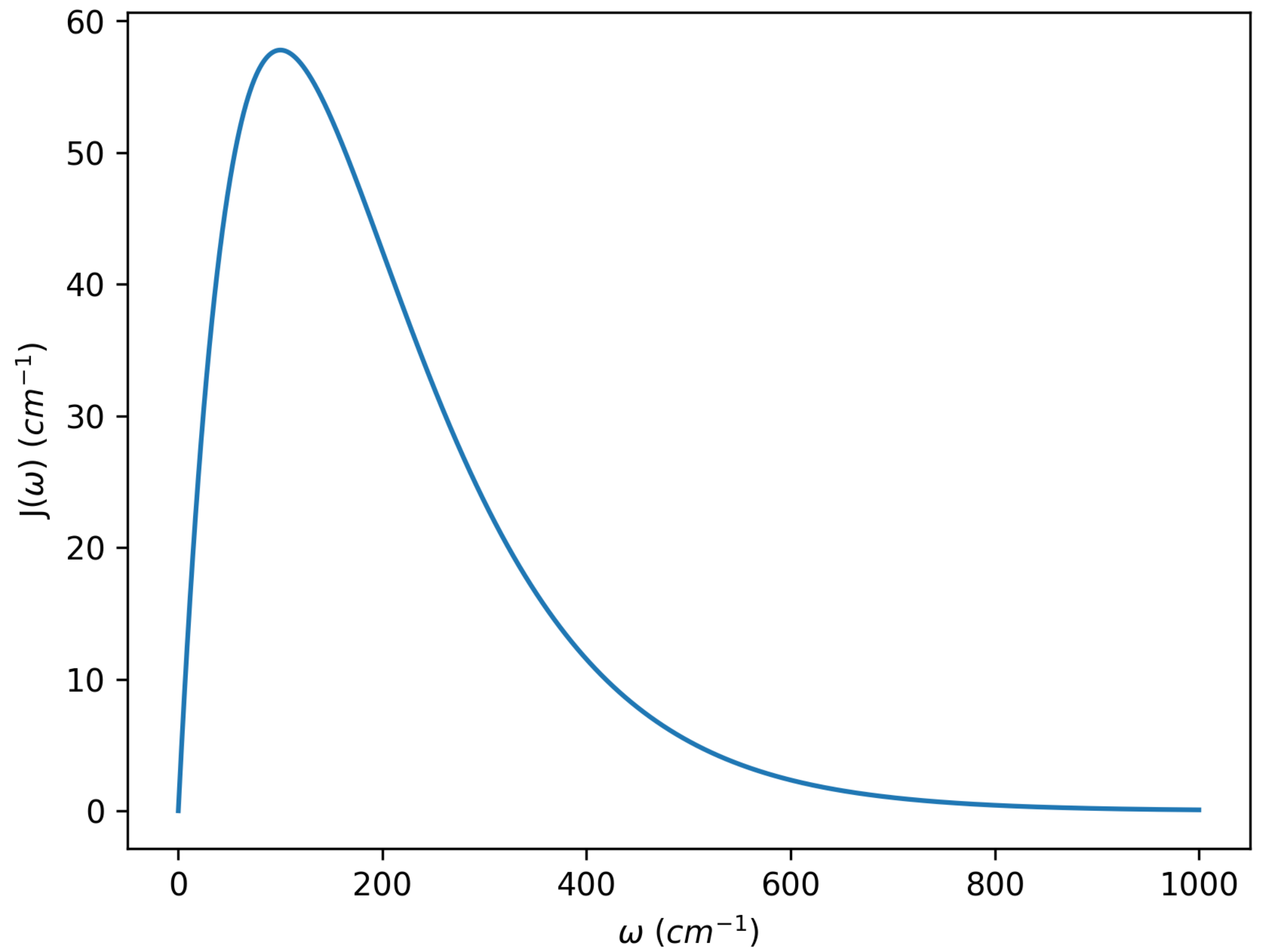
Moderate exciton-phonon coupling strength

$$g = 87.7cm^{-1}, \quad J(\Delta E) = g/2$$

Non-Markovian bath

$$J(\pm\Delta E) \not\approx const.$$





Implementation of Q-TEDOPA

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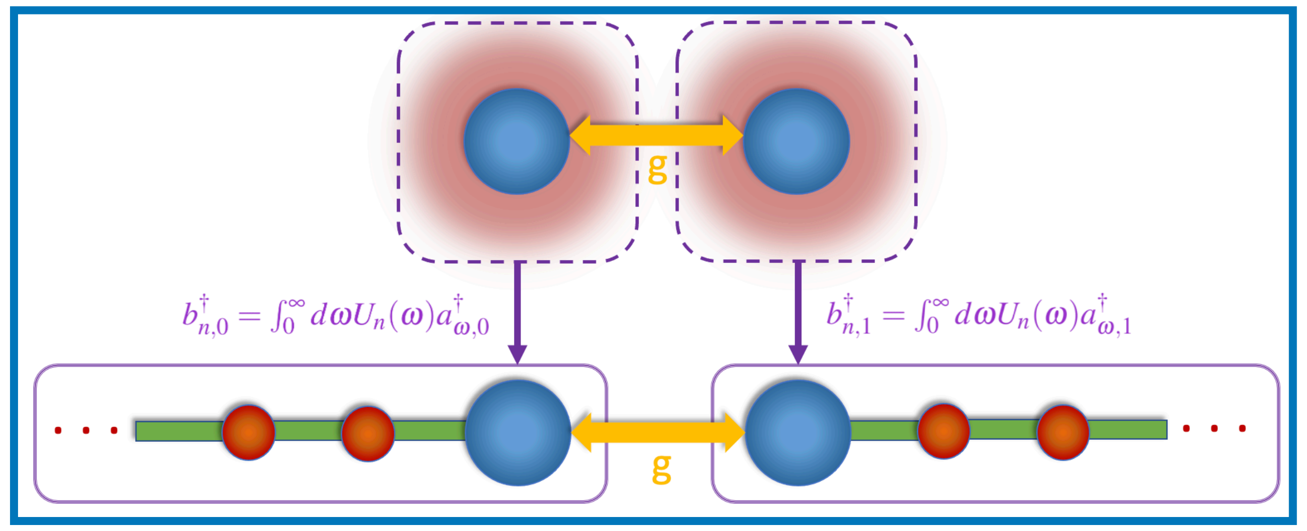

12 qubits, ~400 CNOT gates → **Quantum error mitigation**

1. Symmetry verification.
2. Qiskit Measurement Error Mitigation (MEM)
3. Randomized Compiling (RC).
4. Zero-Noise (exponential) Extrapolation (ZNE)

- Hashim, Akel, et al. *arXiv preprint arXiv:2010.00215* (2020).
- Endo, Suguru, Simon C. Benjamin, and Ying Li. *Physical Review X* 8.3 (2018): 031027.

Implementation of Q-TEDOPA

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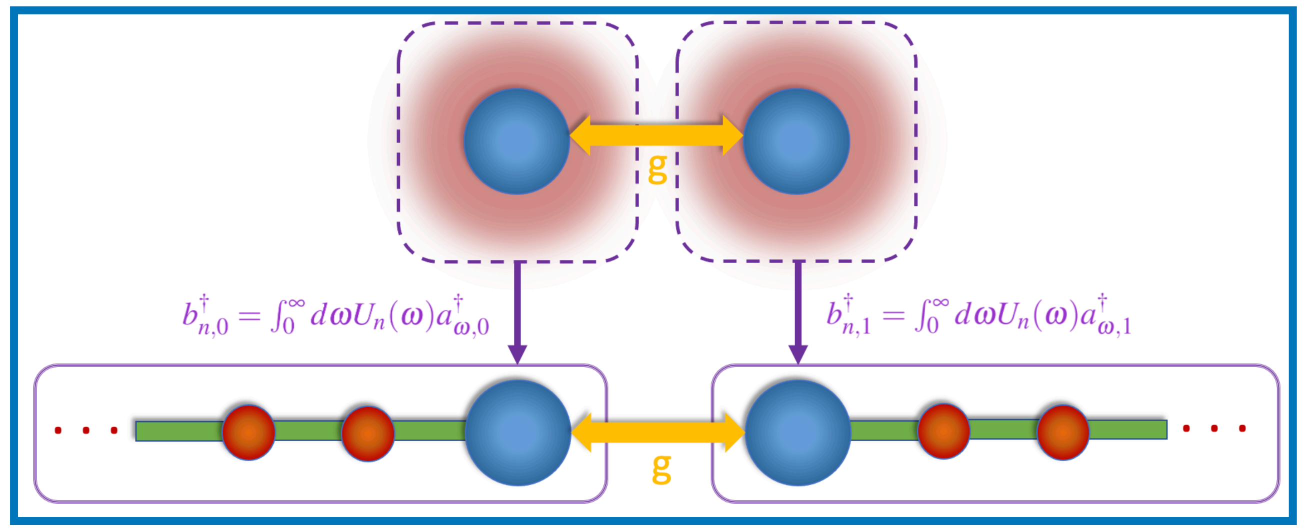
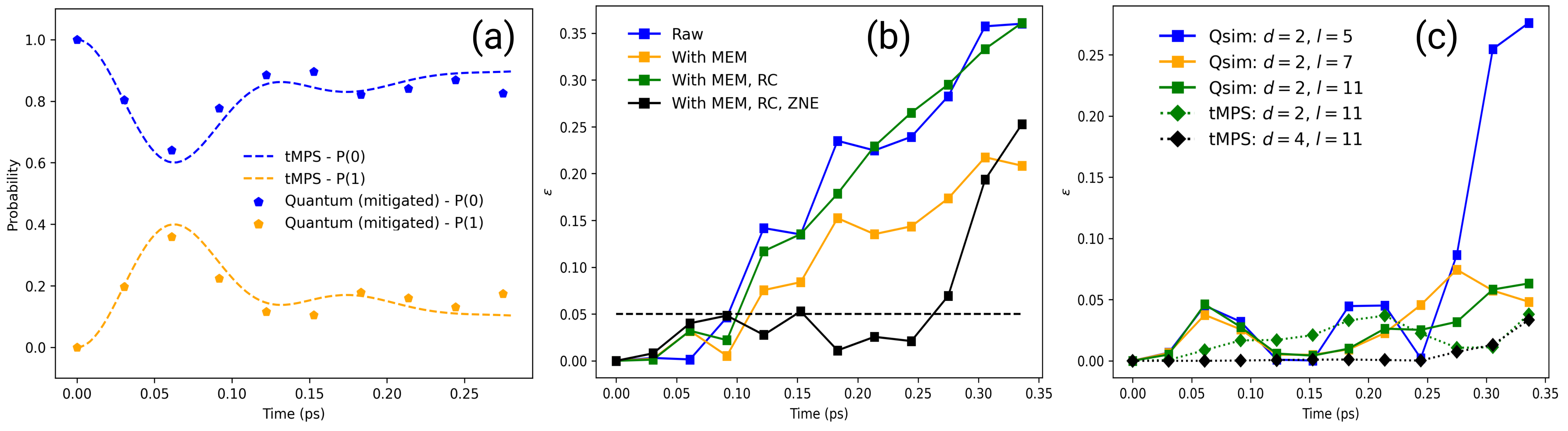


12 qubits, ~400 CNOT gates

$$\varepsilon = \sum_x |\mathscr{P}_{exact}(x) - \mathscr{P}_{sim}(x)| = 2d_{TV}$$



Part II

Quantum error mitigation

José D. Guimarães and Carlos Tavares | 2022

Quantum error mitigation (QEM)

Why?

- Quantum error correction is not “NISQ-friendly”.

Quantum error mitigation (QEM)

Why?

- Quantum error correction is not “NISQ-friendly”.
- Quantum error mitigation techniques rely on classical post-processing techniques to partially cancel noise!

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Quantum error mitigation (QEM)

Why?

- Quantum error correction is not “NISQ-friendly”.
- Quantum error mitigation techniques rely on classical post-processing techniques to partially cancel noise!
- Fast growing research field due to the quest for showing quantum advantage with NISQ devices.
- Every large quantum digital simulation requires QEM!

Quantum error mitigation

Towards a layered architecture for quantum error mitigation

- To be presented at IEEE Quantum Software 2022.
- Paper publication coming soon.

Towards a layered architecture for error mitigation in quantum computation

José D. Guimarães
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Carlos Tavares
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ctavares@inesctec.pt

***Abstract*—In the past few years, the first commercially available quantum computers have emerged, in an early stage of development, the so-called *Noisy Intermediate-Scale Quantum* (NISQ) era. Although these devices are still very prone to errors of different natures, they also have shown to deal successfully with small computational problems. Nowadays, one of the challenges in quantum computation is exactly to be able to show that quantum computers are useful, whereby mitigating the effects of the faulty hardware is pivotal. Recently, a wide range of quantum error mitigation techniques have been proposed and successfully implemented, alternative to quantum error correction codes. Herein, we discuss several types of noise in a quantum computer and techniques available to mitigate them, as well as their limitations and conditions of applicability. We also suggest an hierarchy for them, towards the conception of a layered software framework of error mitigation techniques, and implement some of them in a quantum simulation of the Heisenberg model on an IBM quantum computer, improving the fidelity of the simulation by 2.8x.**

***Index Terms*—Quantum computing, quantum error mitigation, IBMQ, quantum simulation.**

correction codes [8, 9], and new universal quantum computer models [8, 10].

Errors in quantum computations come from various sources, for instance, qubit relaxation, faulty implementation of quantum gates, unwanted *correlations* between the qubits in the quantum device, i.e., the so-called *cross-talking* [11], qubit readout errors, or errors induced by outside environment, which scramble the information about the state of the qubits, a process generally known as *decoherence* [12, 13].

Quantum error correction codes are the *canonical* way to suppress noise in quantum computers, however the high overhead of qubits necessary to perform the state correction has not yet been achieved in current quantum computers [14]. The study of quantum errors can be framed in the research field of *open quantum systems* [12], which has greatly contributed to further understanding of their nature, providing insight to the conception of more tailored solutions to their mitigation [15, 16]. As a result, there have been appearing

Quantum error mitigation

Towards a layered architecture for quantum error mitigation

- To be presented at IEEE Quantum Software 2022.
- Paper publication coming soon.
- Based on the IBM Quantum Summer Challenge 2021.
- Layered application of QEM techniques to a quantum simulation of the Heisenberg model.

Towards a layered architecture for error mitigation in quantum computation

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Quantum error mitigation

Layered architecture for QEM

Noise in a quantum computer

Blume-Kohout, Robin, et al. *PRX Quantum* 3.2 (2022): 020335.

SPAM errors | Coherent noise | Incoherent Markovian noise

QEM techniques:

Quantum error mitigation

Layered architecture for QEM

Noise in a quantum computer

	SPAM errors	Coherent noise	Incoherent Markovian noise
QEM techniques:	Qiskit Measurement error mitigation	Randomized Compiling, Dynamical Decoupling, Floquet calibration, symmetry-averaging.	Zero-Noise Extrapolation, Quasi-probability method, Virtual Distillation, symmetry verification

- Qiskit documentation

- Hashim, Akel, et al., Phys. Rev. X 11, 041039 (2021)
- Tripathi, Vinay, et al. *arXiv preprint arXiv:2108.04530* (2021).
- Arute, Frank, et al. *arXiv preprint arXiv:2010.07965* (2020).
- Stanisic, Stasja, et al. *arXiv preprint arXiv:2112.02025* (2021).

- Endo, Suguru, Simon C. Benjamin, and Ying Li. *Physical Review X* 8.3 (2018): 031027.
- Huggins, William J., et al. *Physical Review X* 11.4 (2021): 041036
- Sun, Jinzhao, et al. *Physical Review Applied* 15.3 (2021): 034026
- Endo, Suguru, et al. *Journal of the Physical Society of Japan* 90.3 (2021): 032001.

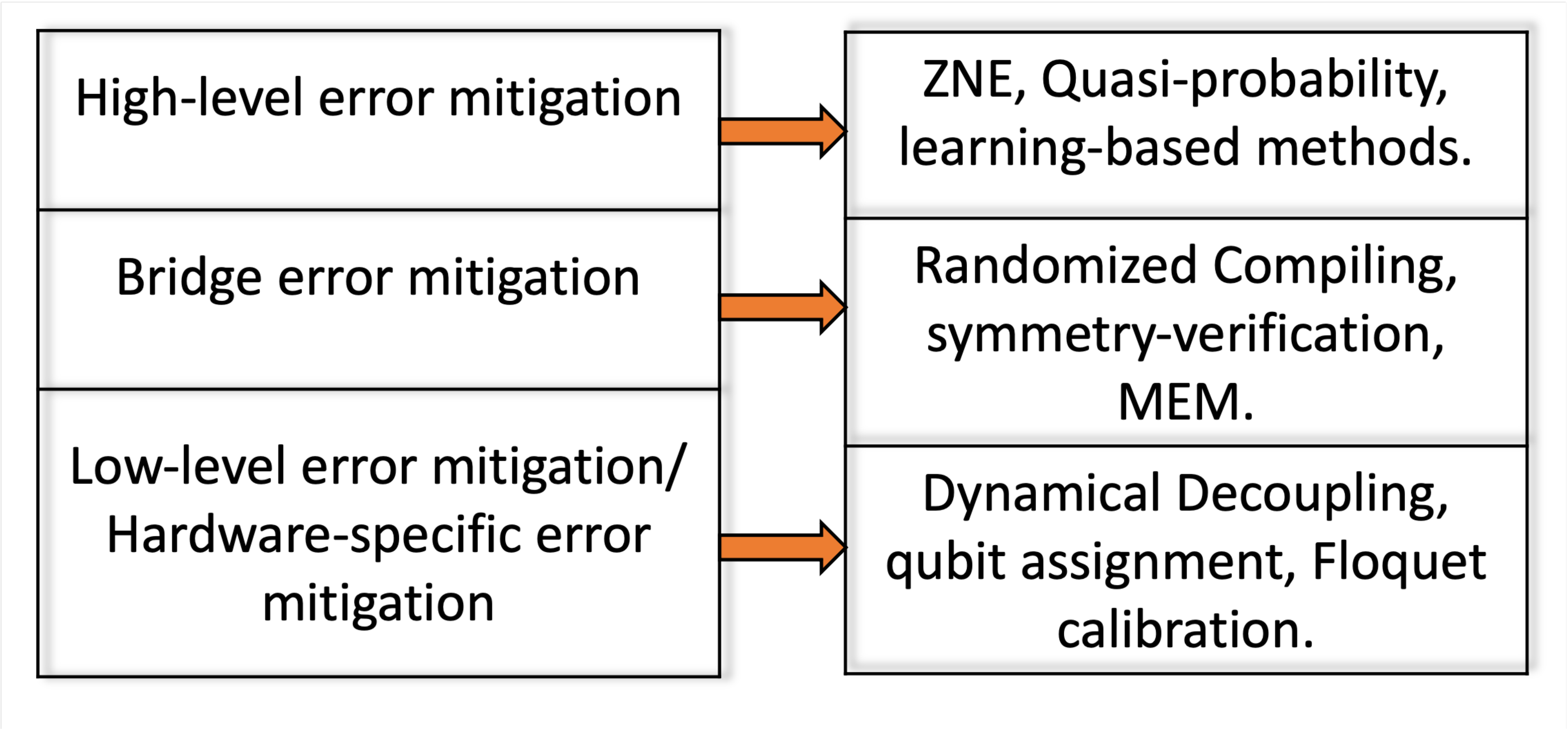
Others: Entanglement forging, qubit assignment, etc.

- Eddins, Andrew, et al. *PRX Quantum* 3.1 (2022): 010309.
- Arute, Frank, et al. *arXiv preprint arXiv:2010.07965* (2020).

Quantum error mitigation

Layered architecture for QEM

Our classification of QEM techniques



Coherent noise

Randomized Compiling, Dynamical Decoupling, Floquet calibration.

Incoherent Markovian noise

Zero-Noise Extrapolation (ZNE), Quasi-probability method, learning-based methods, symmetry verification.

SPAM errors

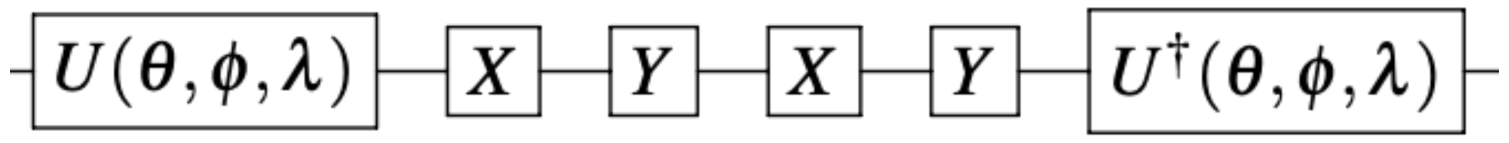
Qiskit Measurement error mitigation (MEM)

Quantum error mitigation

Dynamical Decoupling

Application of consecutive pulses to mitigate qubit cross-talk and decoherence.

- XY4, XX and XY8 sequences.



- Pokharel, Bibek, et al. *Physical review letters* 121.22 (2018): 220502.
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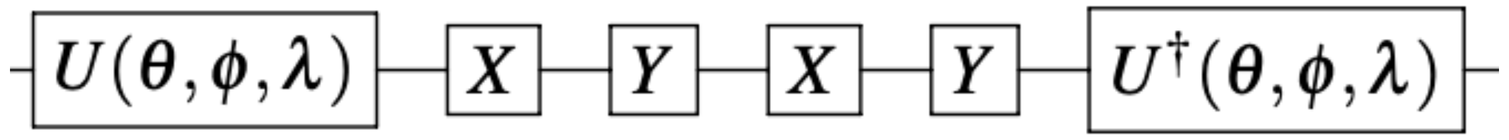

- Short sequences (and short intervals of time) are preferable.

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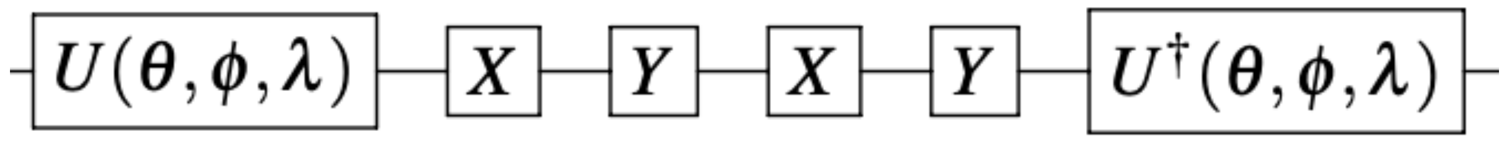

- Short sequences (and short intervals of time) are preferable.

- Improve the technique by using *Correlated Phase Randomization* or *Adaptive Dynamical Decoupling* methods.

Gautam, Akanksha, and Kavita Dorai. *arXiv preprint arXiv:2112.10417* (2021).

Das, Poulami, et al. *MICRO-54: 54th Annual IEEE/ACM International Symposium on Microarchitecture*. 2021.

- Pokharel, Bibek, et al. *Physical review letters* 121.22 (2018): 220502.
- Tripathi, Vinay, et al. *arXiv preprint arXiv:2108.04530* (2021).

Quantum error mitigation

Randomized Compiling

Applies a Pauli channel to every sub-circuit C

$$\mathscr{P}(C) = \sum_j P_j C P_j$$

Tailors coherent noise into stochastic Pauli noise

Quantum error mitigation

Randomized Compiling

Applies a Pauli channel to every sub-circuit C

$$\mathscr{P}(C) = \sum_j P_j C P_j$$

Sub-circuit = Two-qubit gate layer

CNOT error ~ 10^{-2}
Single-qubit gate error ~ 10^{-4}

Quantum error mitigation

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Hashim, Akel, et al., Phys. Rev. X 11, 041039 (2021)

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In a quantum circuit

$$C \rightarrow P_j C P_j', \quad P_j' = C^\dagger P_j C$$

Quantum error mitigation

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$$\mathscr{P}(C) = \sum_j P_j C P_j$$

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Sampled from the Pauli group

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Quantum error mitigation

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$$\mathscr{P}(C) = \sum_j P_j C P_j$$

In a quantum circuit

Sampled from the Pauli group

$$C \rightarrow P_j C P'_j, \quad P'_j = C^\dagger P_j C$$

Generate *L* Pauli-sampled circuits and average the results.

Quantum error mitigation

Randomized Compiling

Applies a Pauli channel to every two-qubit gate layer C

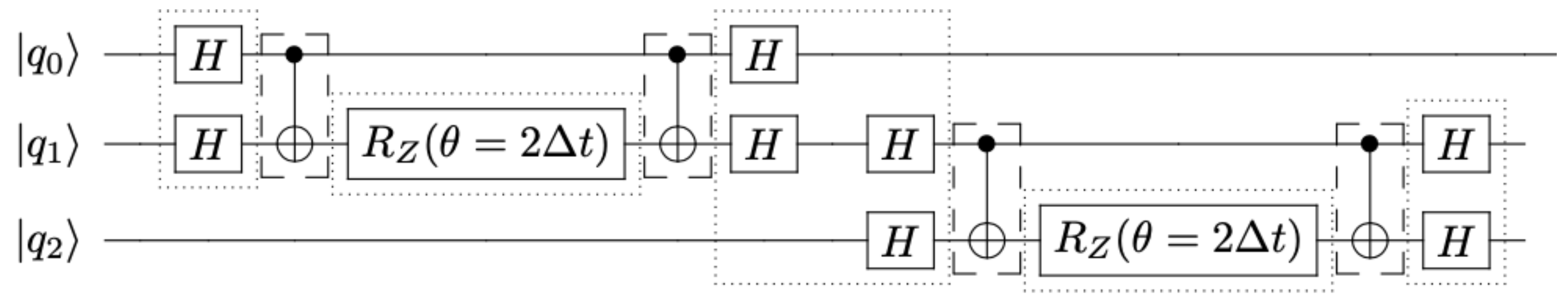
$$\mathscr{P}(C) = \sum_j P_j C P_j$$

In a quantum circuit

Sampled from the Pauli group

$$C \rightarrow P_j C P_j', \quad P_j' = C^\dagger P_j C$$

We applied Randomized Compiling to all CNOT gates in the circuits!



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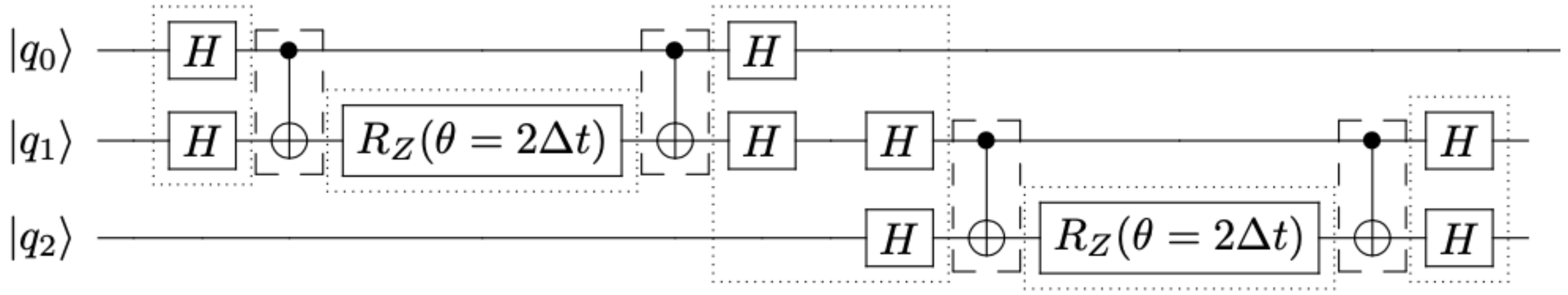
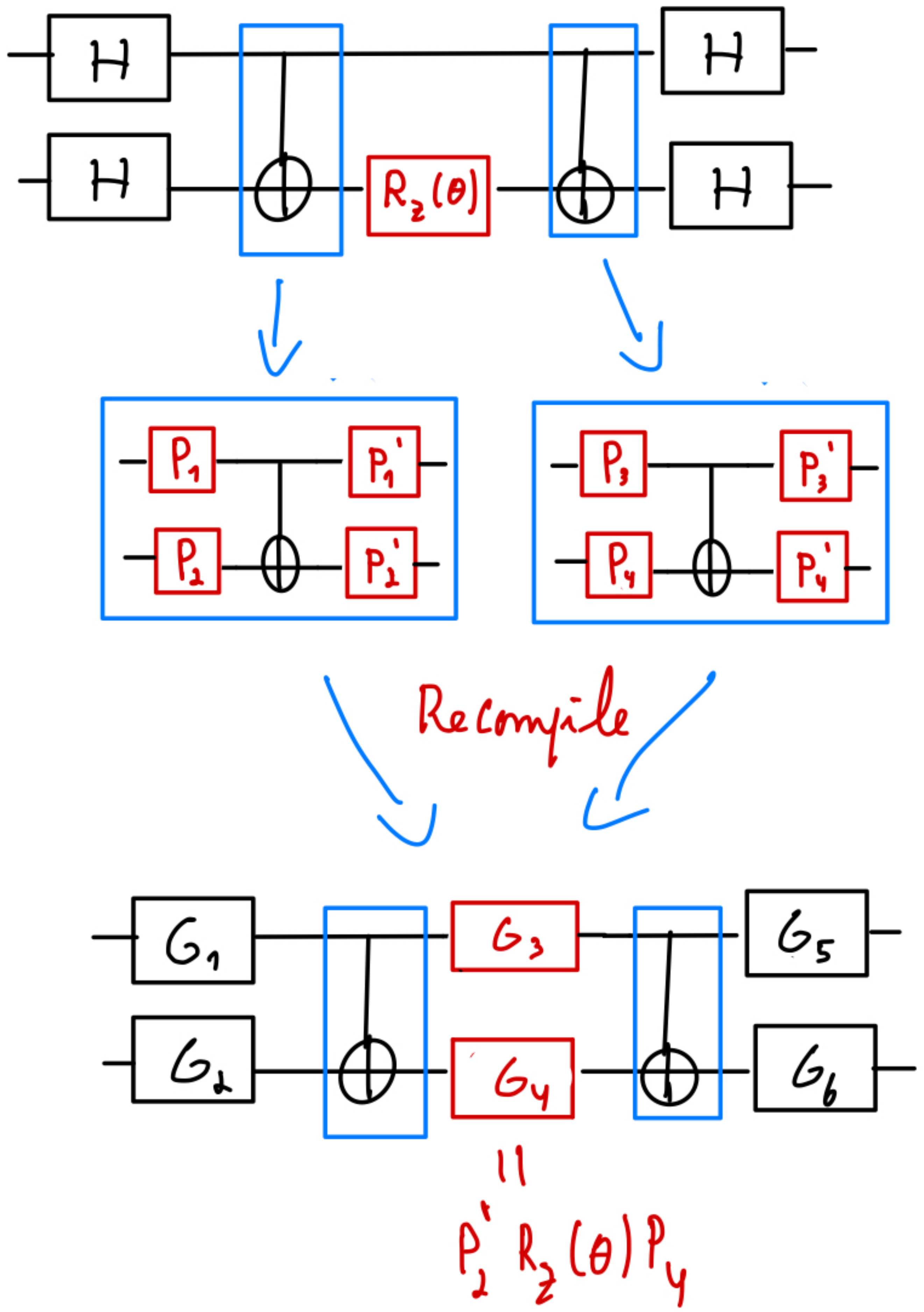




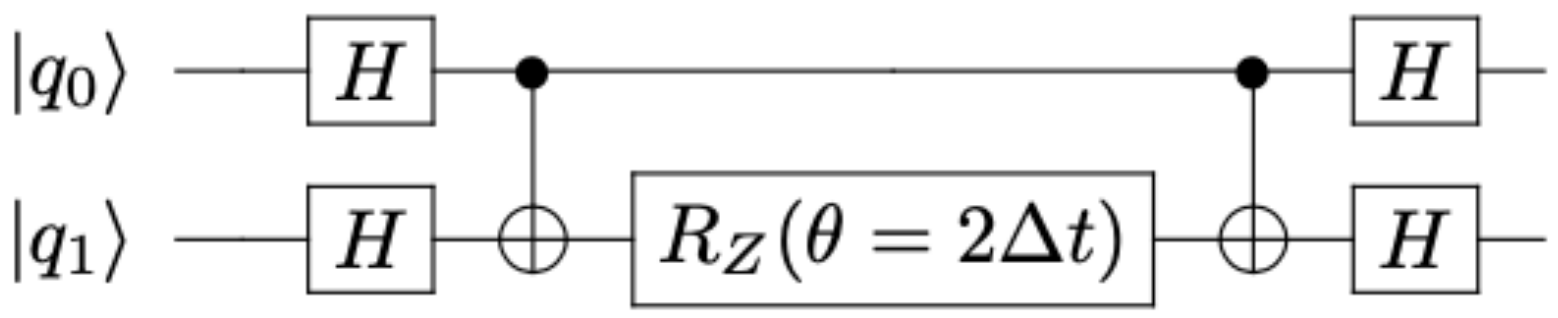
Quantum error mitigation

Zero-Noise extrapolation

1. Execute the quantum circuit.

Endo, Suguru, Simon C. Benjamin, and Ying Li. *Physical Review X* 8.3 (2018): 031027.

Average error $\mathcal{E}$



Output: $\langle O(\varepsilon)\rangle$

Quantum error mitigation

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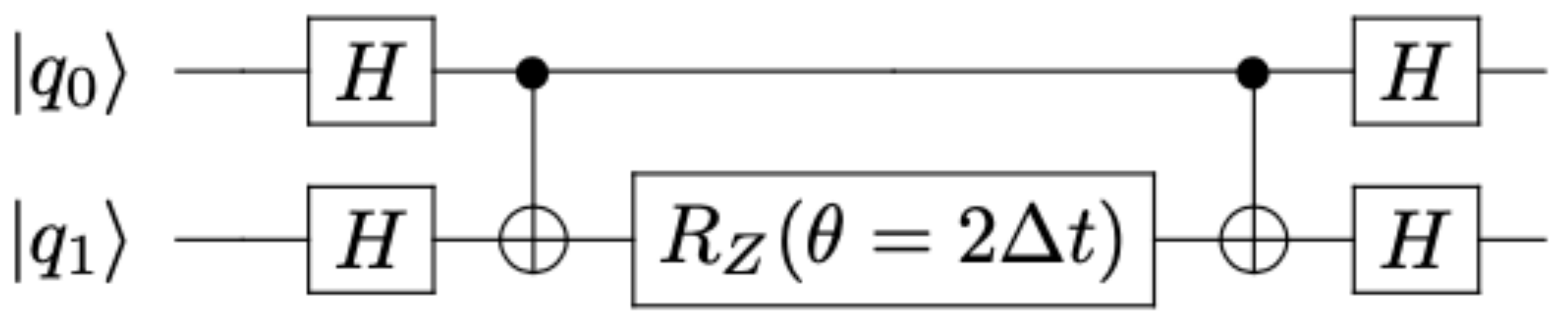
Average error ε

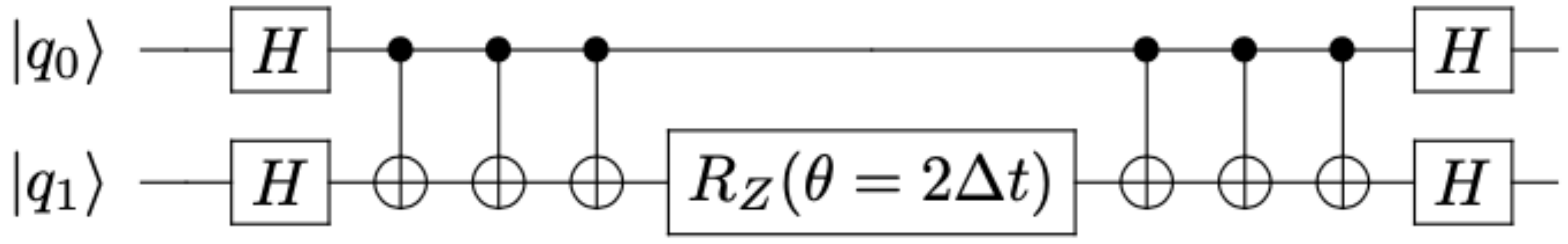


Output: $\langle O(\varepsilon)\rangle$

2. Execute the quantum circuit with increased average error rate.

Average error

$$\varepsilon' = 3\varepsilon$$



Output: $\langle O(\varepsilon')\rangle$

Quantum error mitigation

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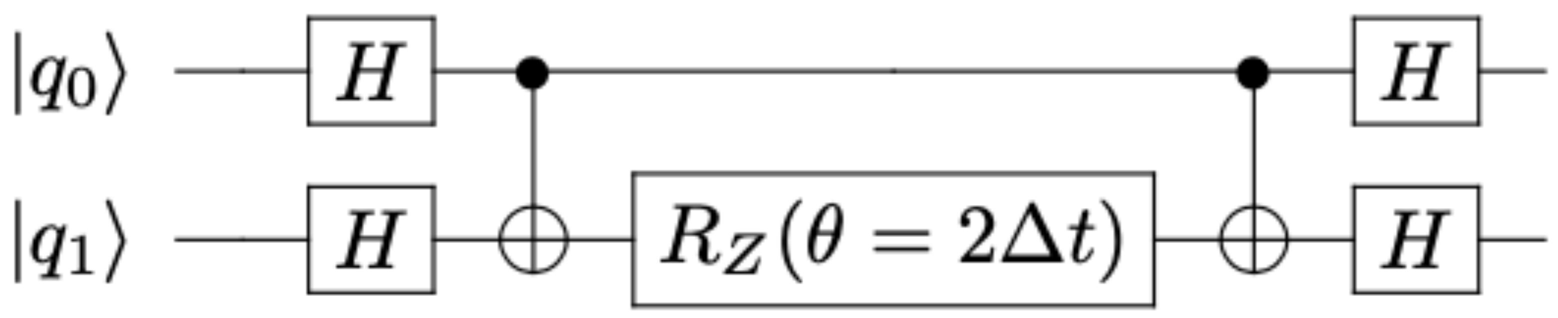
Endo, Suguru, Simon C. Benjamin, and Ying Li. *Physical Review X* 8.3 (2018): 031027.

Average error ε



Output: $\langle O(\varepsilon) \rangle$

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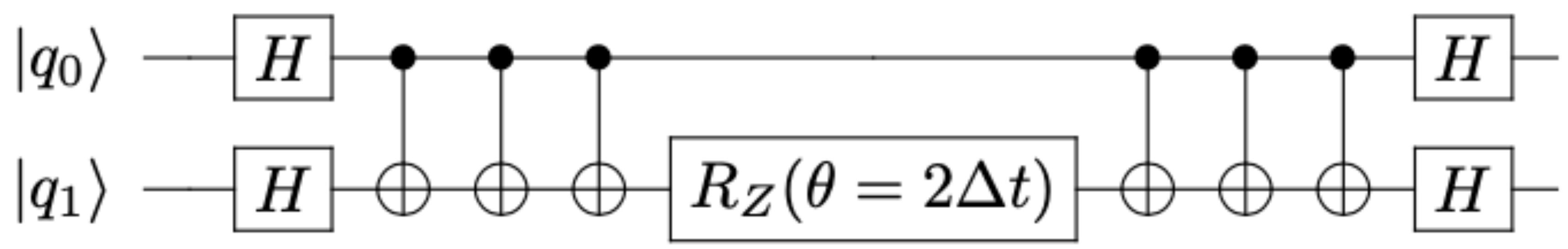


Output: $\langle O(\varepsilon') \rangle$

Methods to increase the error rate

Pascuzzi, Vincent R., et al. *Physical Review A* 105.4 (2022): 042406.

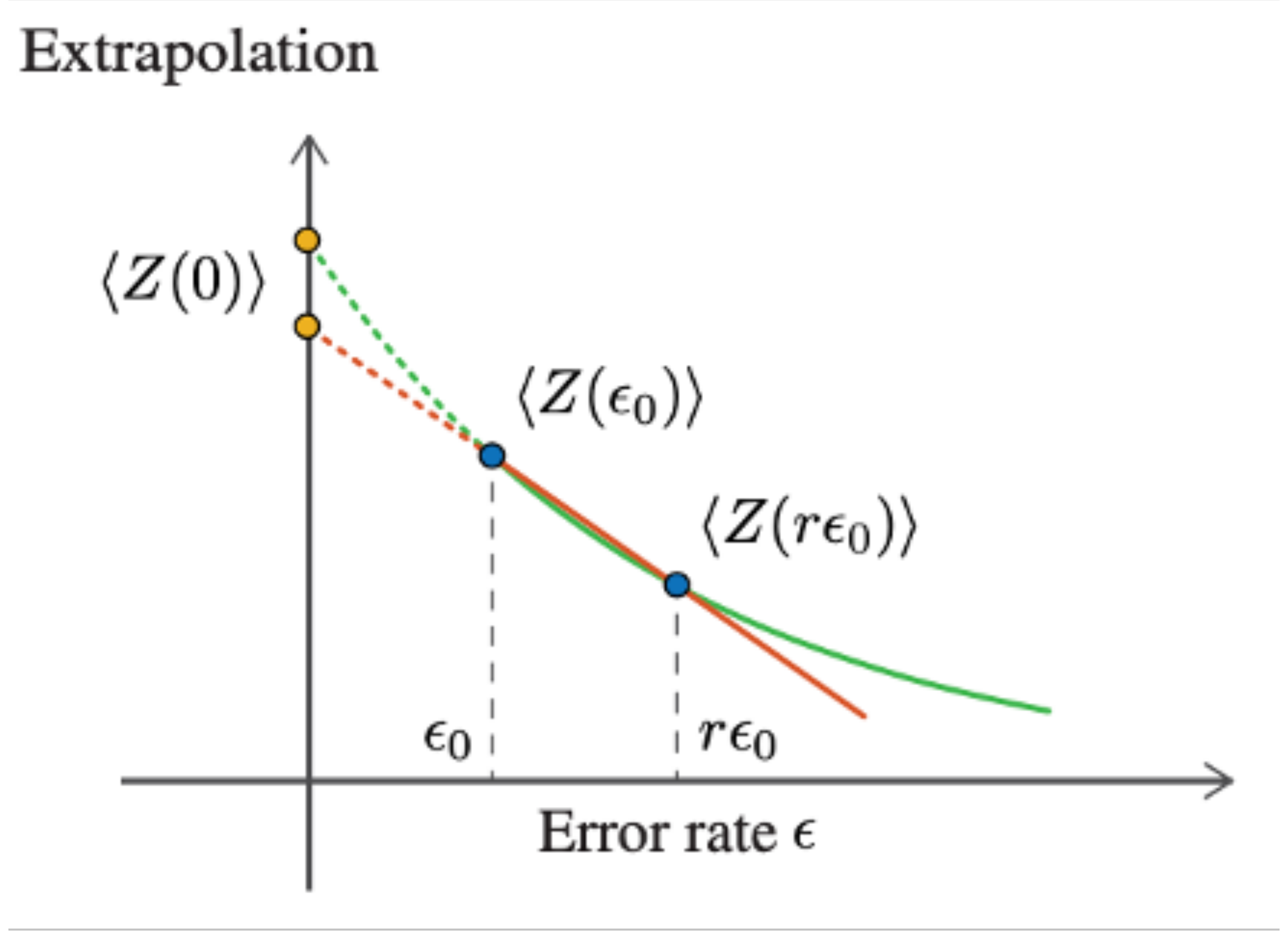
Fixed Identity Insertion Method (FIIM) or Random Identity Insertion Method (RIIM)

Quantum error mitigation

Zero-Noise extrapolation

3. Extrapolate to the Zero-Noise result $\langle O(0) \rangle$.

Endo, Suguru, Simon C. Benjamin, and Ying Li. *Physical Review X* 8.3 (2018): 031027.



Quantum error mitigation
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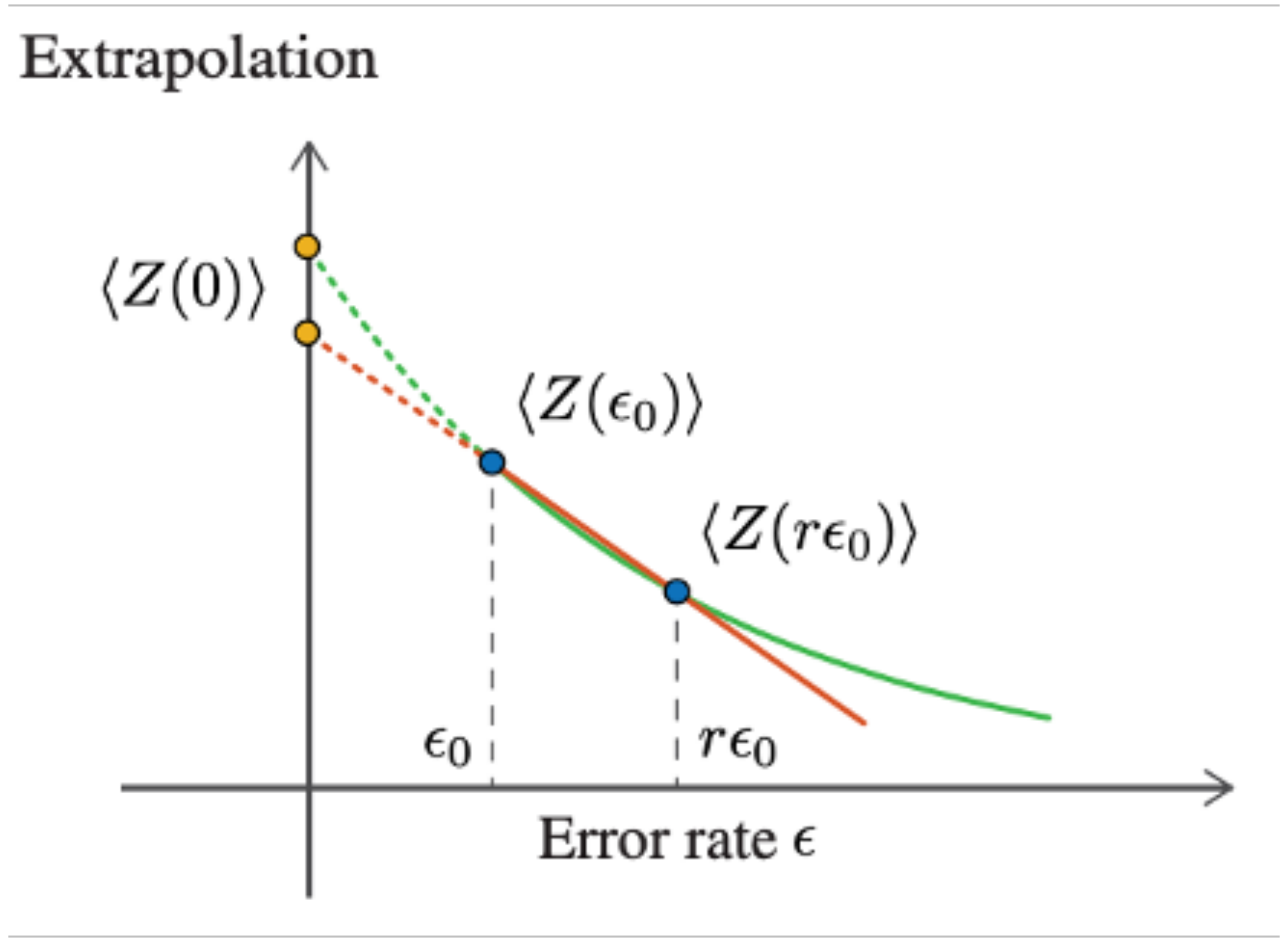
Ansatz to use?

- Linear extrapolation.
- Richardson extrapolation.
- Exponential extrapolation.



Giurgica-Tiron, Tudor, et al. *2020 IEEE International Conference on Quantum Computing and Engineering (QCE)*. IEEE, 2020.

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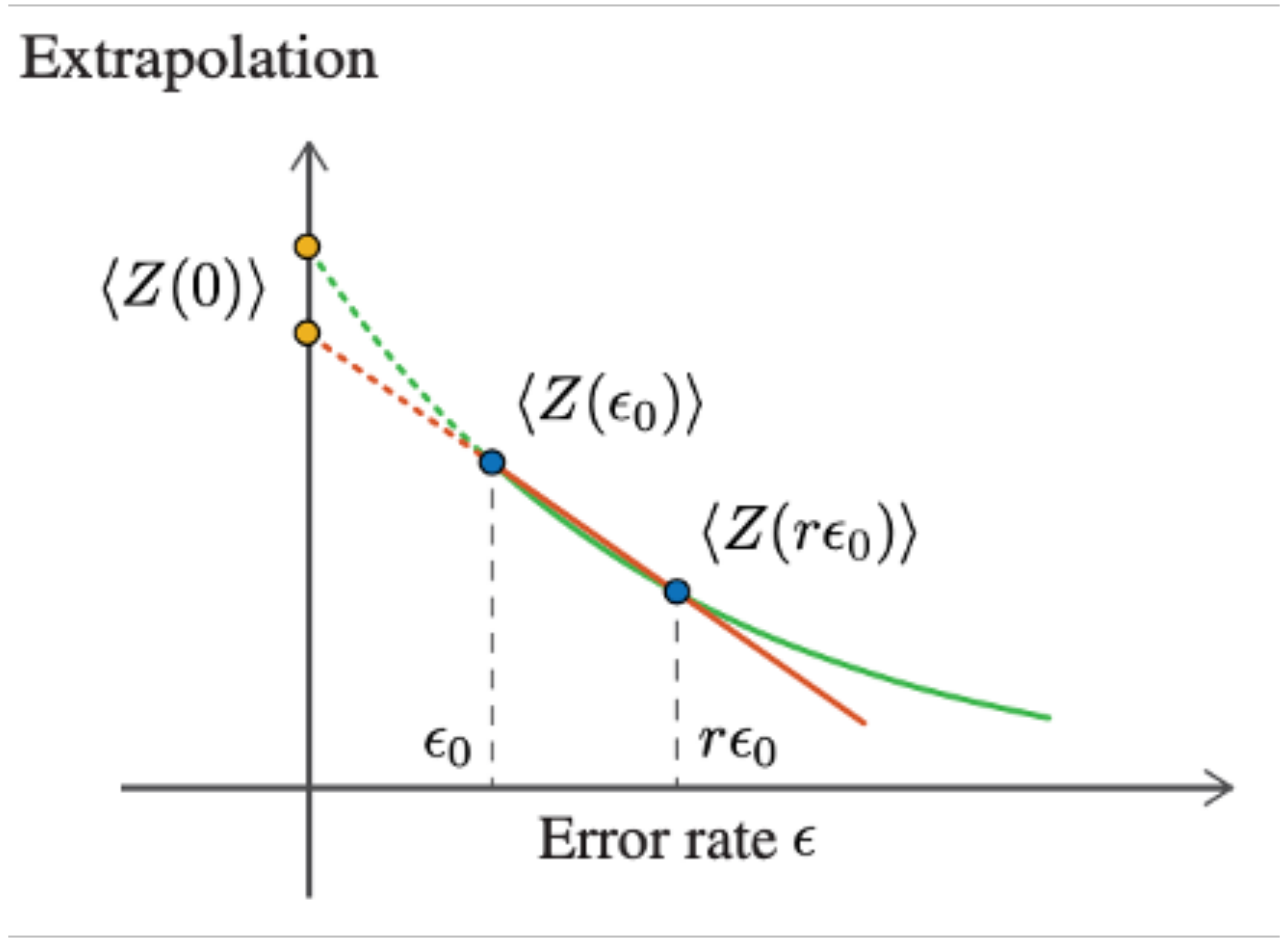

Very efficient at mitigating incoherent (stochastic) noise!

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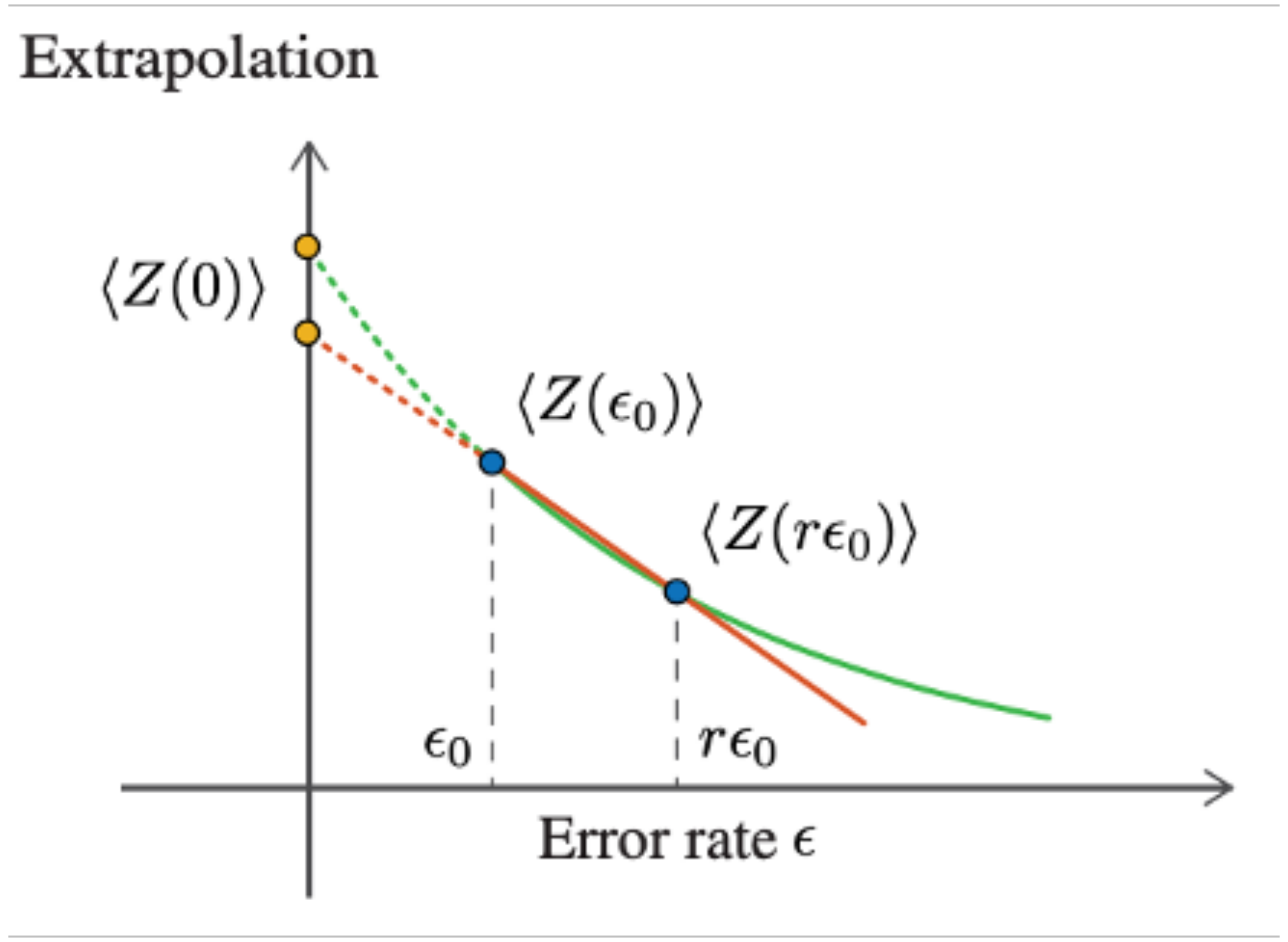

Very efficient at mitigating incoherent (stochastic) noise!

Agnostic to the underlying noise model in the quantum computer!

Heisenberg model simulation

Outline

- Hamiltonian:

$$H_{Heis} = J \sum_{\langle ij \rangle}^{N} \left(X_i X_j + Y_i Y_j + Z_i Z_j \right),$$

$$H_{Heis3} = X_0 X_1 + X_1 X_2 + Y_0 Y_1 + Y_1 Y_2 + Z_0 Z_1 + Z_1 Z_2 \,.$$

Heisenberg model simulation

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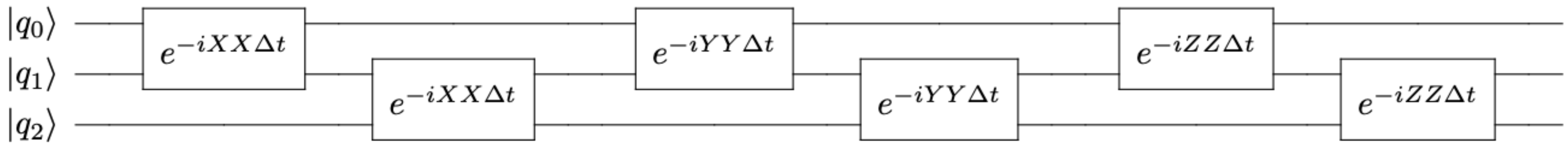
- Trotter decomposition of the evolution operator (4 iterations).

$$U_{Heis3}(t) \approx \left[\exp\left(\frac{-it}{n} H_{Heis3}^{(ZZ)} \right) \exp\left(\frac{-it}{n} H_{Heis3}^{(YY)} \right) \exp\left(\frac{-it}{n} H_{Heis3}^{(XX)} \right) \right]^n$$



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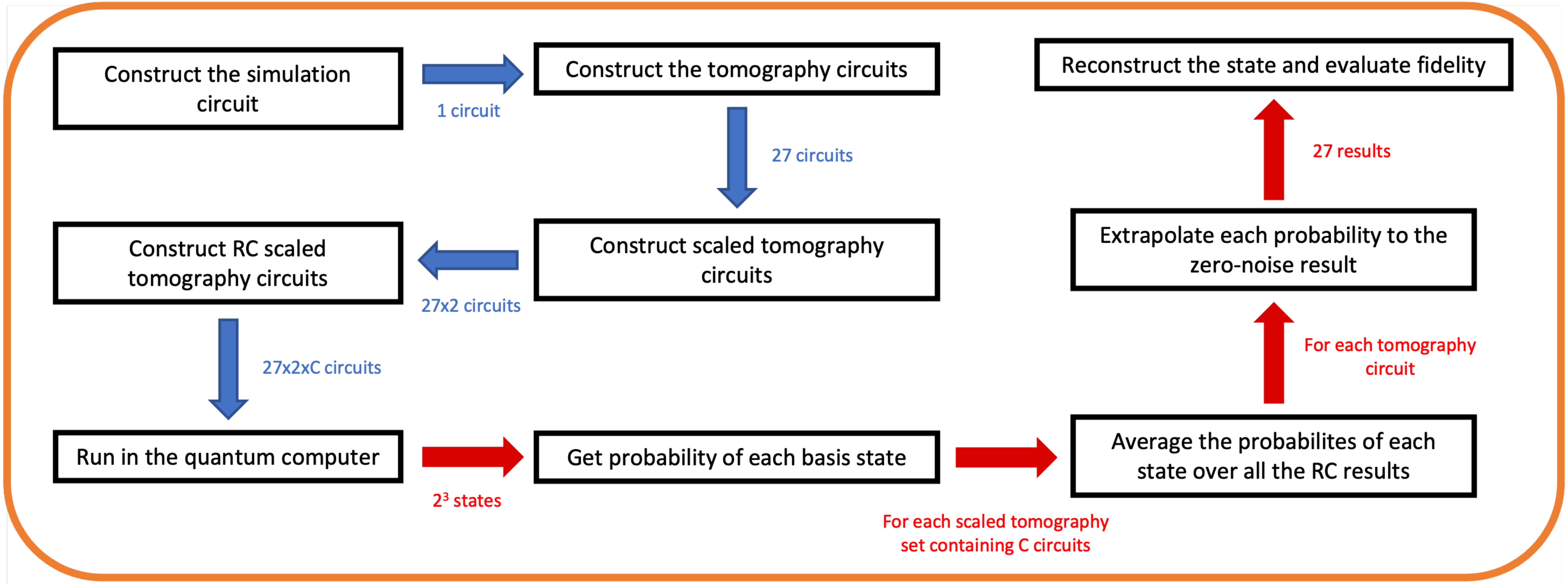
- Implement state tomography and compute fidelity of the simulation.

Heisenberg model simulation

Workflow



Heisenberg model simulation

Discussion

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Heisenberg model simulation

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Heisenberg model simulation

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Heisenberg model simulation

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- **Overall improvement:** $F = 0.2782 \rightarrow 0.7866.$

Thanks for your attention!

José D. Guimarães | 2022

Universidade do Minho

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LABORATORY