On Pauli-based computation

Group: Quantum and Linear Optical Computation (INL)
PhD Project: Optimizing models of hybrid quantum/classical computation

F. C. R. Peres | 7th of July 2021

Supervisor: Professor Ernesto Galvão
Co-supervisor: Professor João Lopes dos Santos

Models of Quantum Computation

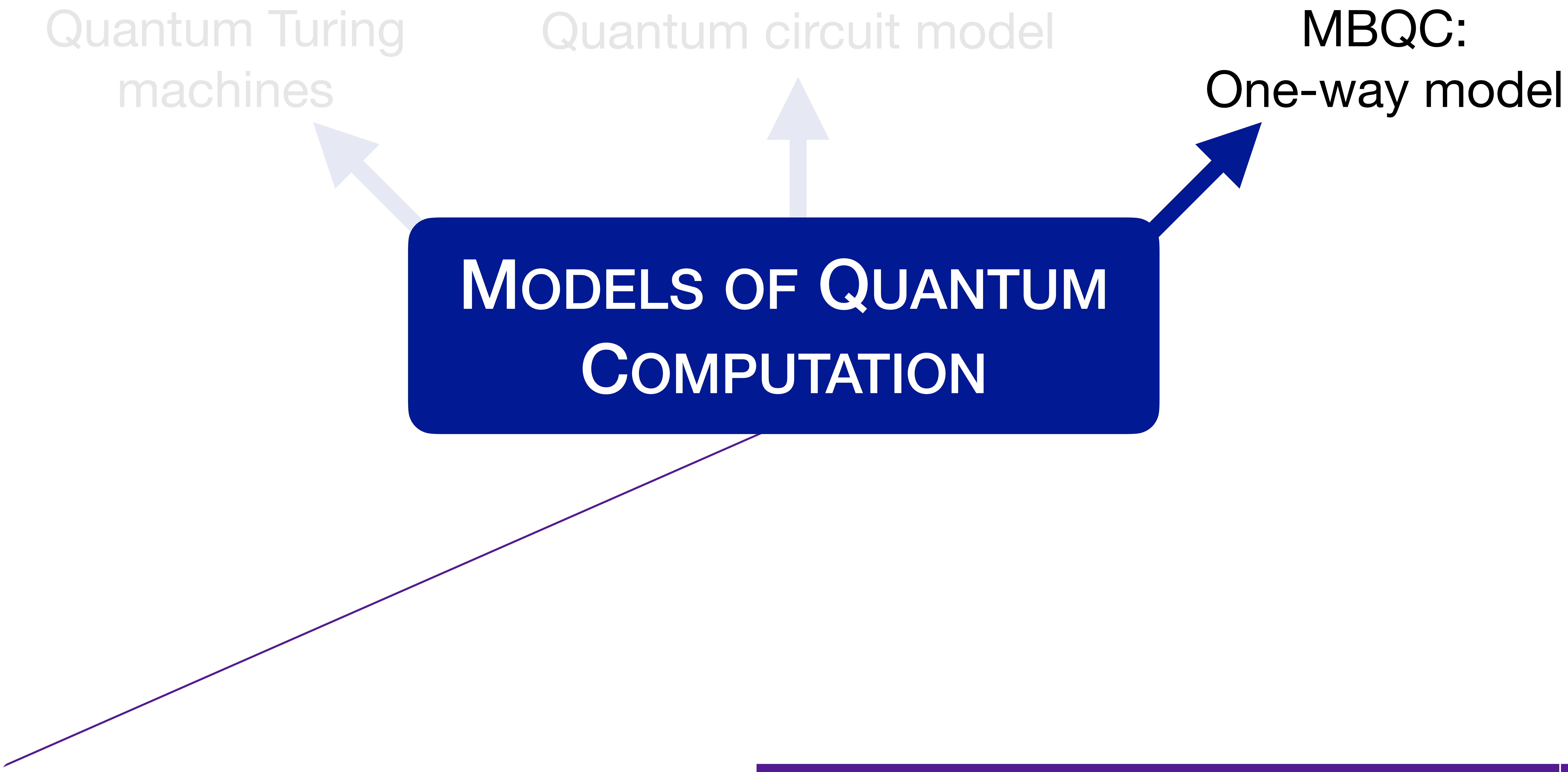
Quantum Turing machines

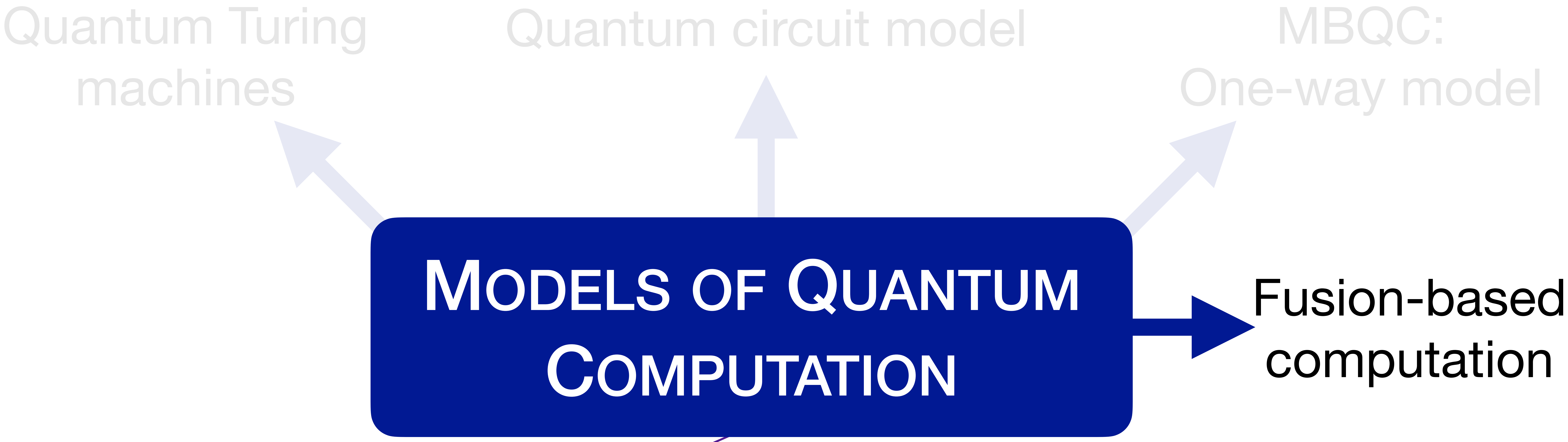
Models of Quantum Computation

Quantum Turing machines

Quantum circuit model

Models of Quantum Computation


Quantum Turing machines
Quantum circuit model
MBQC:
One-way model
MODELS OF QUANTUM COMPUTATION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Fusion-based computation

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MBQC:
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Boson sampling

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Quantum Turing machines

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Models of Quantum Computation

Fusion-based computation

MBQC: Pauli-based computation


Why...Why...


Why...Why...

- Different models highlight different features

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➡ one-way model:

✓ forefront role of entanglement.

Presentation overview

1. Introductory concepts

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2. PBC: universality and resource minimization

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2. PBC: universality and resource minimization

3. PBC and hybrid computation

Key definition: [PAULI OPERATORS ON n QUBITS]

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Concept: [Pauli group on n qubits]

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Example: [Pauli Group - $n = 1$]

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$$\mathcal{P}_1 = \langle X, \ Y, \ Z \rangle$$

or

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$$\mathscr{P}_1 = \langle X, \ Y, \ Z \rangle$$

or

$$\mathscr{P}_1 = \langle X, Z \rangle \text{ \& phase } i$$

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$2n$ generators

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$$\mathscr{P}_n = \langle X_1, X_2, \ldots, X_n, Z_1, Z_2, \ldots, Z_n \rangle$$

Definition: [CLIFFORD UNITARY]

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$$C\mathscr{P}_n C^\dagger = \mathscr{P}_n \Leftrightarrow CP_iC^\dagger = P_j\,.$$

Generators:

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- Hadamard (H);

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- Controlled-NOT (CX) gates.

Action of the Hadamard gate:

$$X \longrightarrow HXH^\dagger = Z$$

$$H = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$Z \longrightarrow HZH^\dagger = X$$

Action of the phase gate:

$$X \longrightarrow SXS^{\dagger} = Y$$

$$S = \mathrm{diag}\,(1, i)$$

$$Z \longrightarrow SZS^{\dagger} = Z$$

Action of the controlled-NOT gate:

$$(X \otimes I) \longrightarrow (X \otimes X)$$

$$(I \otimes X) \longrightarrow (I \otimes X)$$

$$CX_{12} = \begin{pmatrix} I & 0 \\ 0 & X \end{pmatrix}$$

$$(Z \otimes I) \longrightarrow (Z \otimes I)$$

$$(I \otimes Z) \longrightarrow (Z \otimes Z)$$

Definition: [Stabilizer state of n qubits]

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$$P_i |\psi\rangle = |\psi\rangle, \ \forall P_i \in \mathcal{S} = \langle P_1, \ldots, P_n \rangle$$

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$$\frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

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$$\mathcal{S} = \langle X \otimes X,\, Z \otimes Z \rangle$$

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$$|0\rangle \otimes \frac{|0\rangle + e^{i\pi/2}|1\rangle}{\sqrt{2}}$$

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$$|0\rangle \otimes \frac{|0\rangle + e^{i\pi/2}|1\rangle}{\sqrt{2}} \qquad \mathcal{S} = \langle Z \otimes I, I \otimes Y \rangle$$

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Eigenvector of

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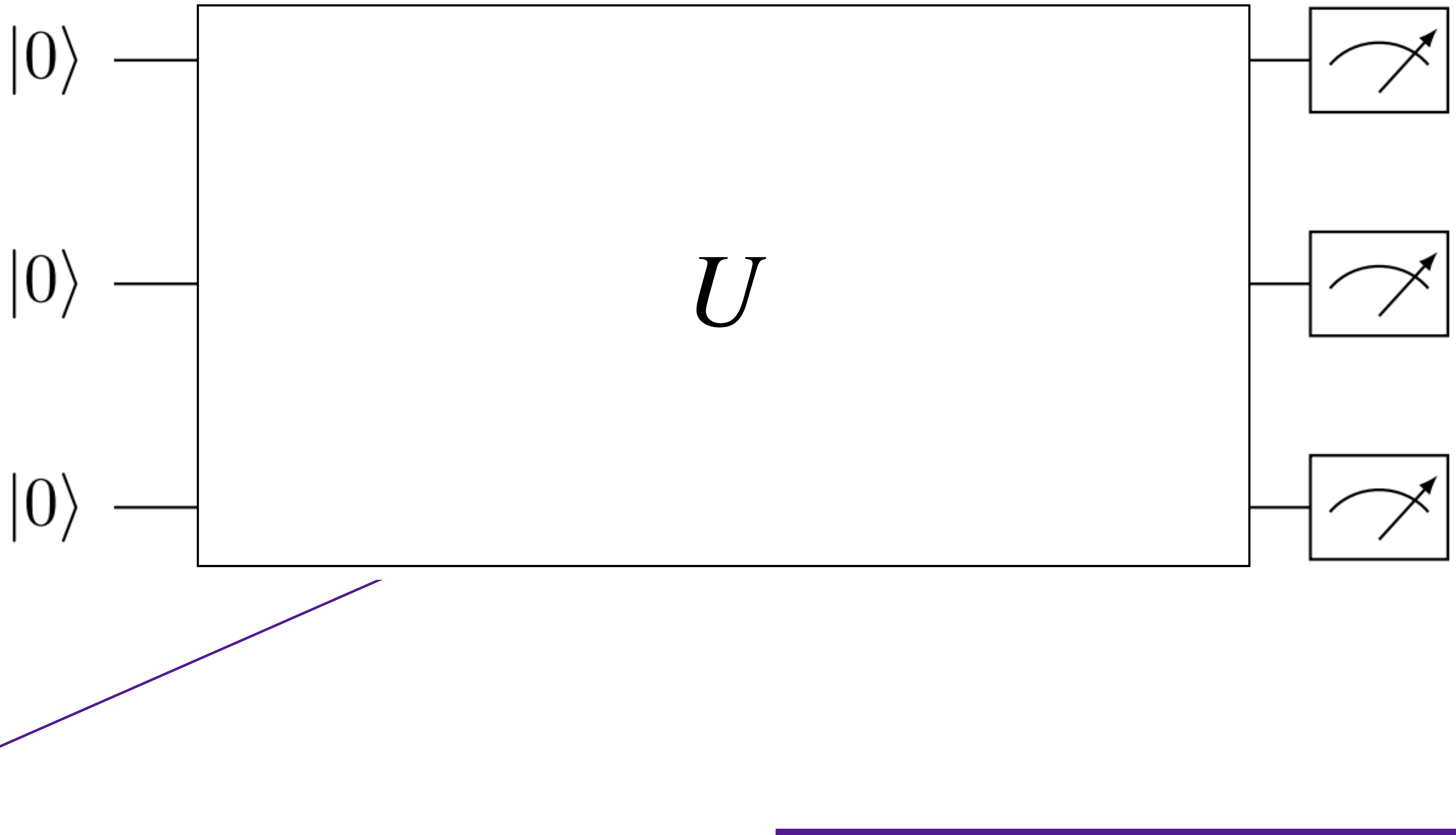
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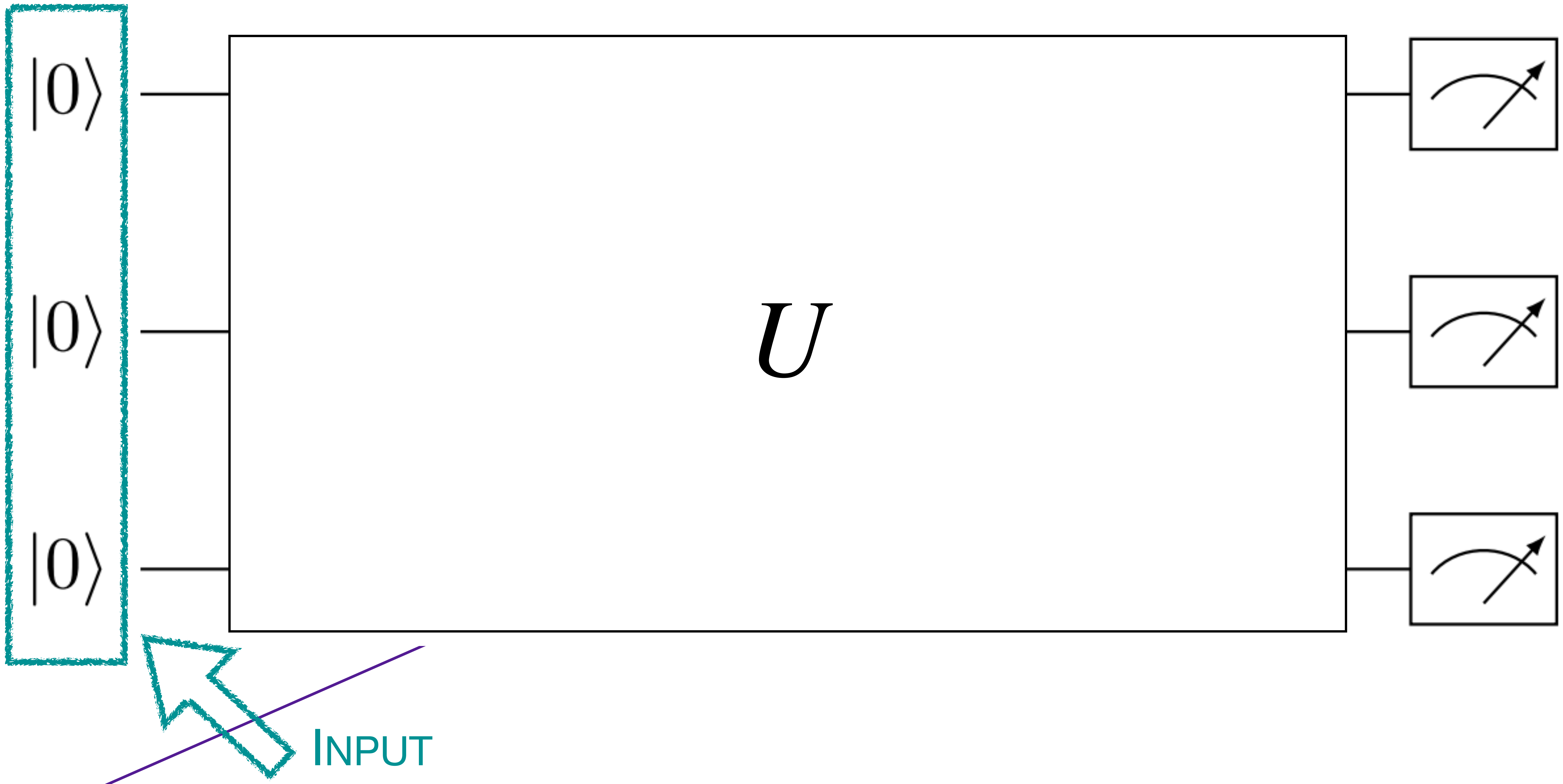
$|0\rangle$
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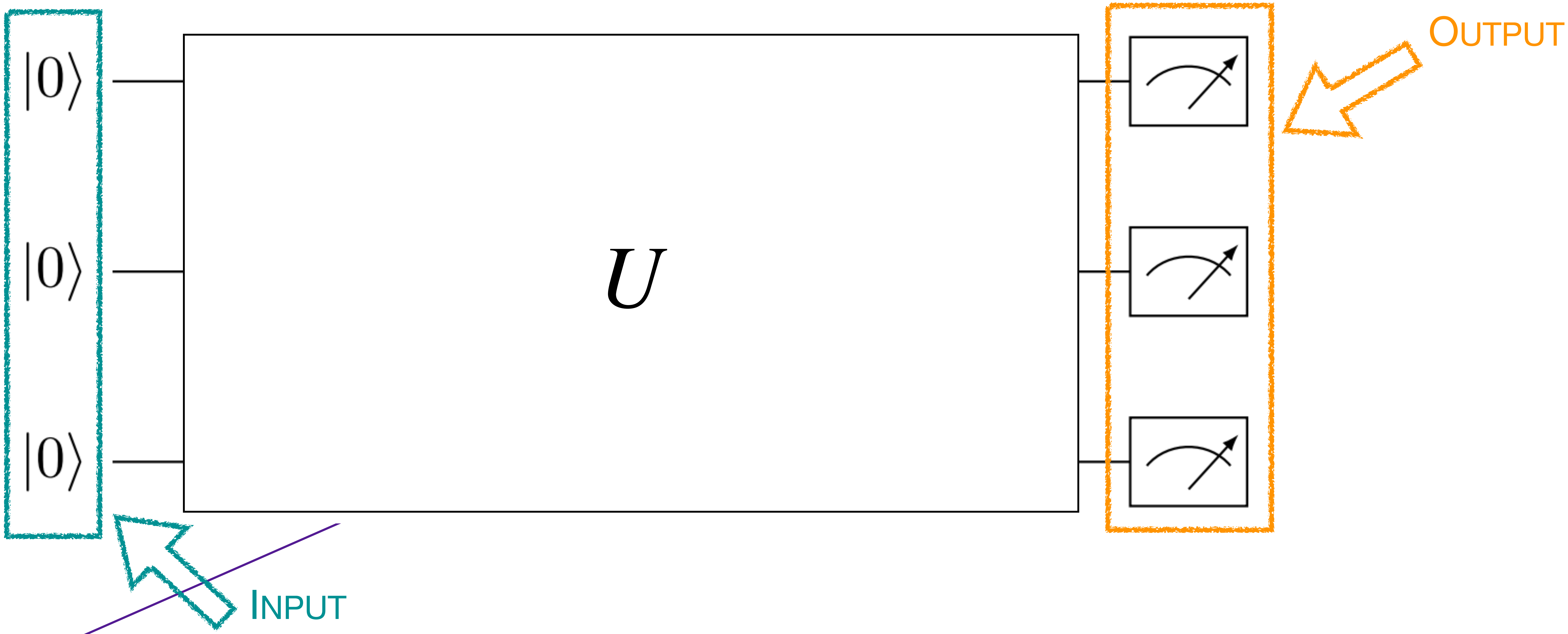
INPUT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OUTPUT
INPUT

Quantum circuits with:

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- stabilizer state inputs,

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- stabilizer state inputs,
- Clifford gates,

Quantum circuits with:

- stabilizer state inputs,
- Clifford gates,
- and Pauli measurements

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are efficiently classically simulatable.

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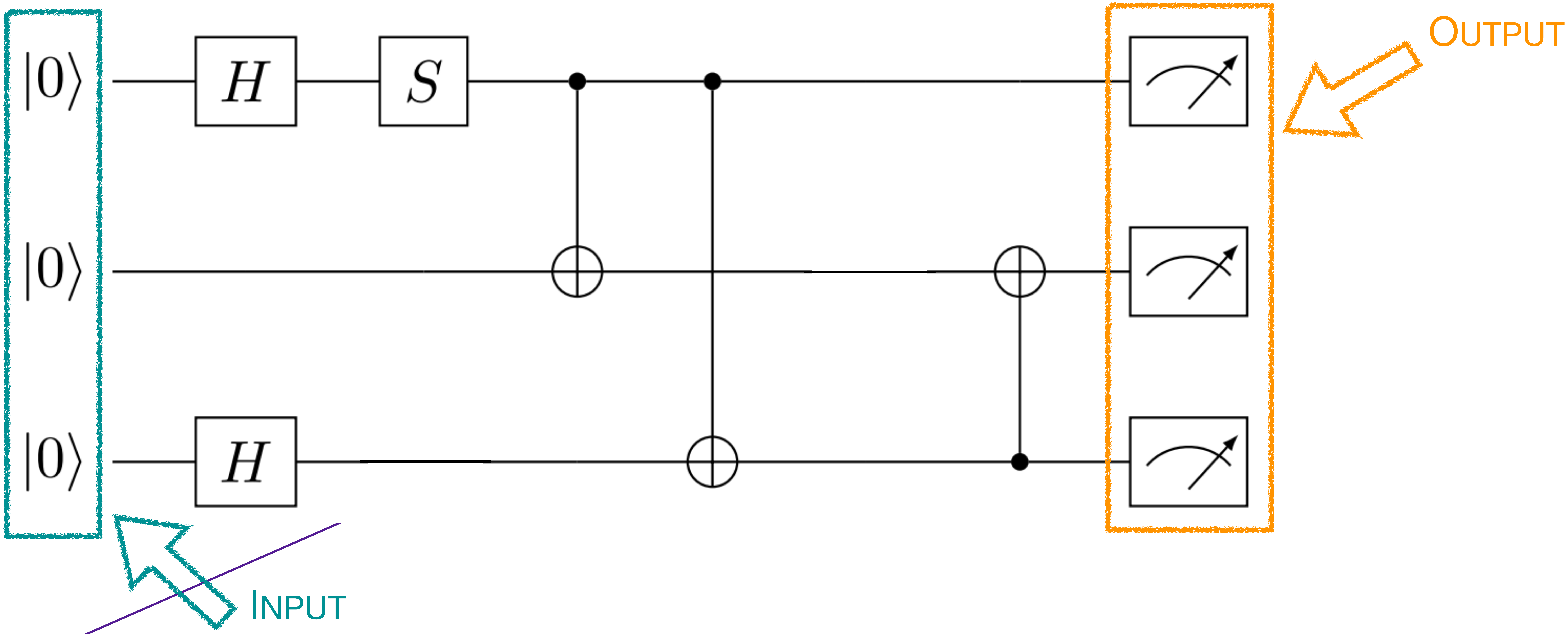
GOTTESMAN-KNILL THEOREM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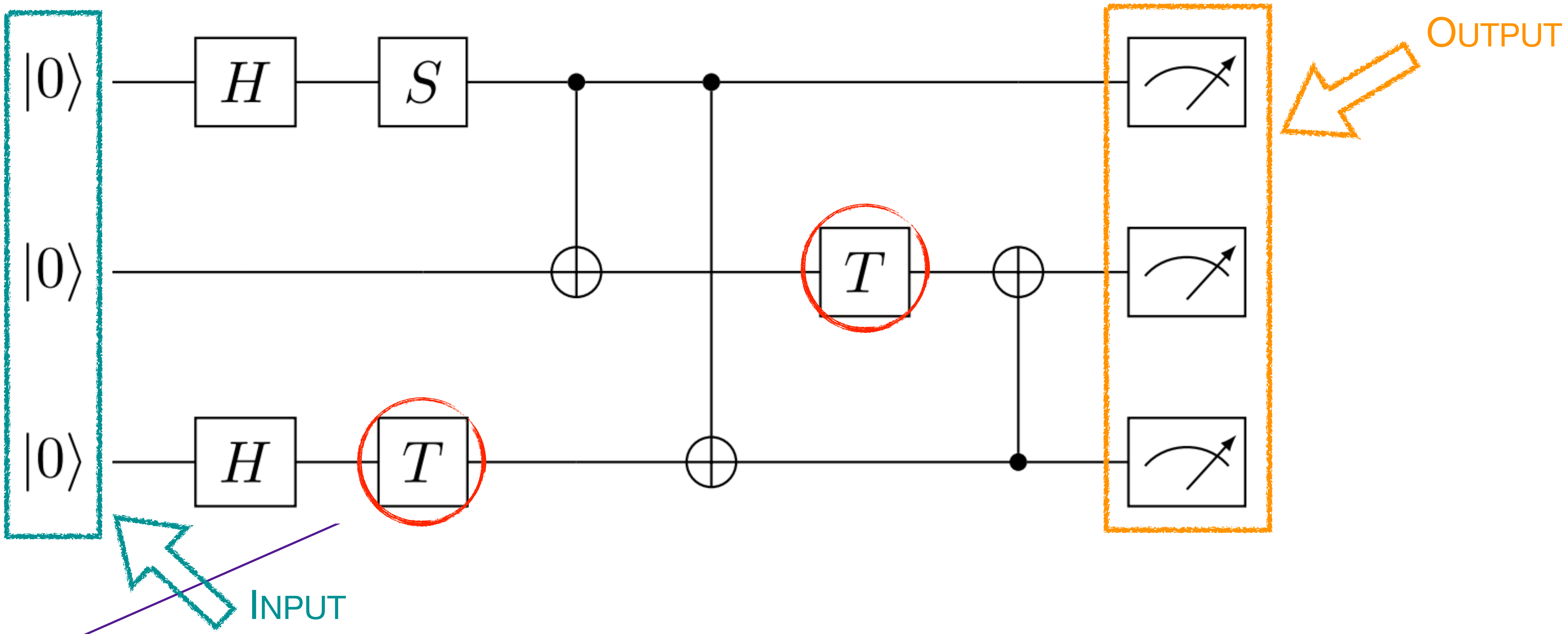
H
S
H
INPUT
OUTPUT

But…

Clifford+T circuits are universal for quantum computation!

$$T = \mathrm{diag}(1, e^{i\pi/4})\,.$$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H
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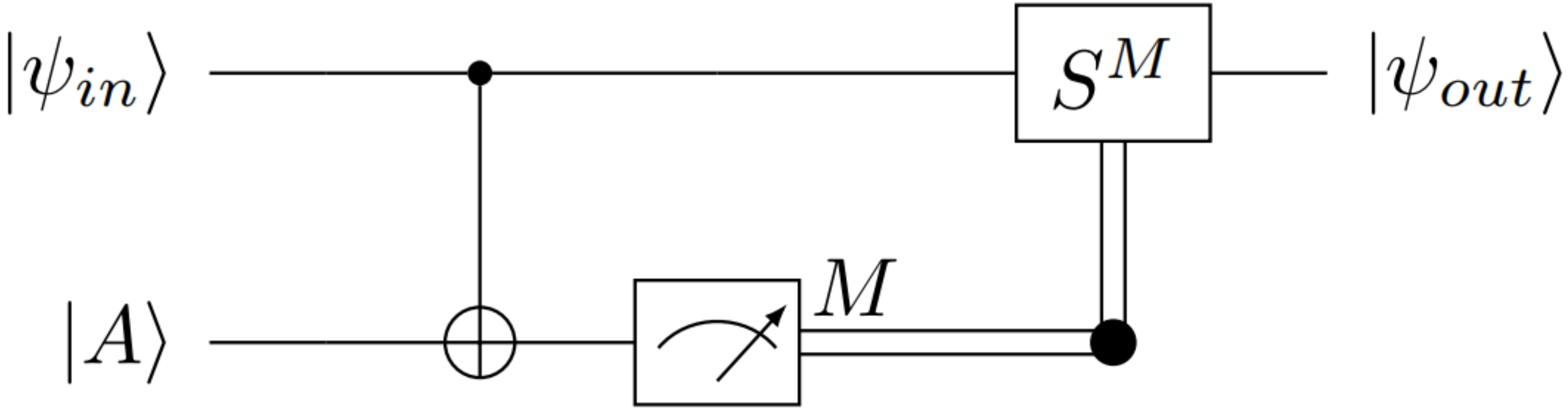
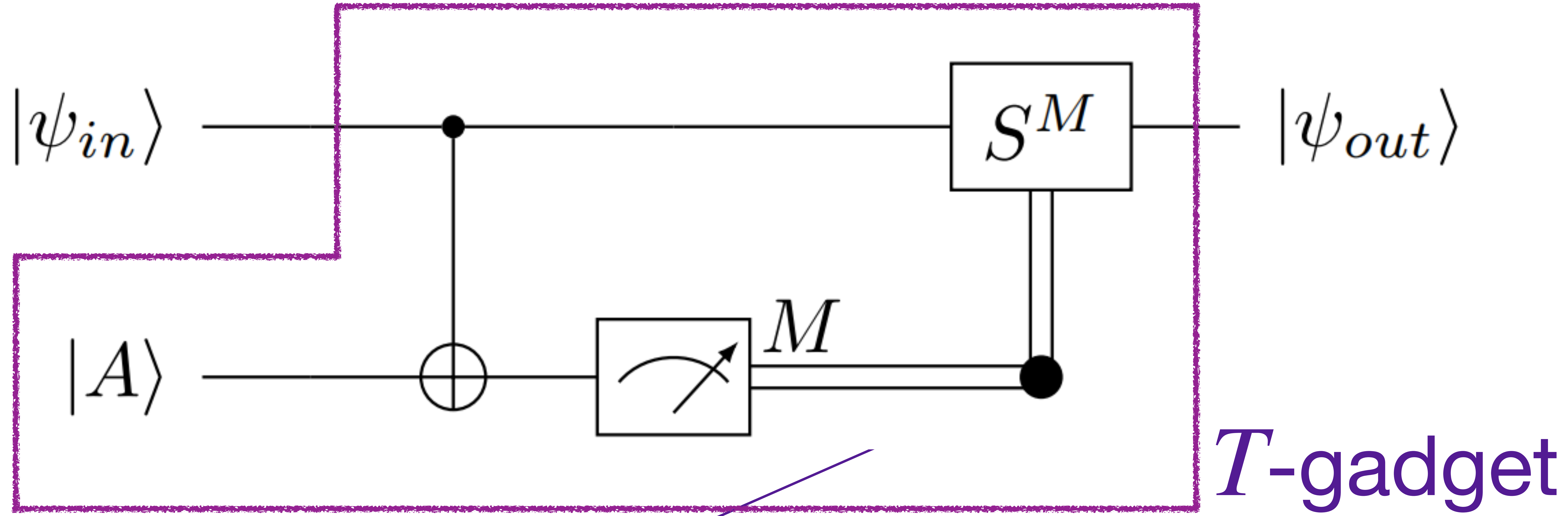
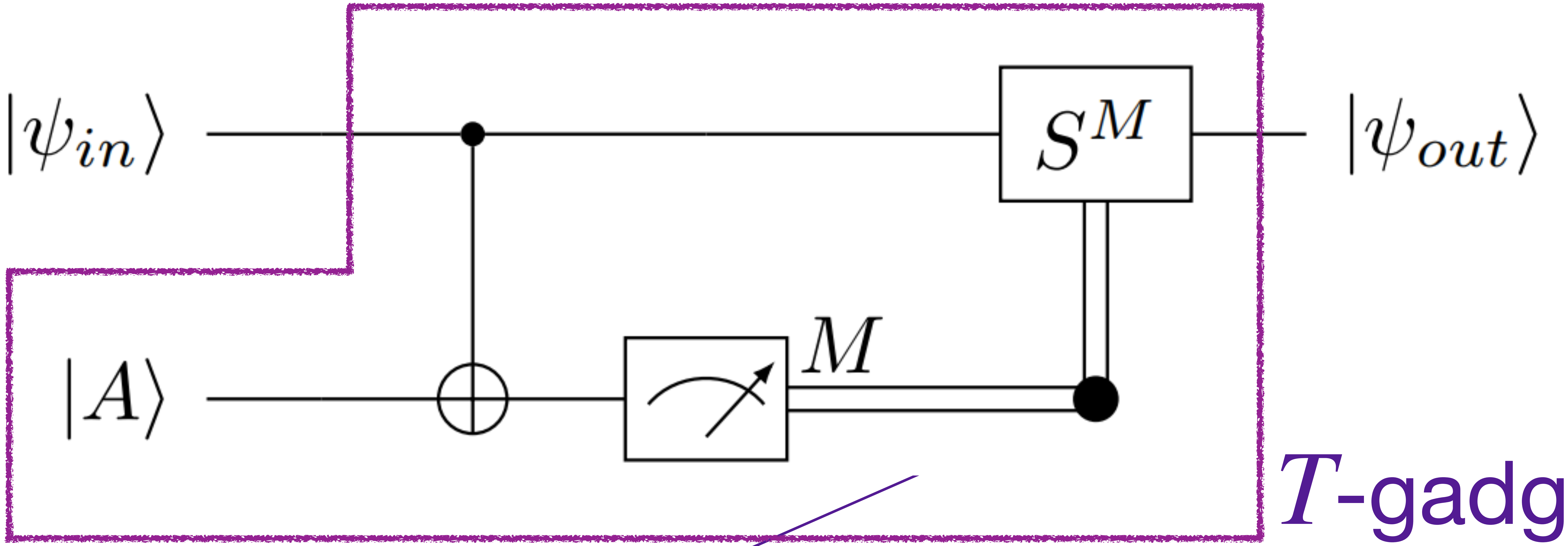
T
T
OUTPUT
INPUT

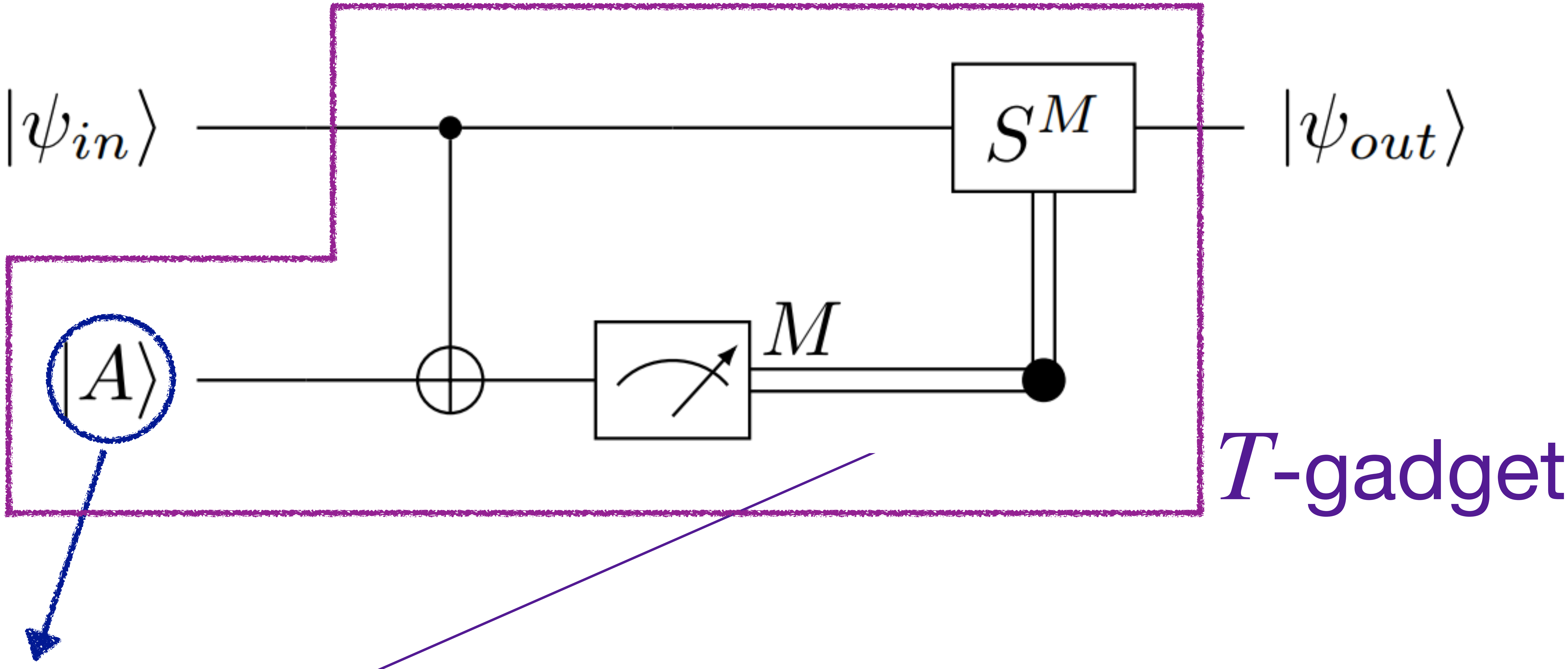

$|\psi_{in}\rangle$
S^M
$|\psi_{out}\rangle$
$|A\rangle$
M


$|\psi_{in}\rangle$
$|A\rangle$
M
S^M
$|\psi_{out}\rangle$
T-gadget



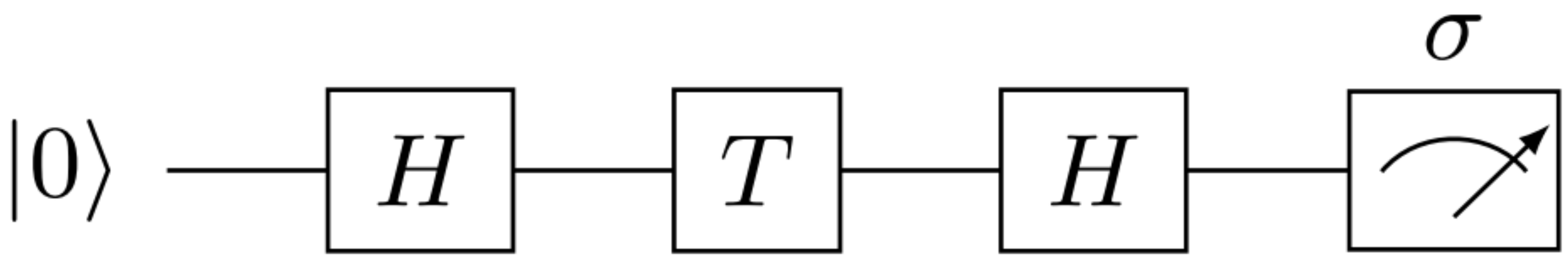
T-gadget

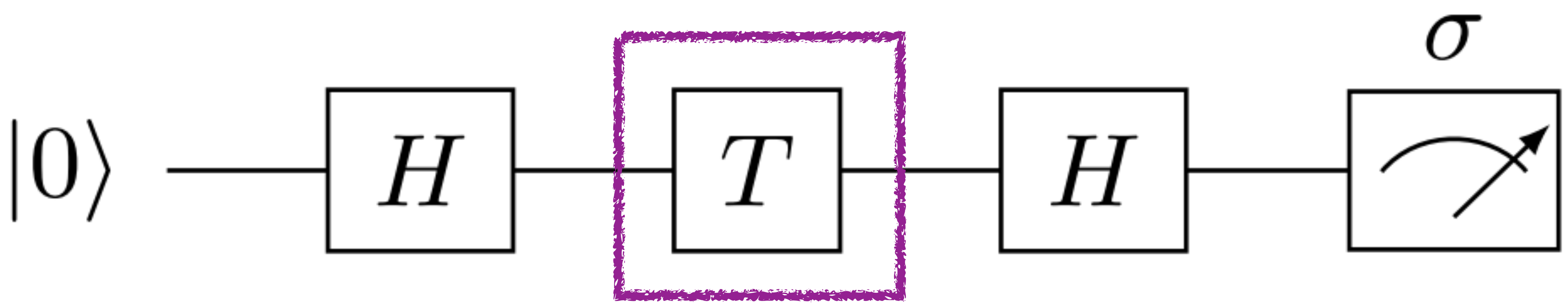
$$\left|\psi_{out}\right\rangle = T\left|\psi_{in}\right\rangle$$



$$|A\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + e^{i\pi/4}|1\rangle\right)$$

$$|\psi_{out}\rangle = T|\psi_{in}\rangle$$


σ
|0⟩
H
T
H


|0⟩
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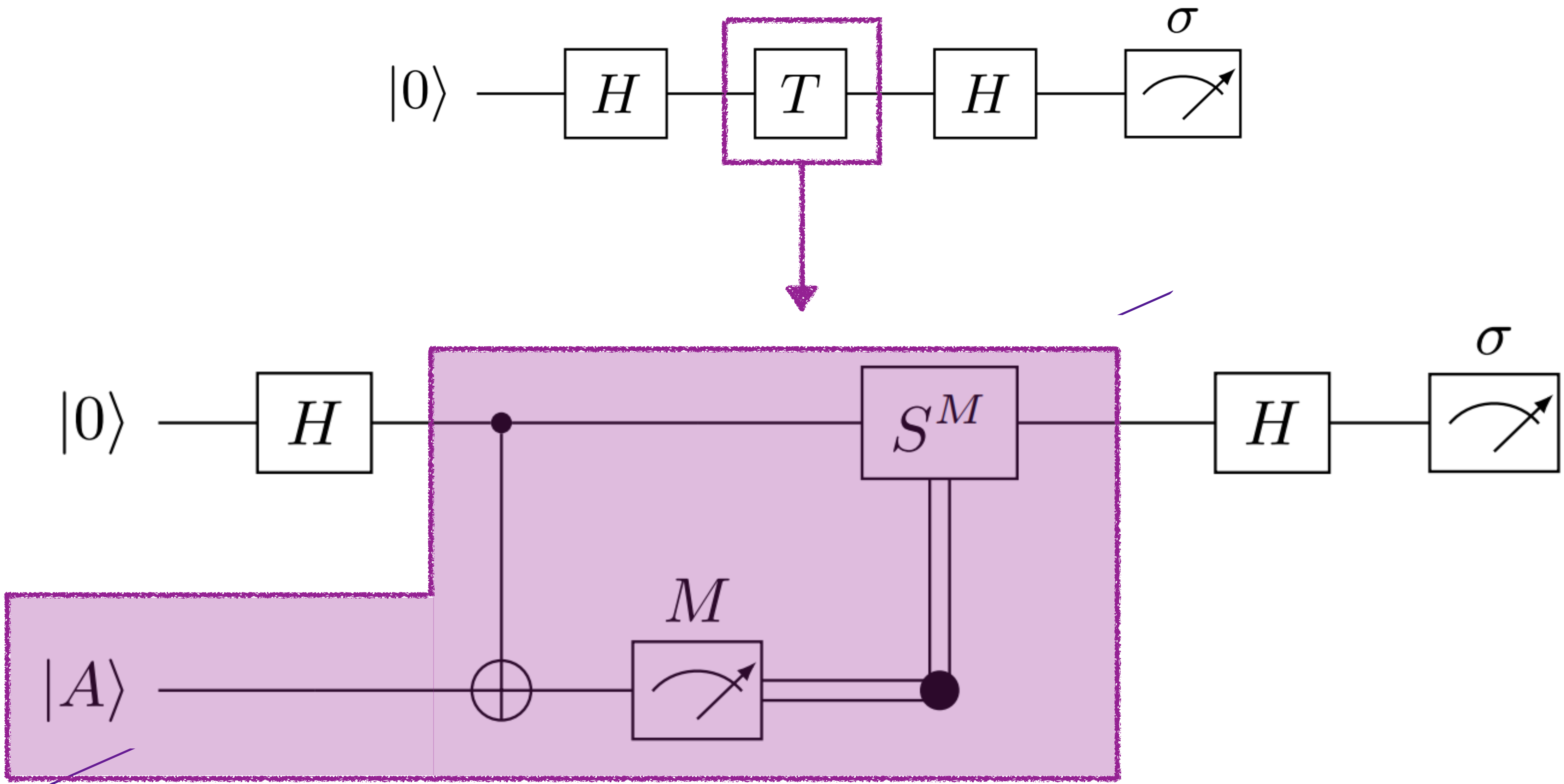
σ



$|0\rangle$
H
T
H
σ
$|0\rangle$
H
S^M
H
σ
M
$|A\rangle$

Pauli-based computation:

S. Bravyi, G. Smith, and J. A. Smolin, Phys. Rev. X **6**, 021043 (2016), arXiv:1506.01396.

Definition: [PAULI-BASED MODEL OF COMPUTATION]

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- Input: product state $|A\rangle^{\otimes t}$

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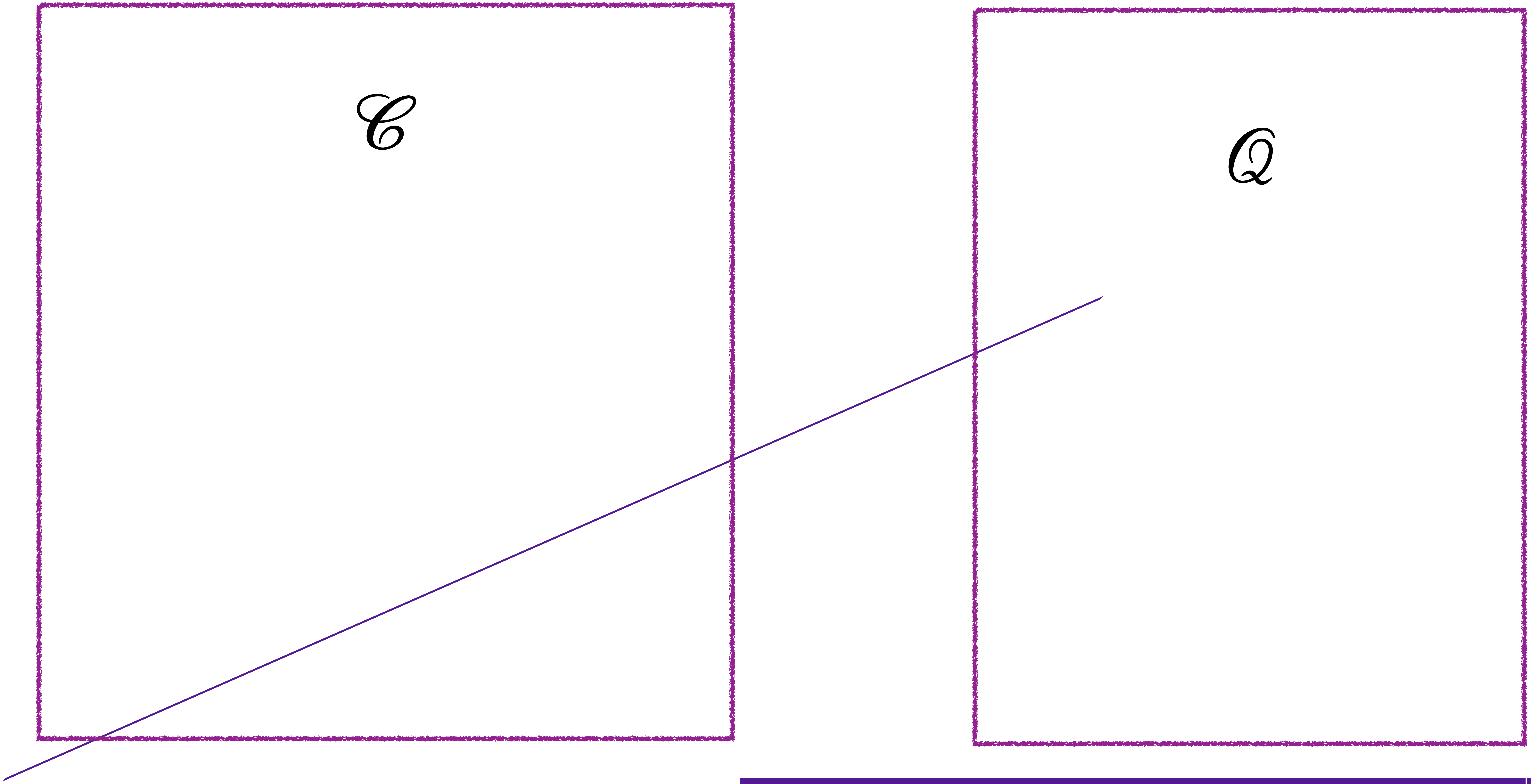
- Input: product state $|A\rangle^{\otimes t}$
- Steps: measurements of **independent and pairwise commuting** Pauli operators $P_i \in \mathscr{P}_t$

Theorem: Any Clifford+T quantum circuit $\mathscr{C}$ with t T gates can be simulated by a standard PBC $\mathcal{Q}$.


$\mathscr{C}$
$\mathscr{Q}$

$\mathscr{C}$

- $|0\rangle^{\otimes n}$

$\mathscr{Q}$

$\mathscr{C}$

- $|0\rangle^{\otimes n}$
- $t\ T$ gates

$\mathscr{Q}$

$\mathscr{C}$

- $|0\rangle^{\otimes n}$
- t T gates
- w Z-measurements

$\mathscr{Q}$

$\mathscr{C}$

- $|0\rangle^{\otimes n}$
- t T gates
- w Z-measurements

$\longrightarrow$

$\mathscr{Q}$

$\mathscr{C}$

- $|0\rangle^{\otimes n}$
- t T gates
- w Z-measurements

$\mathscr{Q}$

- $|A\rangle^{\otimes t}$

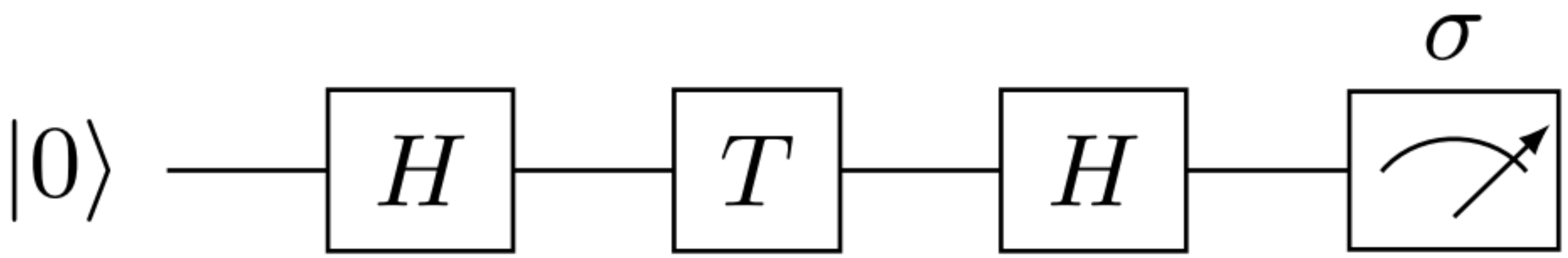
$\mathcal{C}$

- $|0\rangle^{\otimes n}$
- t T gates
- w Z-measurements

$\rightarrow$

$\mathcal{Q}$

- $|A\rangle^{\otimes t}$
- at most t Pauli measurements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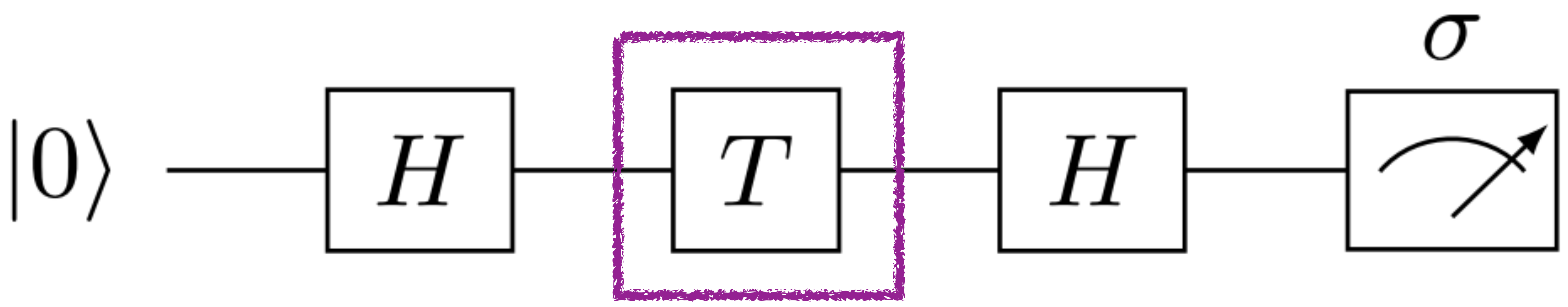
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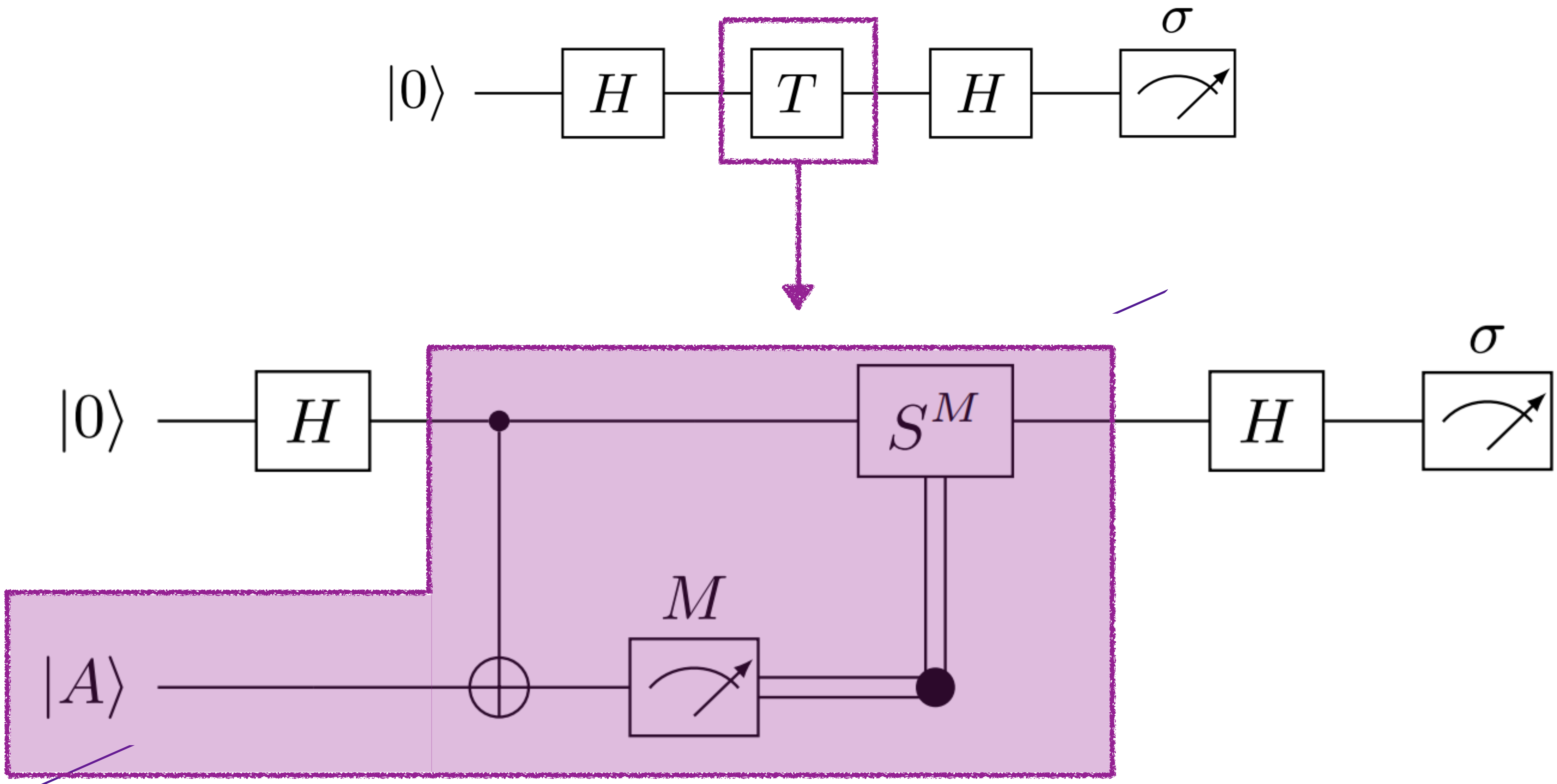
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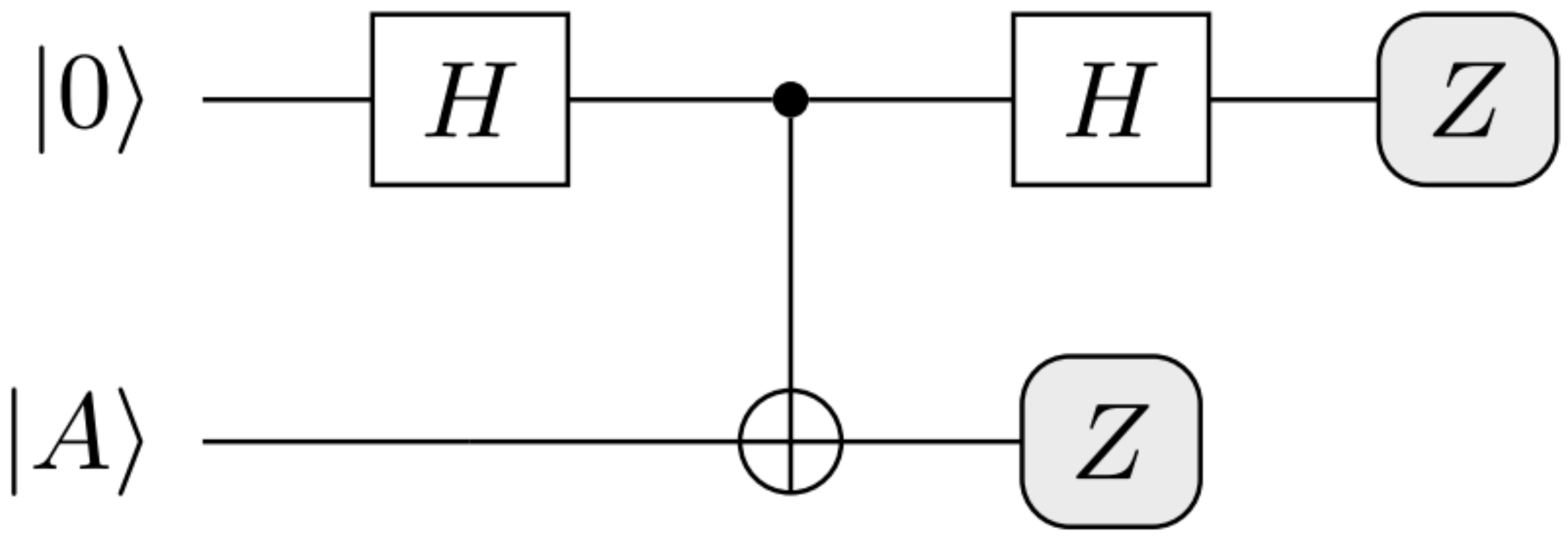
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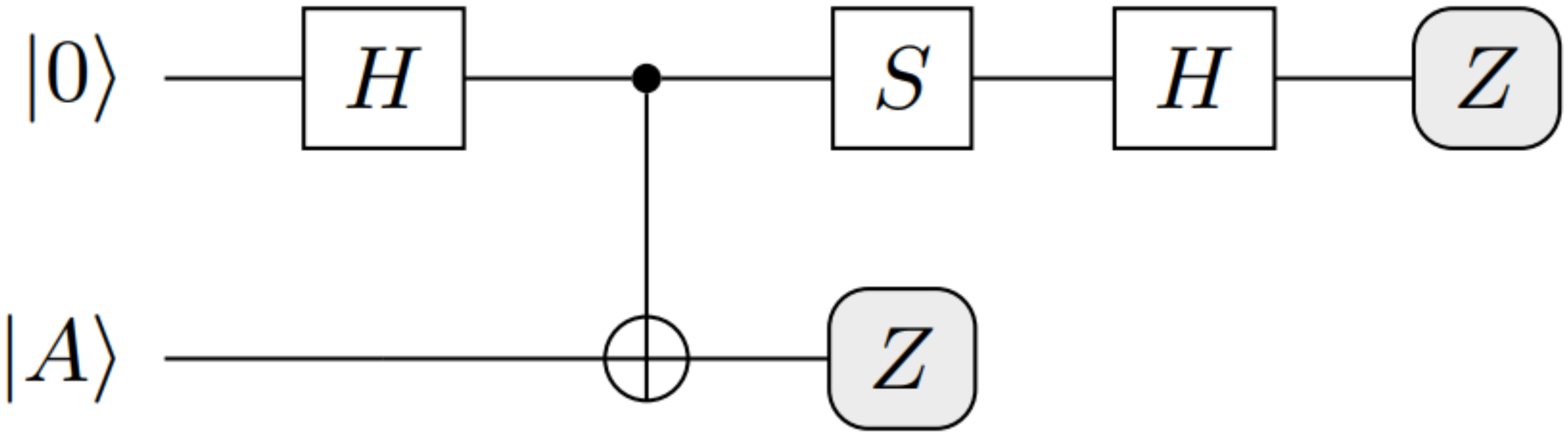
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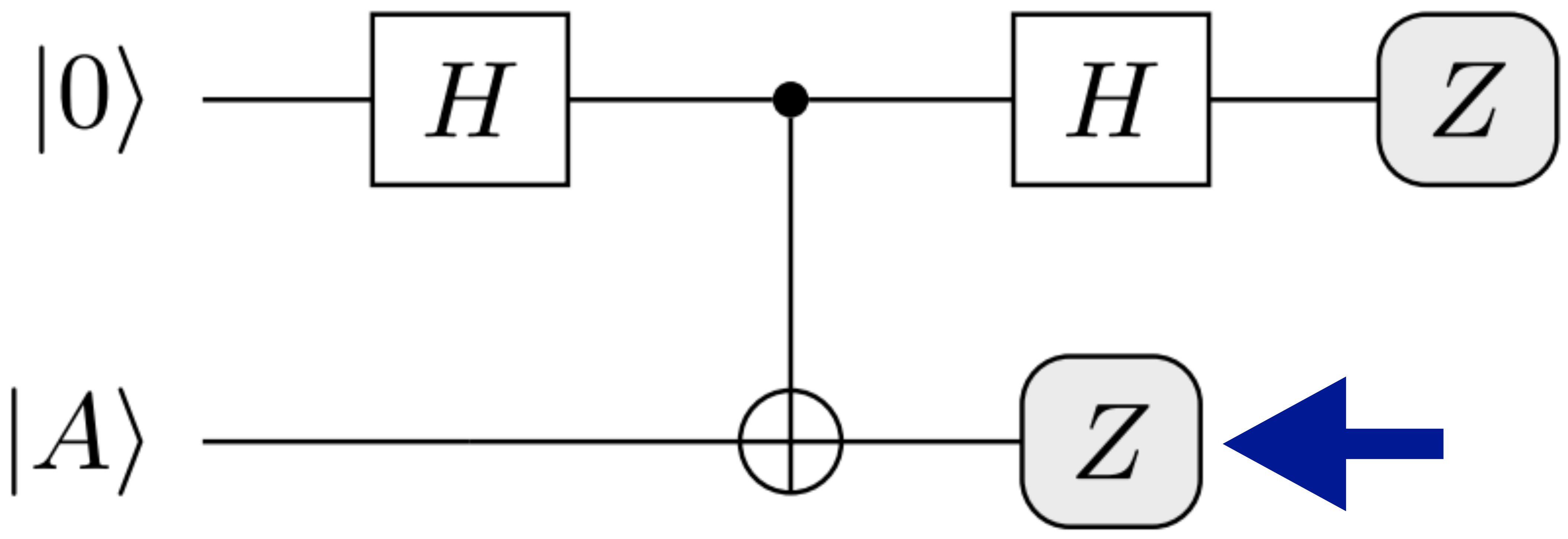
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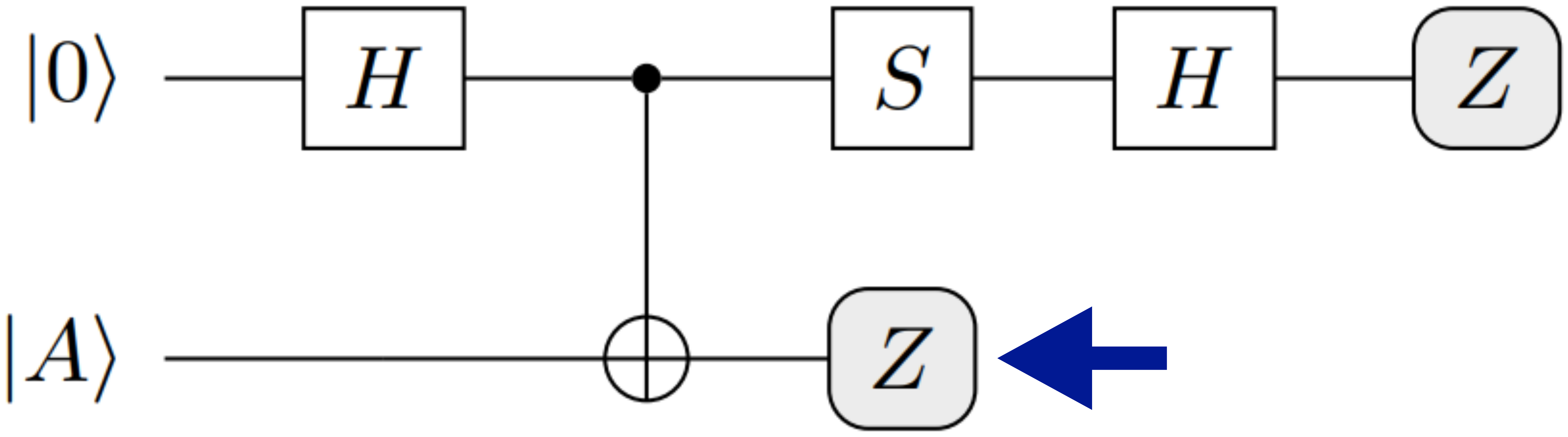
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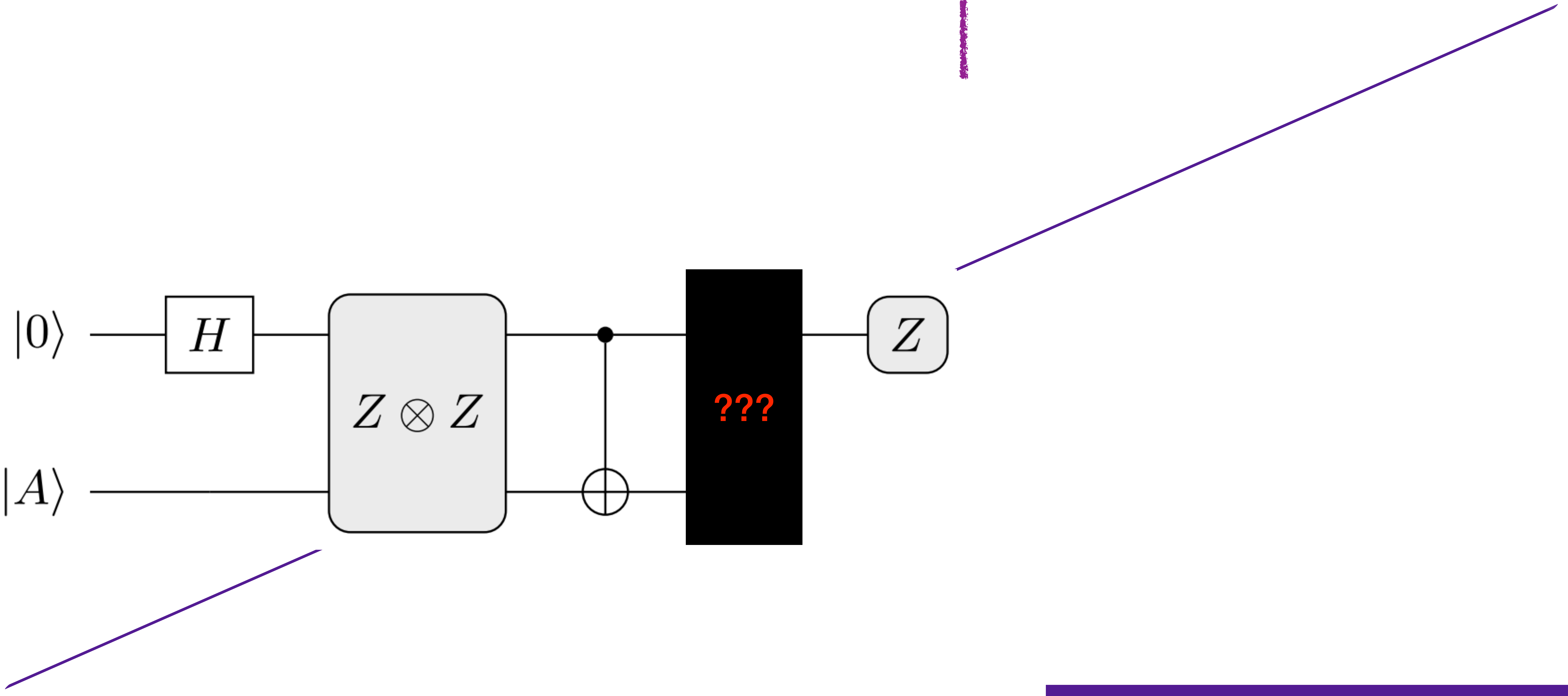
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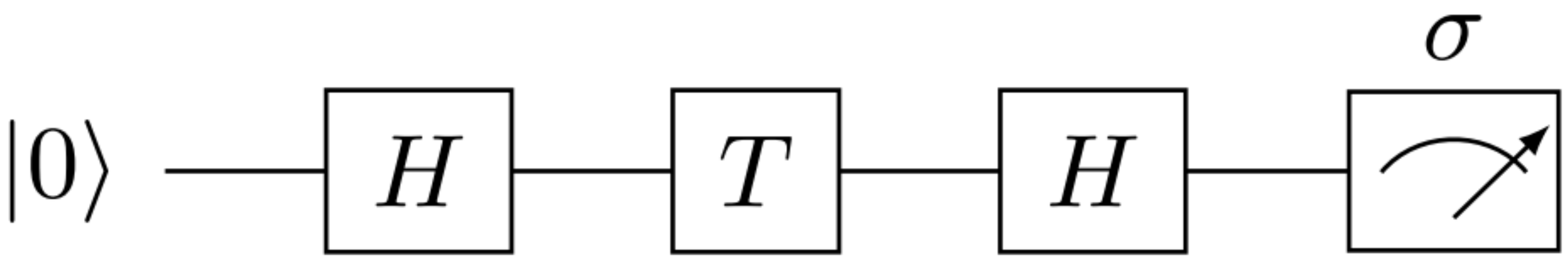
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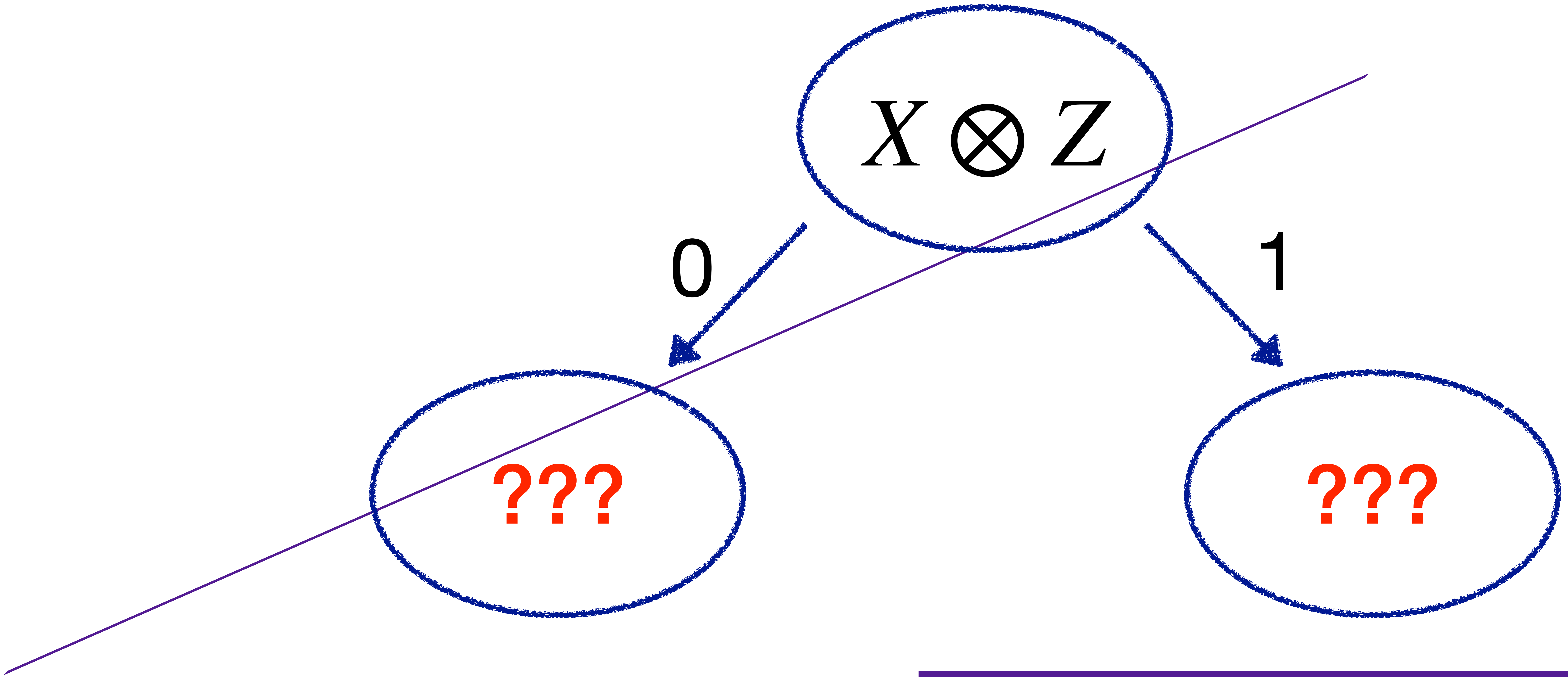
X ⊗ Z
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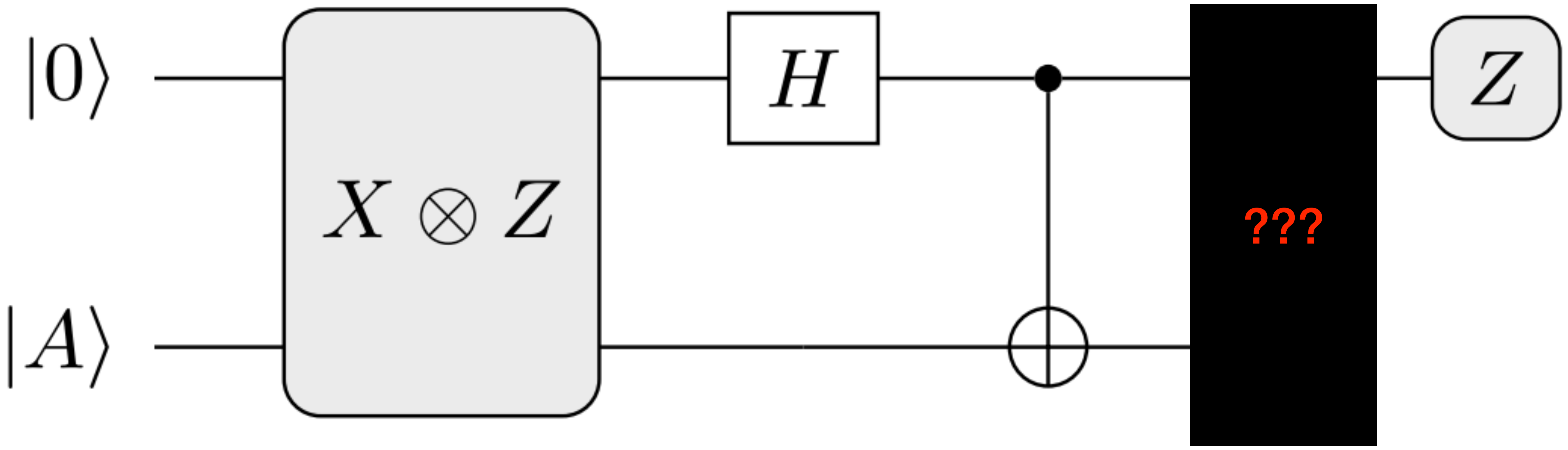
$|0\rangle$
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$X \otimes Z$
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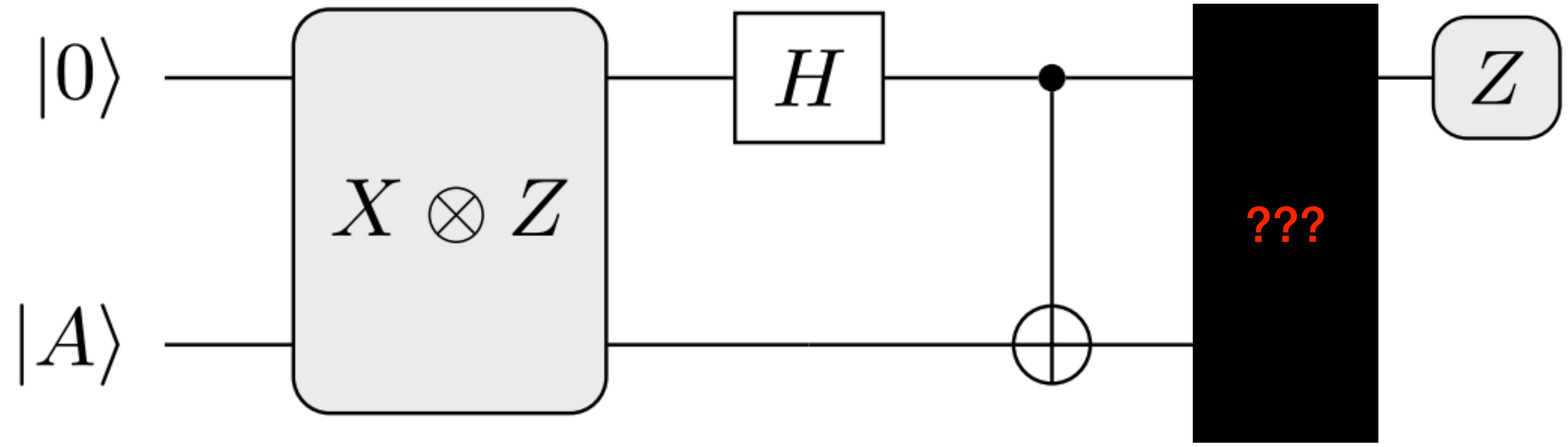
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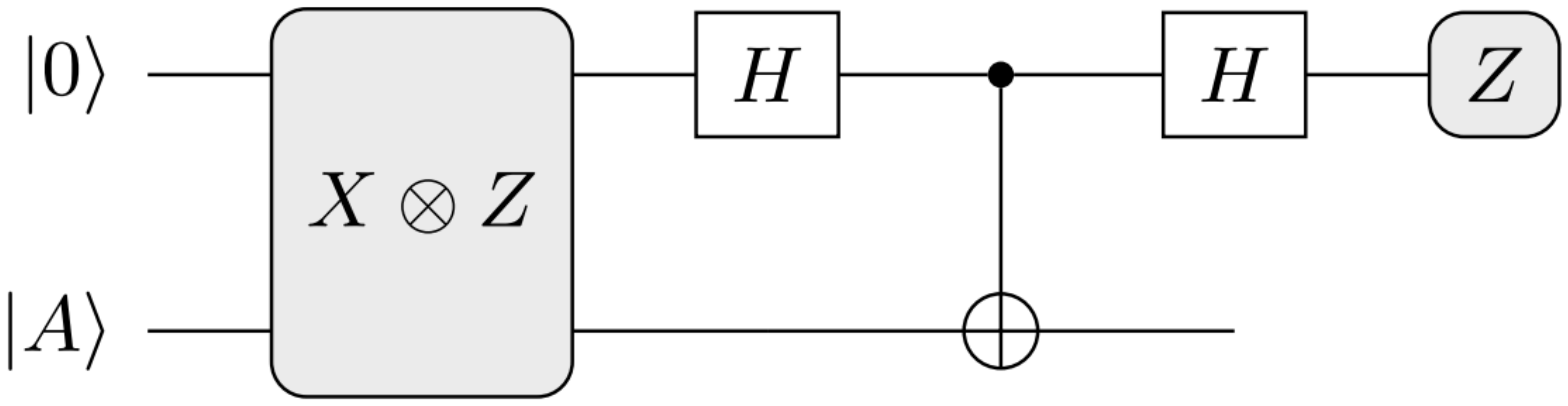
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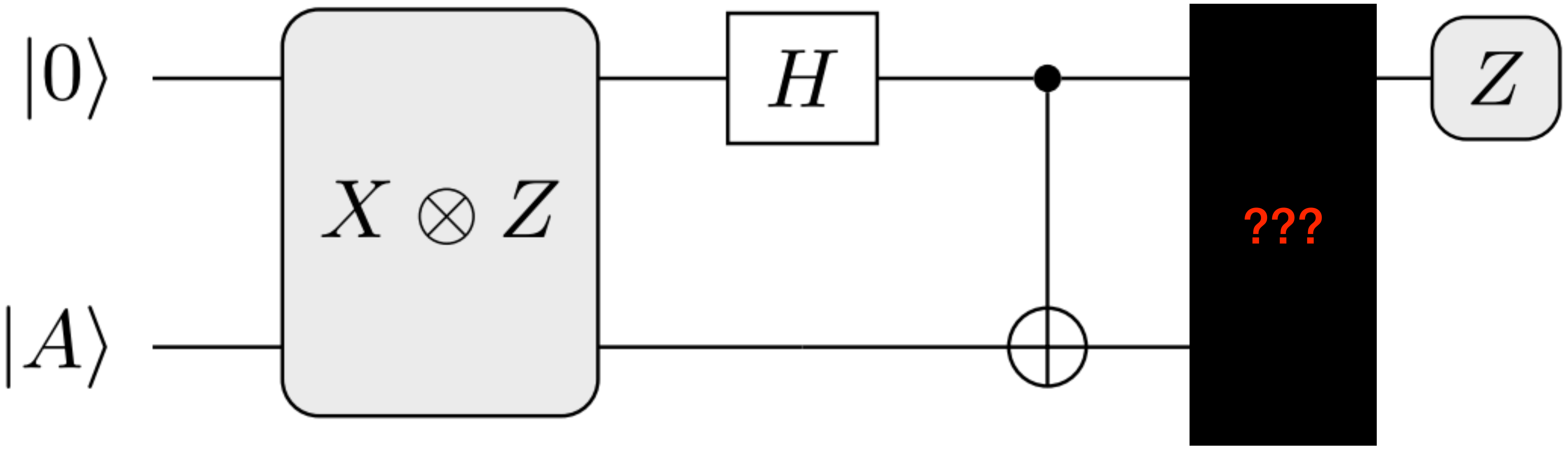
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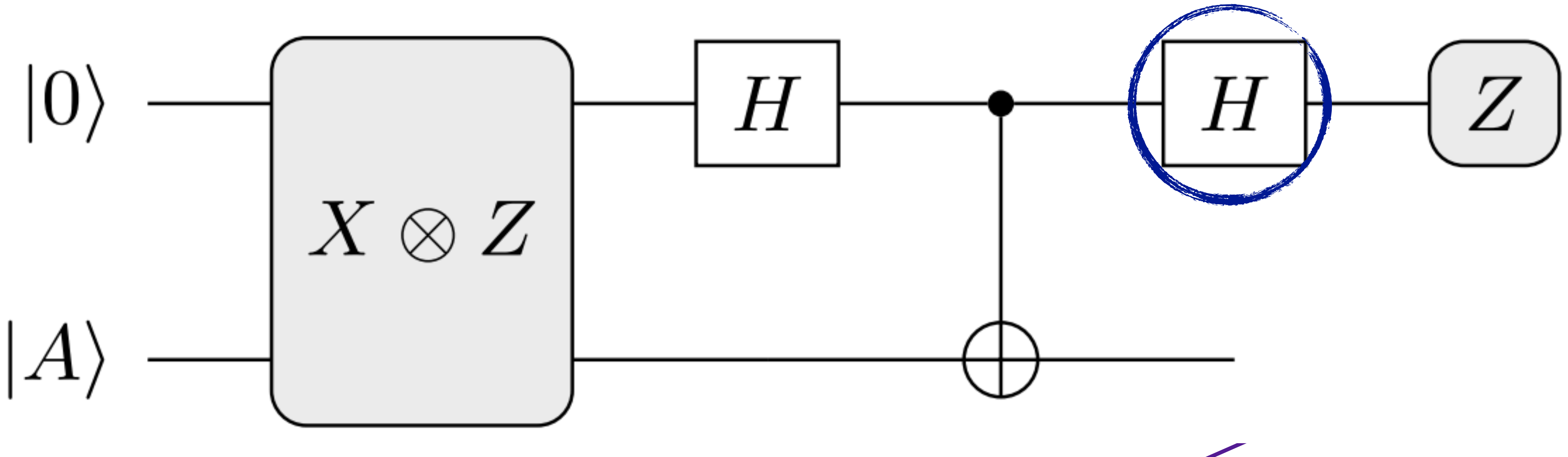
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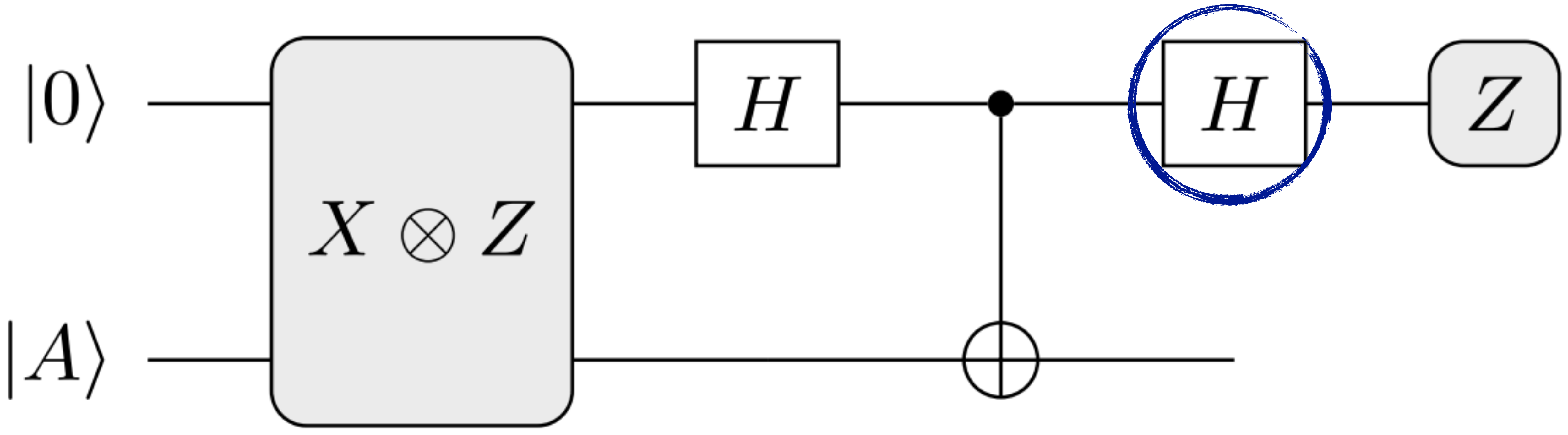
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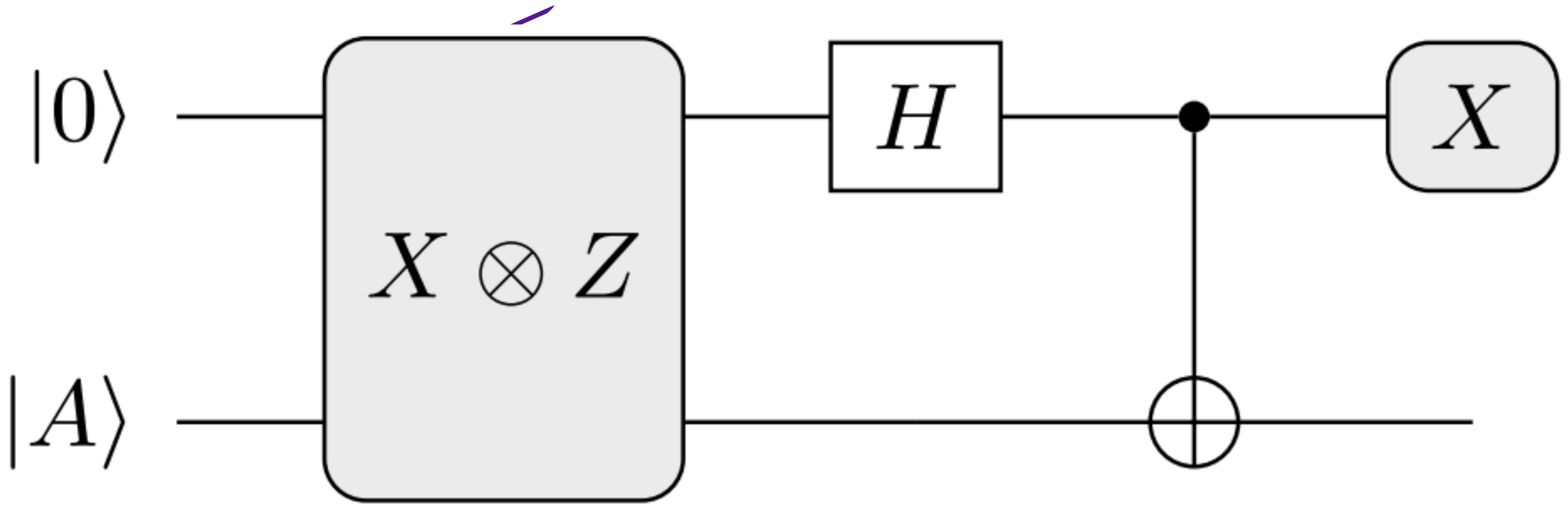
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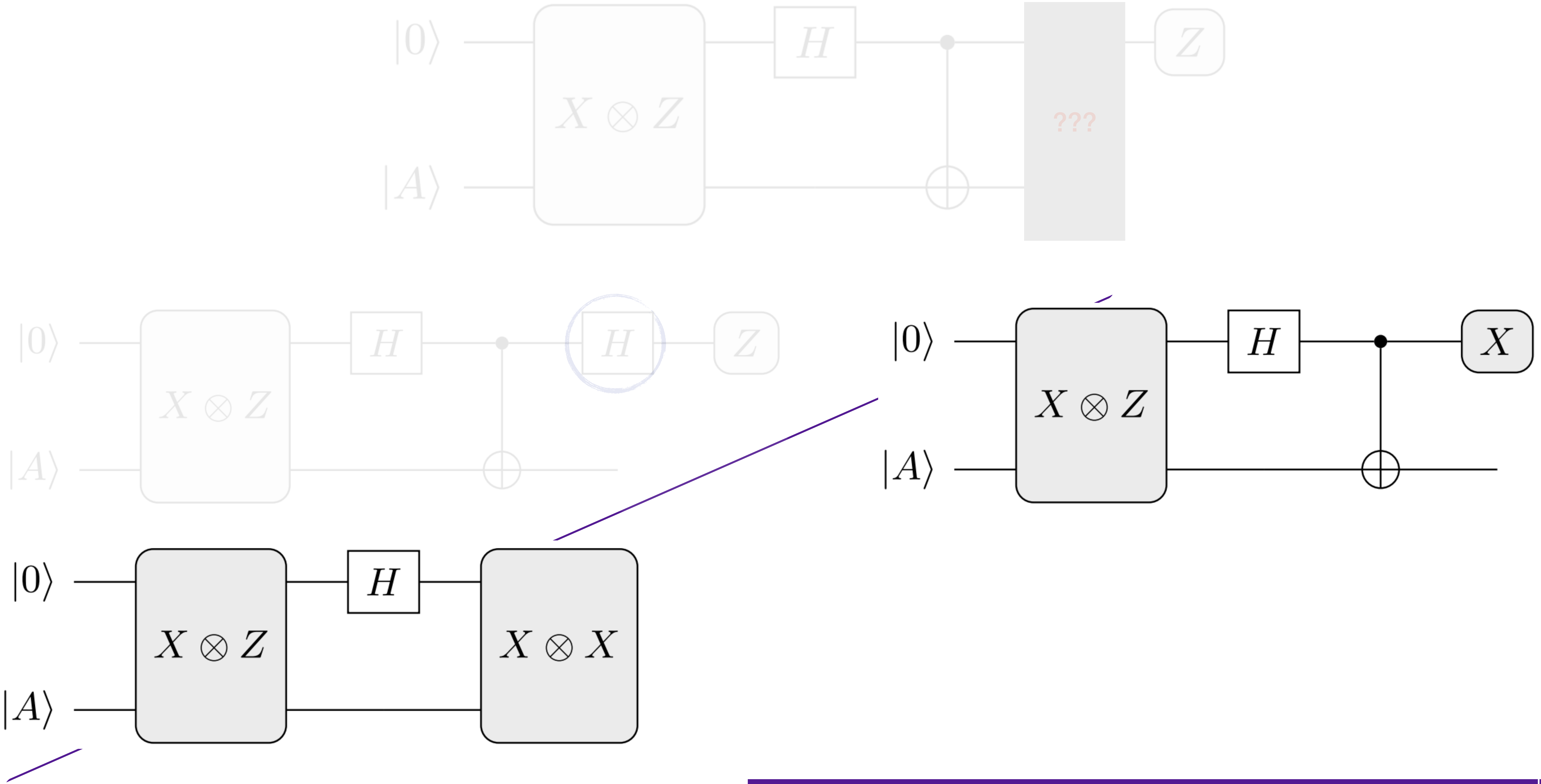
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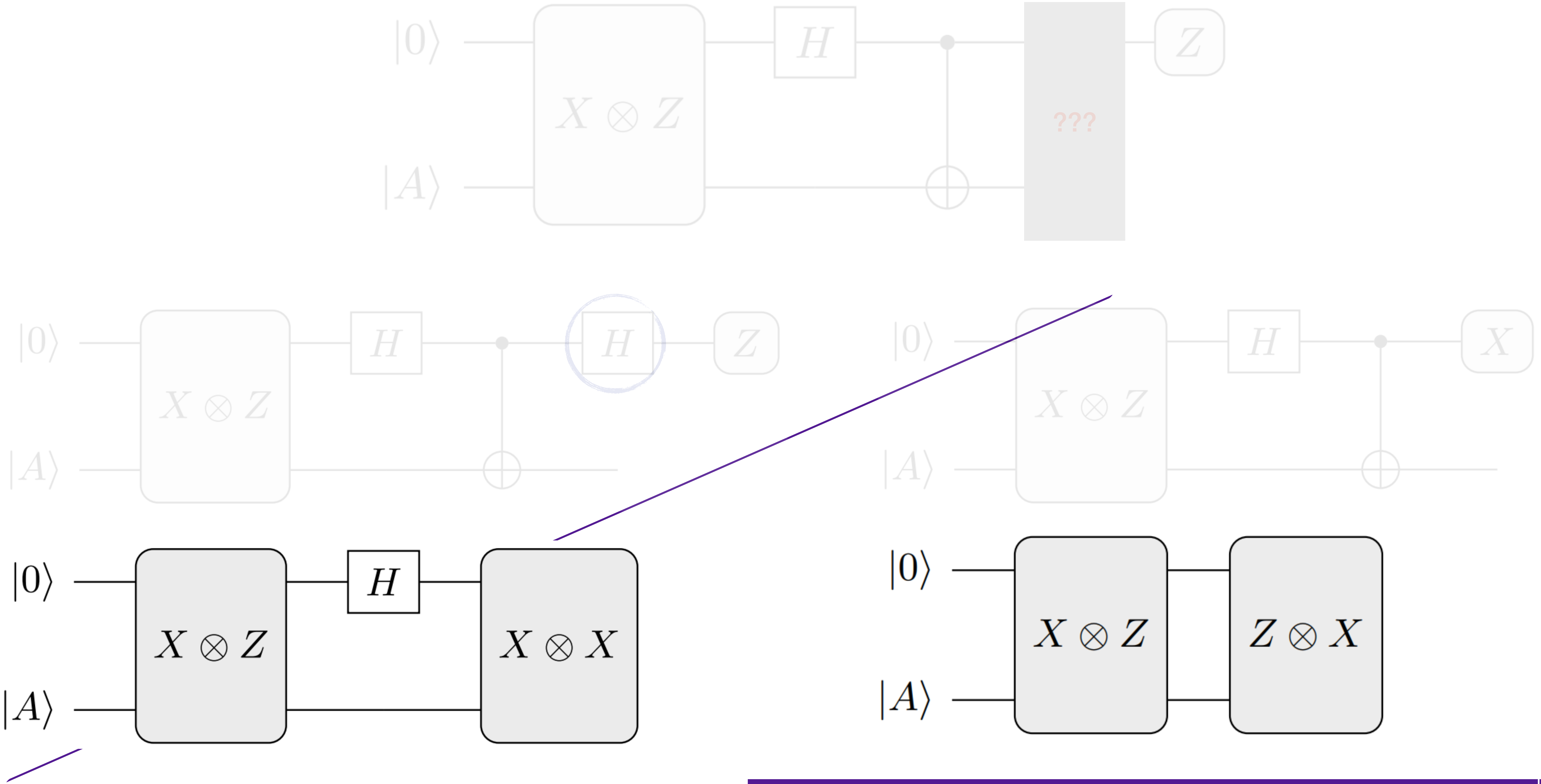
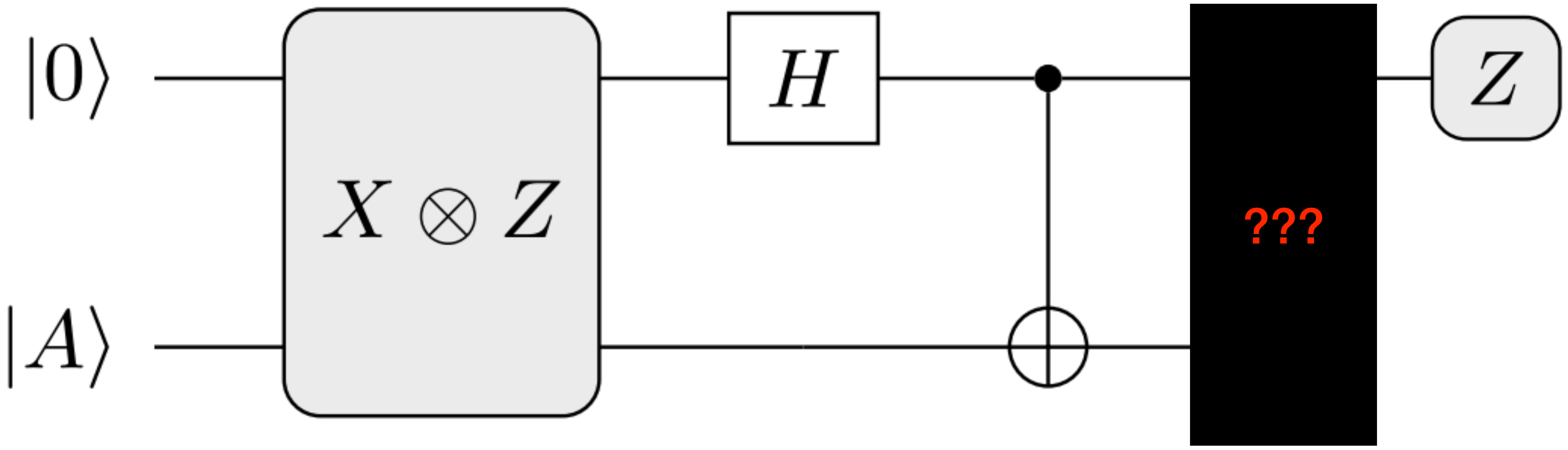
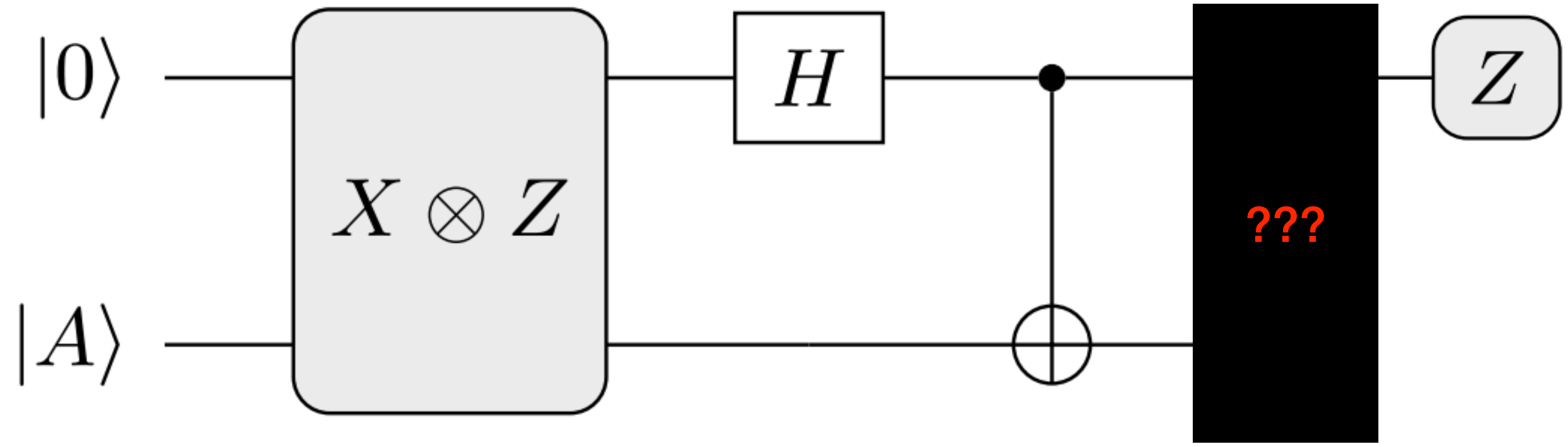
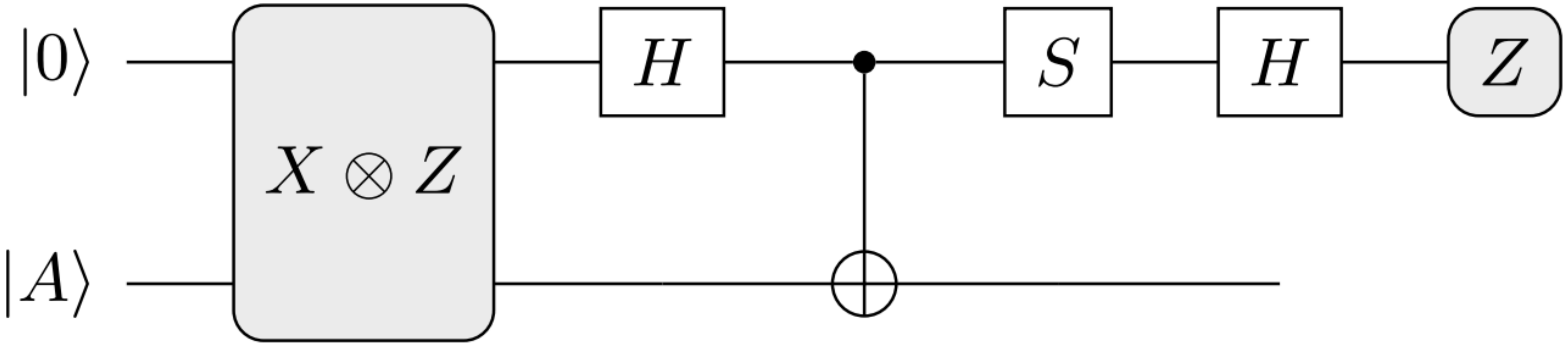
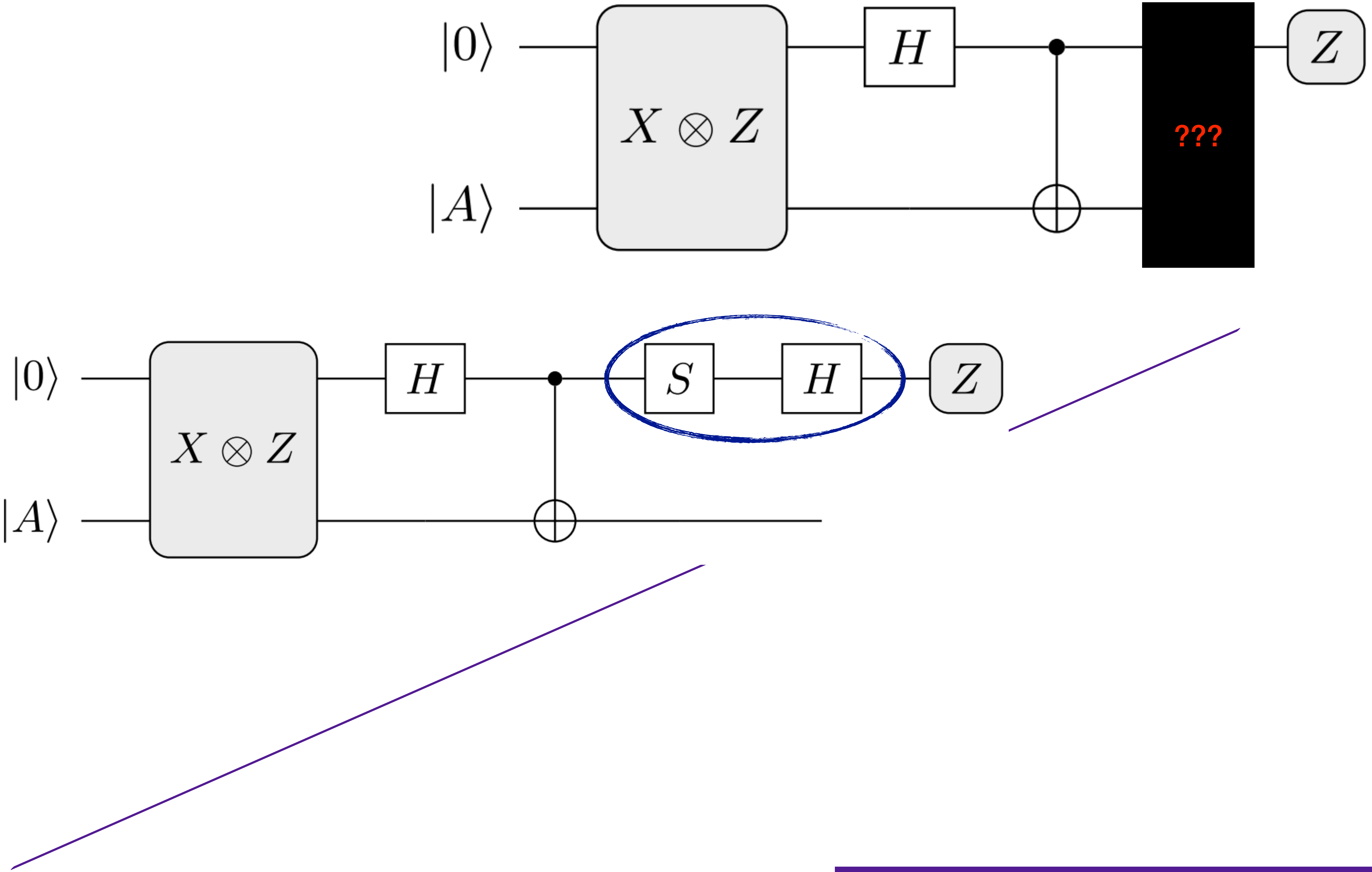
H
X ⊗ X


|0⟩
|A⟩
X ⊗ Z
H
???
Z
|0⟩
|A⟩
X ⊗ Z
H
H
Z
|0⟩
|A⟩
X ⊗ Z
H
X
|0⟩
|A⟩
X ⊗ Z
H
X ⊗ X
|0⟩
|A⟩
X ⊗ Z
Z ⊗ X

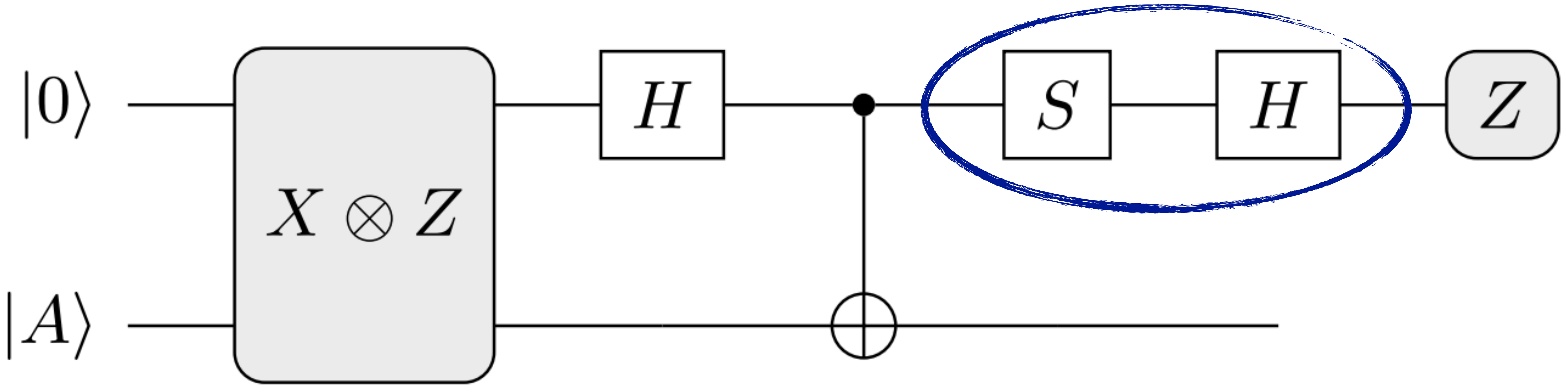

|0⟩
|A⟩
X ⊗ Z
H
???
Z

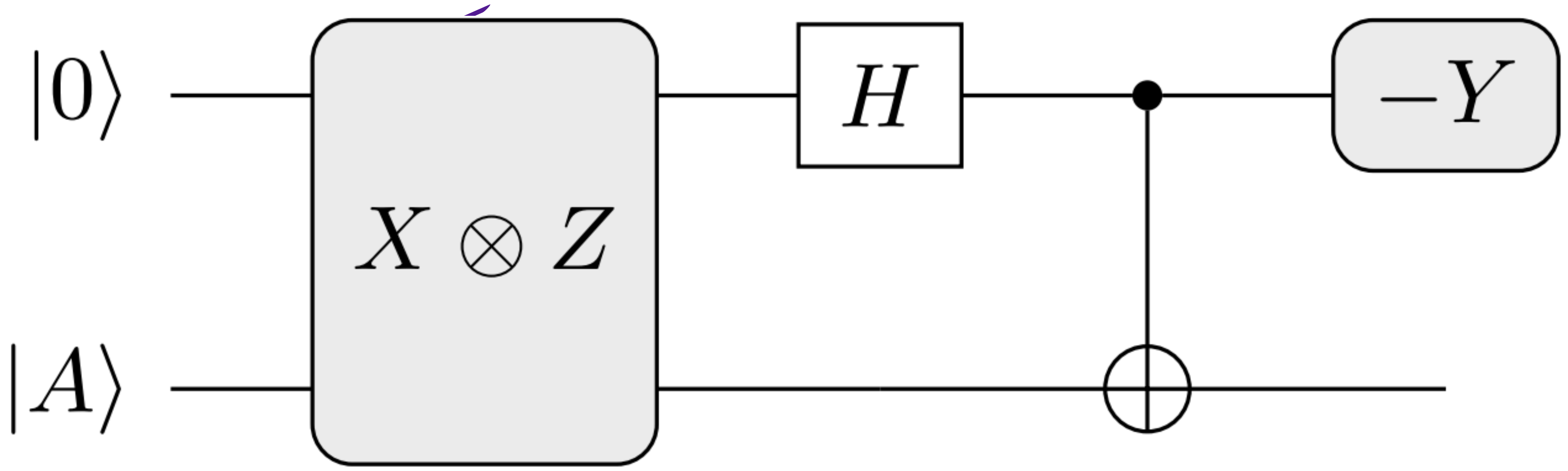

|0⟩
|A⟩
X ⊗ Z
H
???
Z


|0⟩
|A⟩
X ⊗ Z
H
S
H
Z


|0⟩
|A⟩
X ⊗ Z
H
???
Z
|0⟩
|A⟩
X ⊗ Z
H
S
H
Z

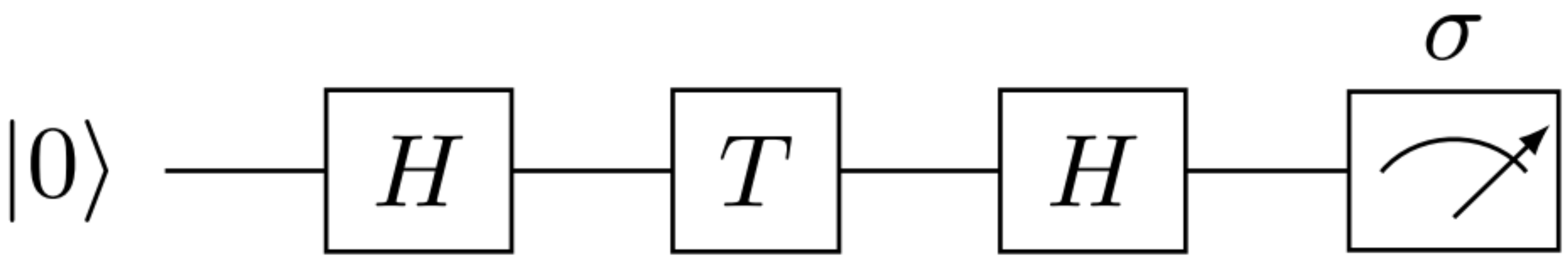
|0⟩
|A⟩
X ⊗ Z
H
???
Z

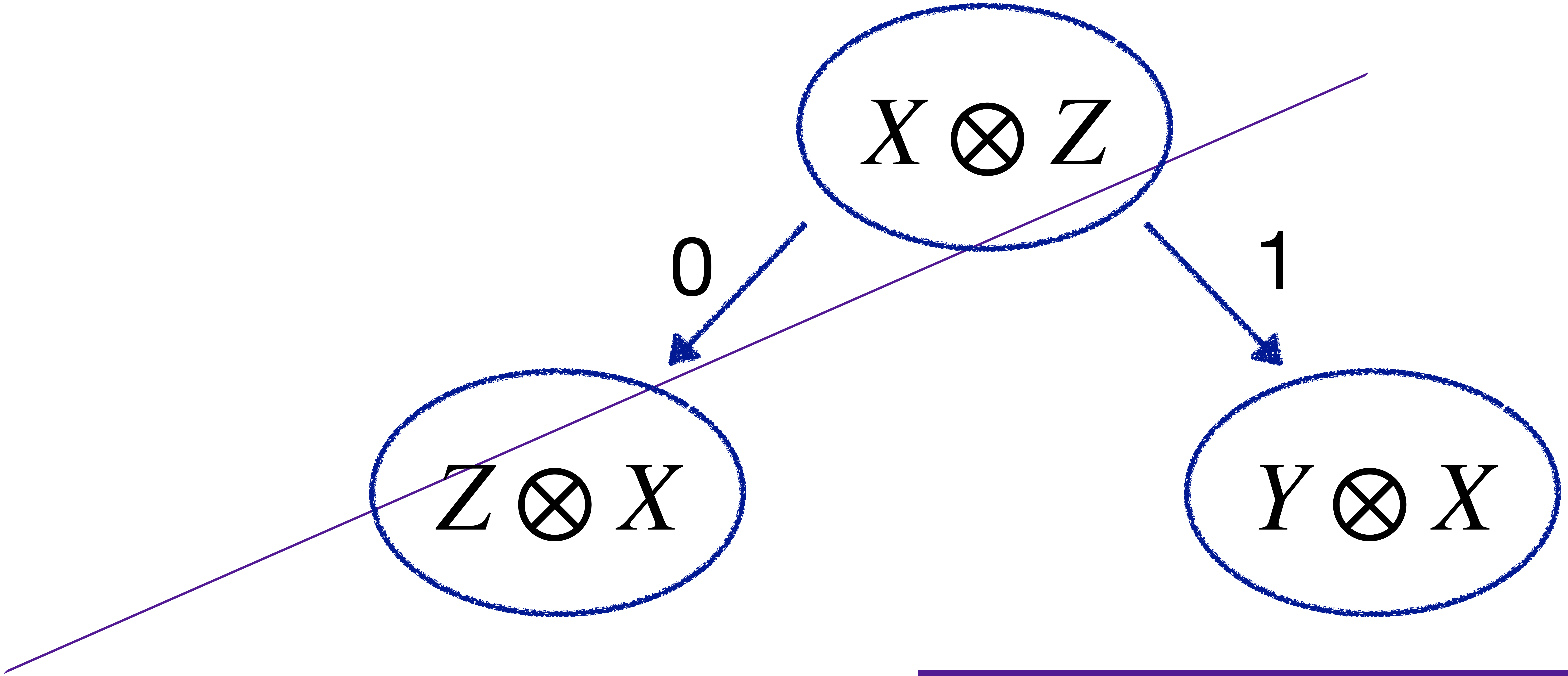

|0⟩
|A⟩
X ⊗ Z
H
S
H
Z


|0⟩
|A⟩
X ⊗ Z
H
−Y

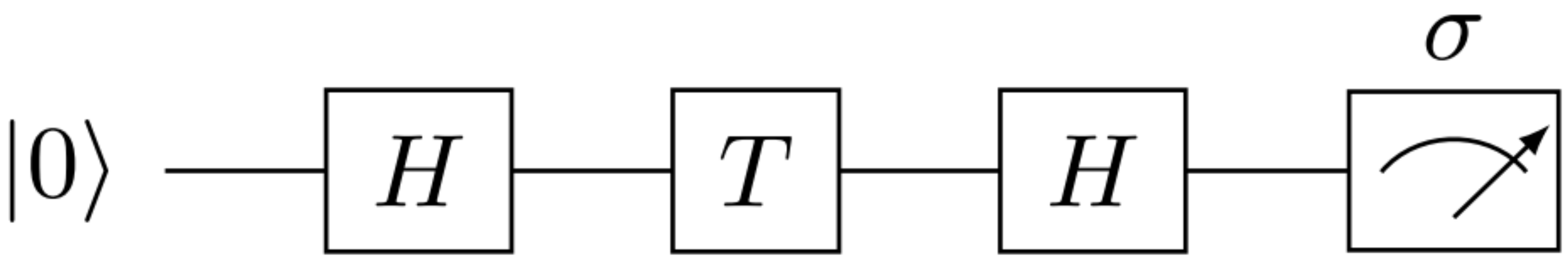

|0⟩
|A⟩
X ⊗ Z
H
???
Z
|0⟩
|A⟩
X ⊗ Z
H
S
H
Z
|0⟩
|A⟩
X ⊗ Z
H
−Y
|0⟩
|A⟩
X ⊗ Z
H
−Y ⊗ X

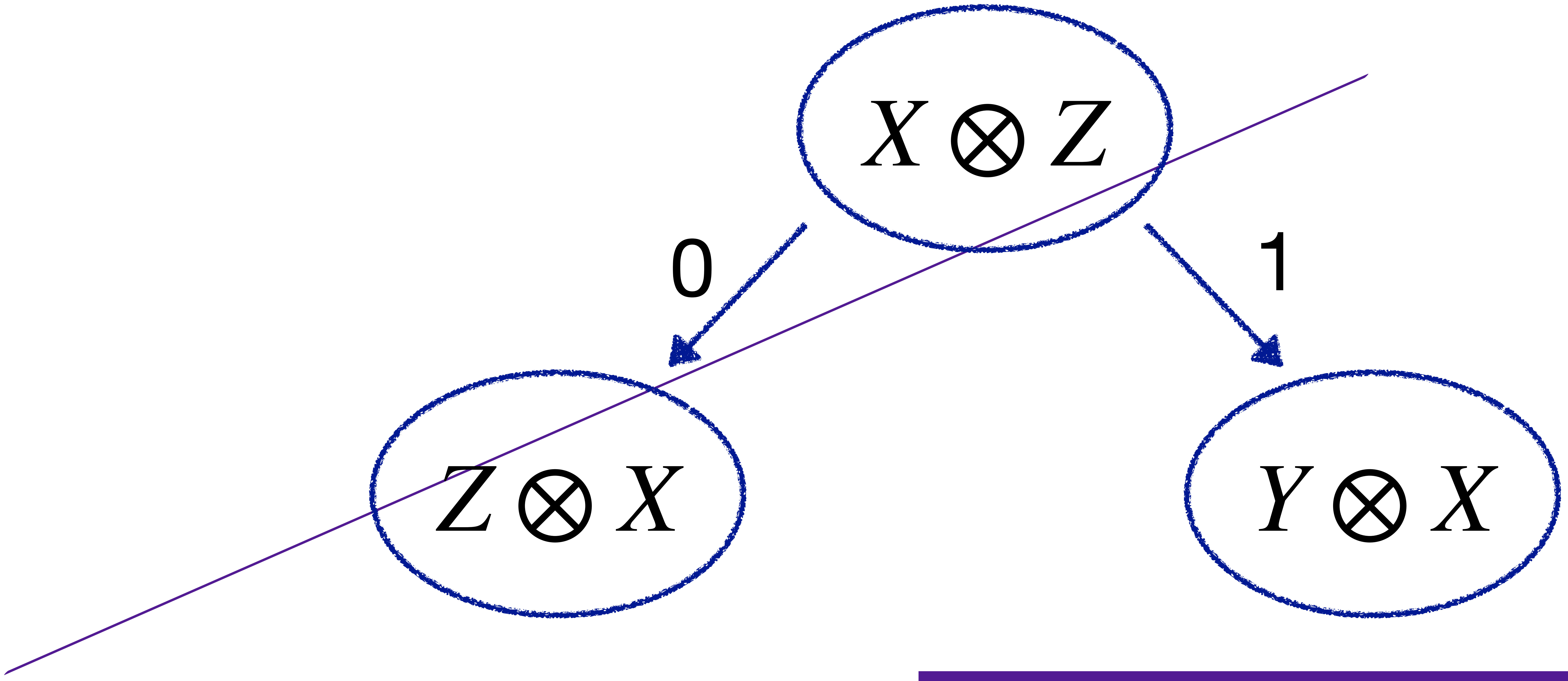
|0⟩
|A⟩
X ⊗ Z
H
???
Z
S
H
Z
−Y
|0⟩
|A⟩
X ⊗ Z
H
−Y ⊗ X
Y ⊗ X

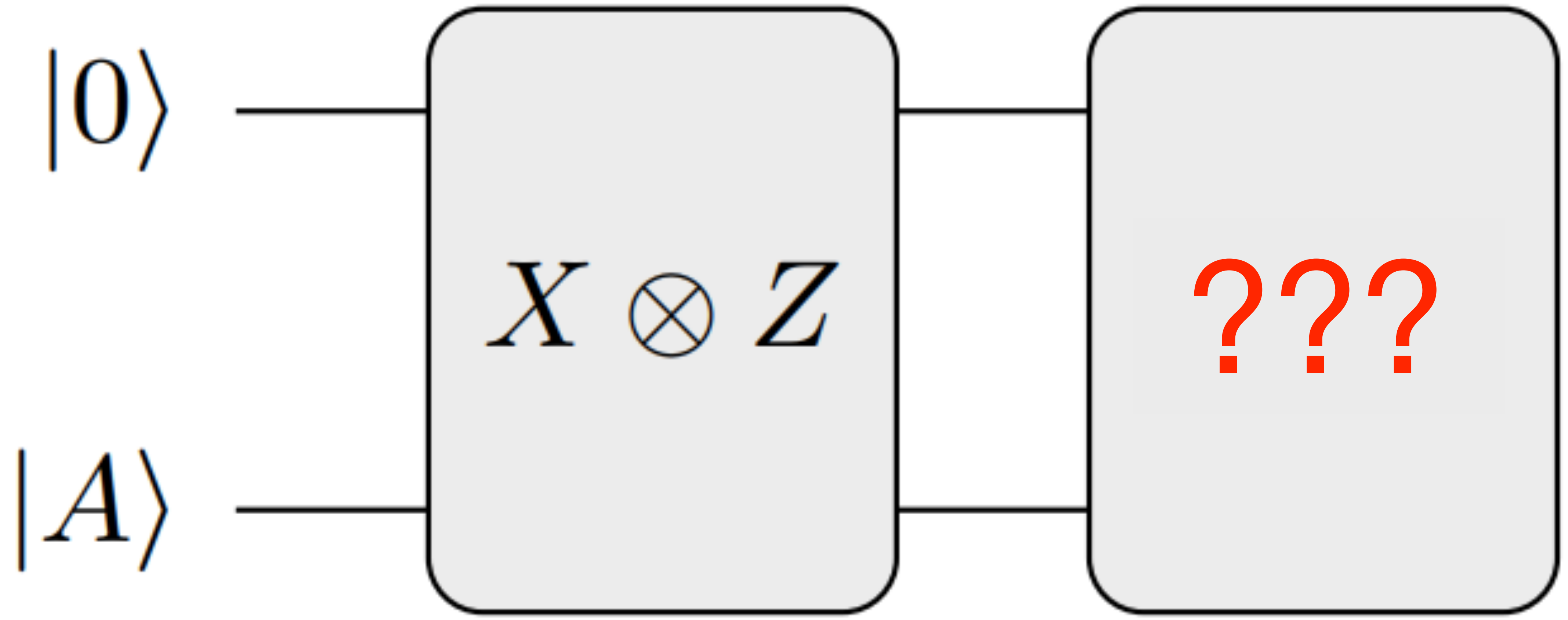

|0⟩
H
T
H
σ


X ⊗ Z
0
1
Z ⊗ X
Y ⊗ X

For a circuit with w measurements and t gates this generalized PBC would have an associated tree with $\mathcal{O}(2^{w+t})$ paths!

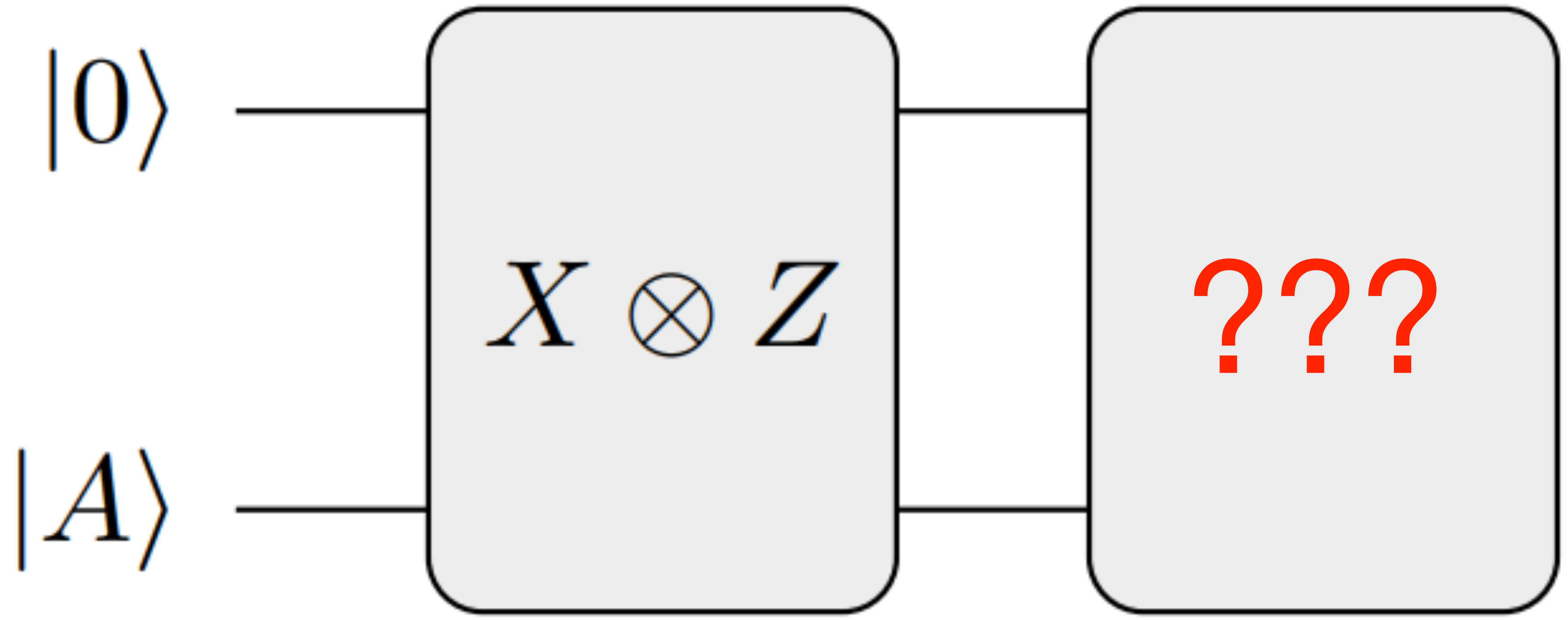

|0⟩
H
T
H
σ


X ⊗ Z
0
1
Z ⊗ X
Y ⊗ X


$|0\rangle$
$|A\rangle$
$X \otimes Z$
???

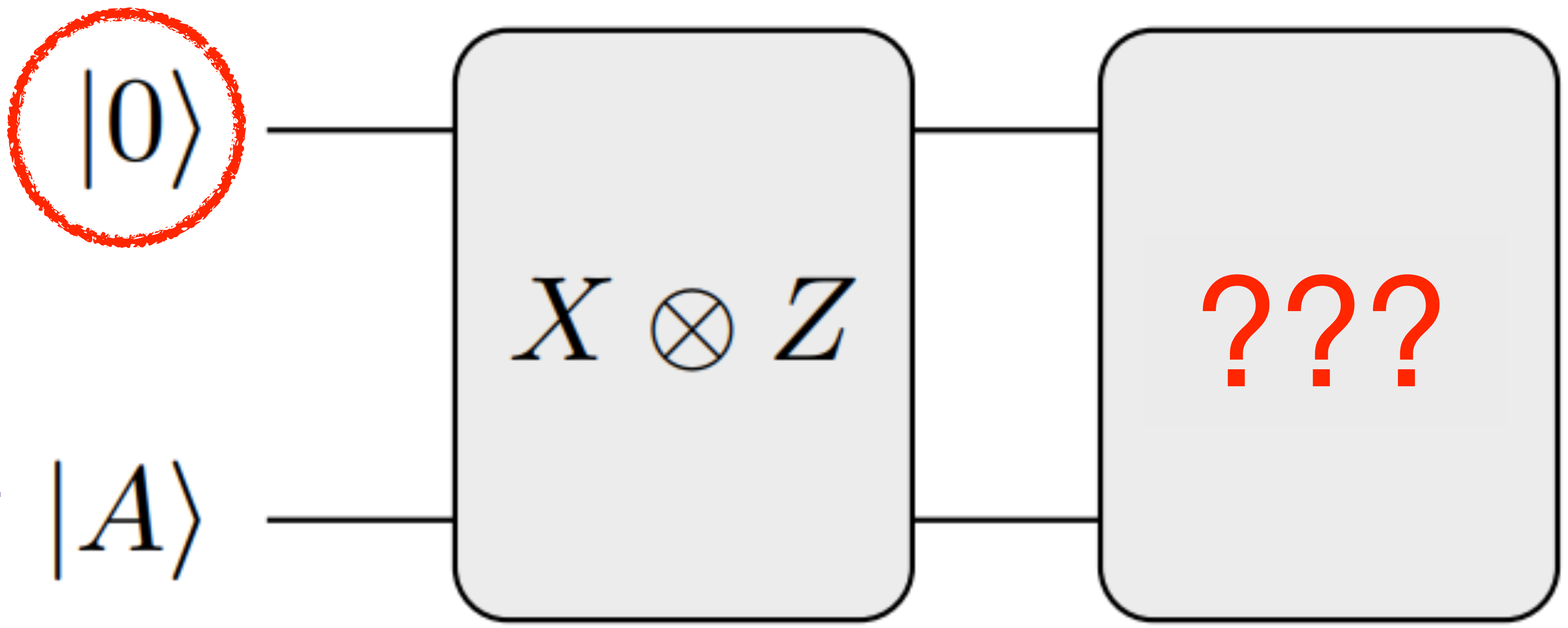
Now…

This does not fit the definition of a standard PBC!



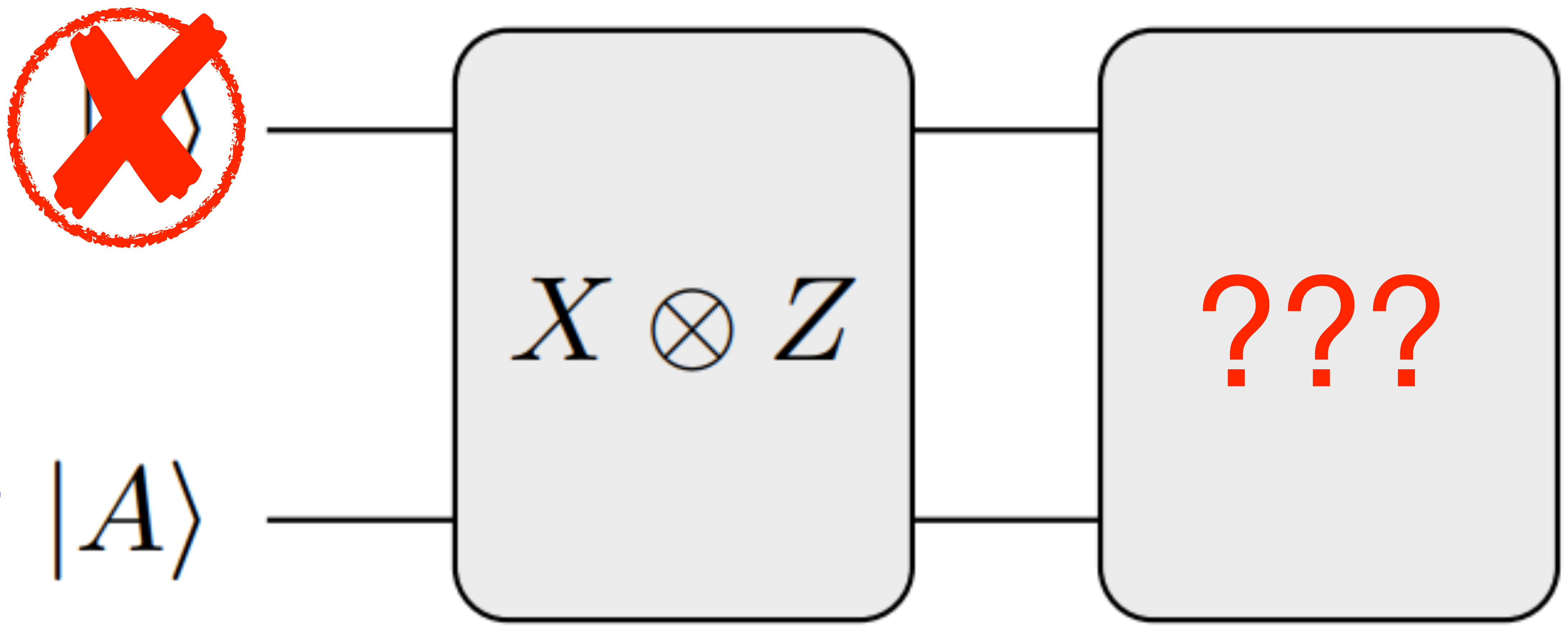
Now…

This does not fit the definition of a standard PBC!



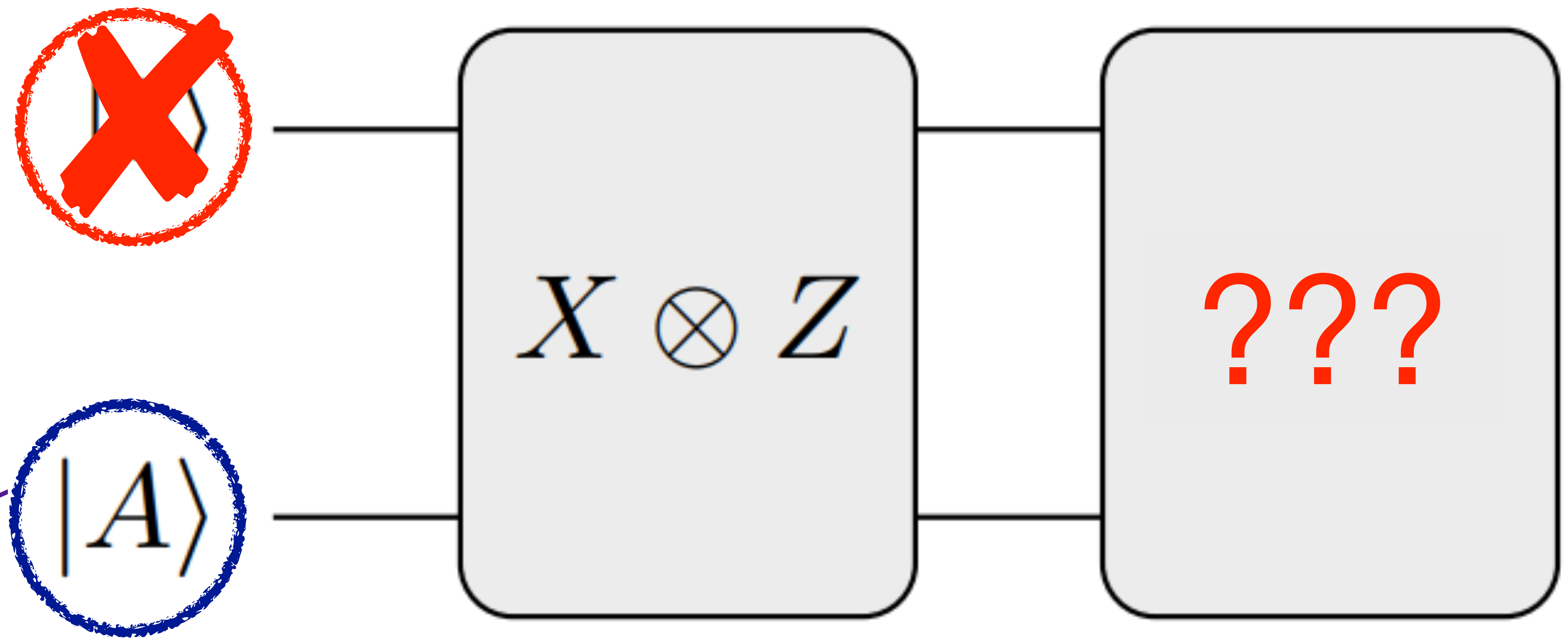
Now…

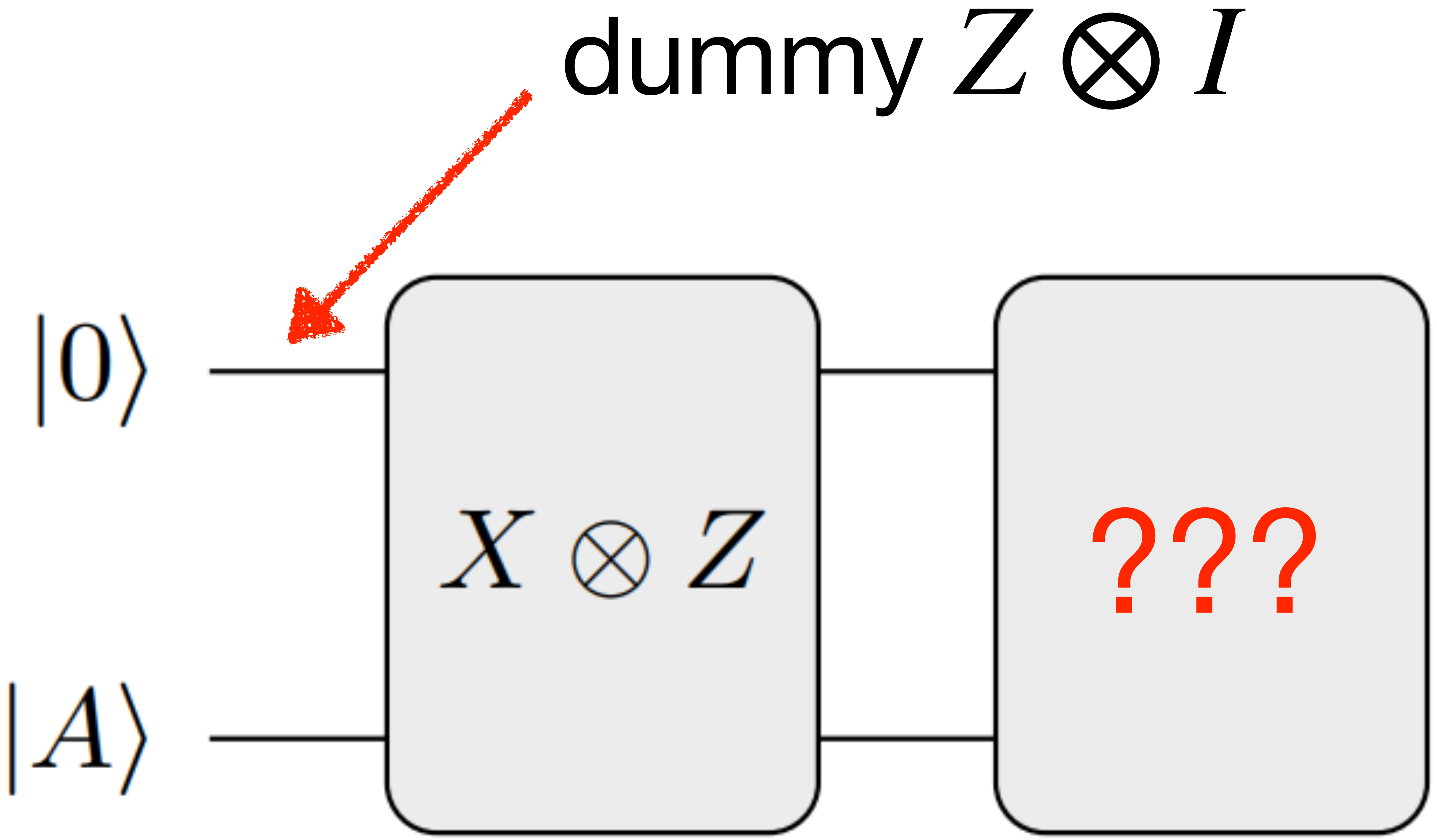
This does not fit the definition of a standard PBC!



Now…

This does not fit the definition of a standard PBC!

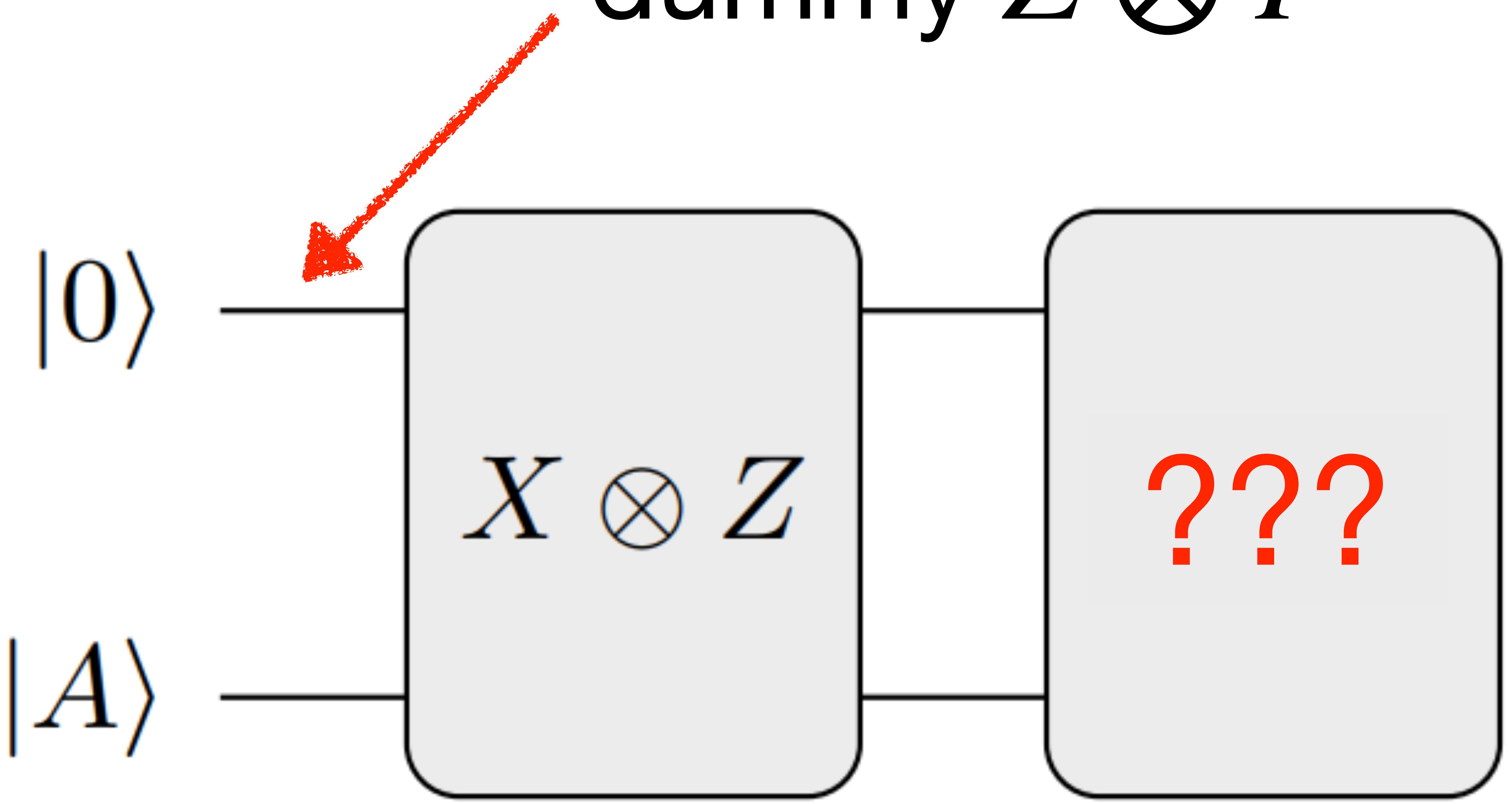




(1) Is P_i dependent on previous measurements?

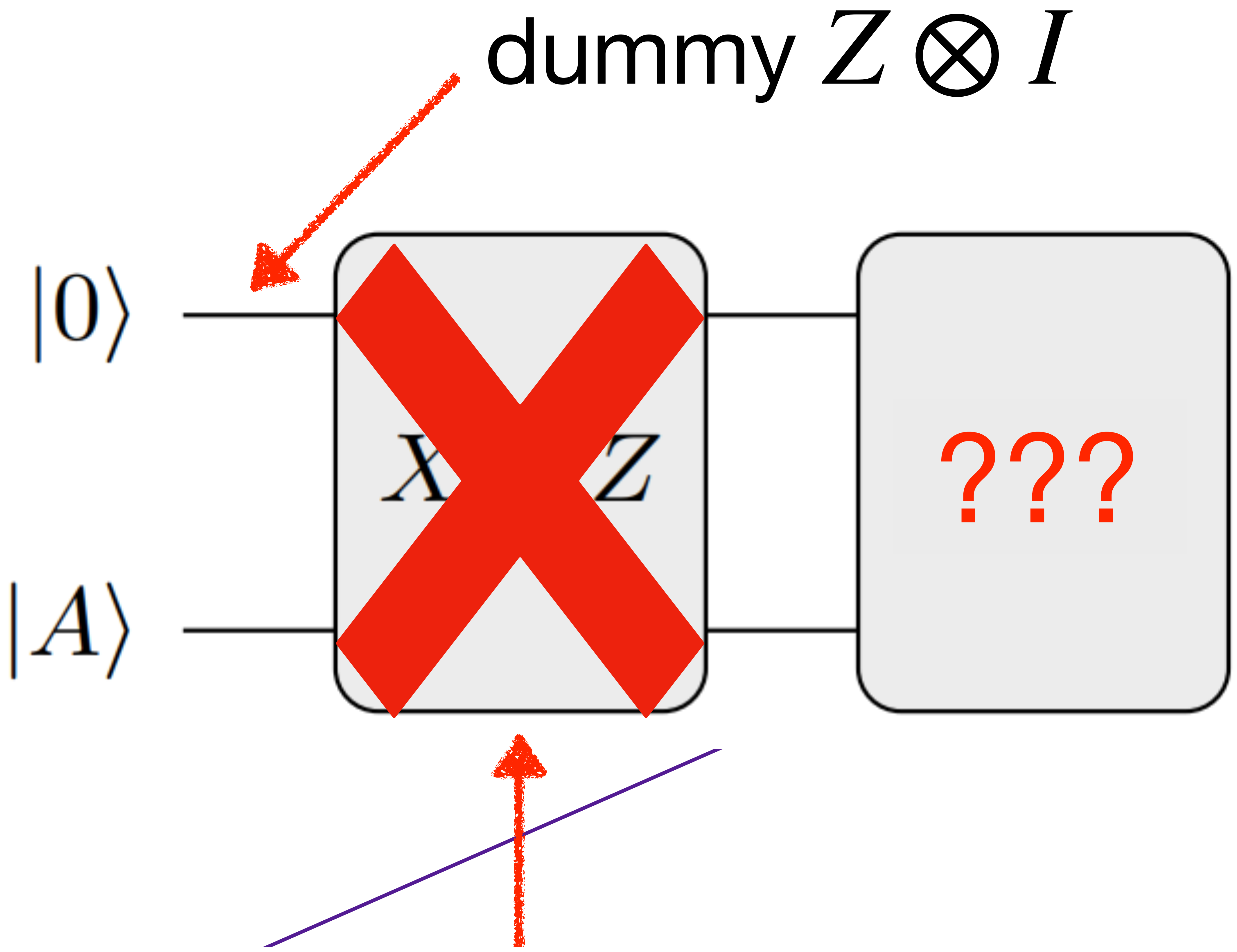
(2) P_i is independent from previous operators.

dummy $Z \otimes I$

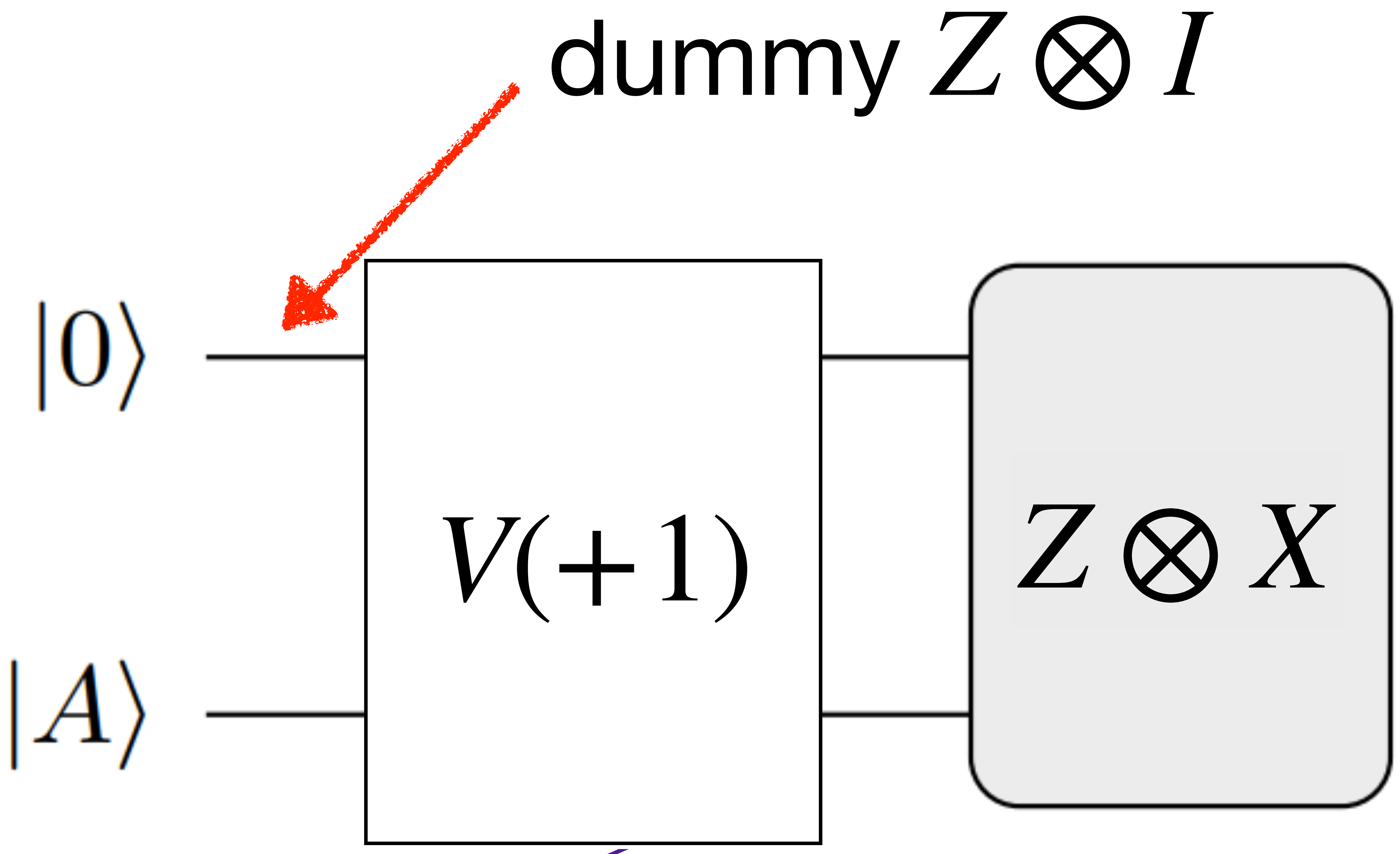


(a) does it commute will all previous measurements?

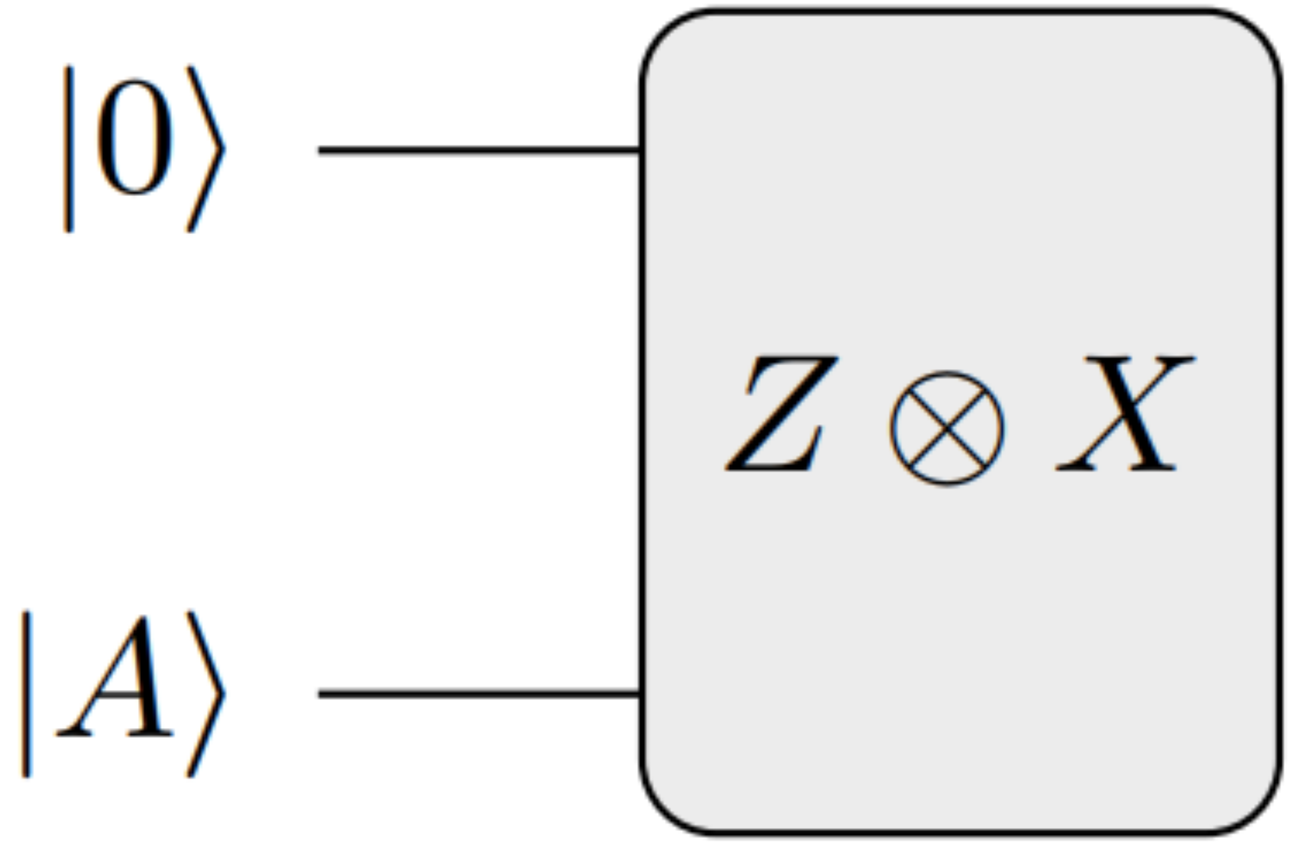
(b) does it anti-commute with a previous measurement?

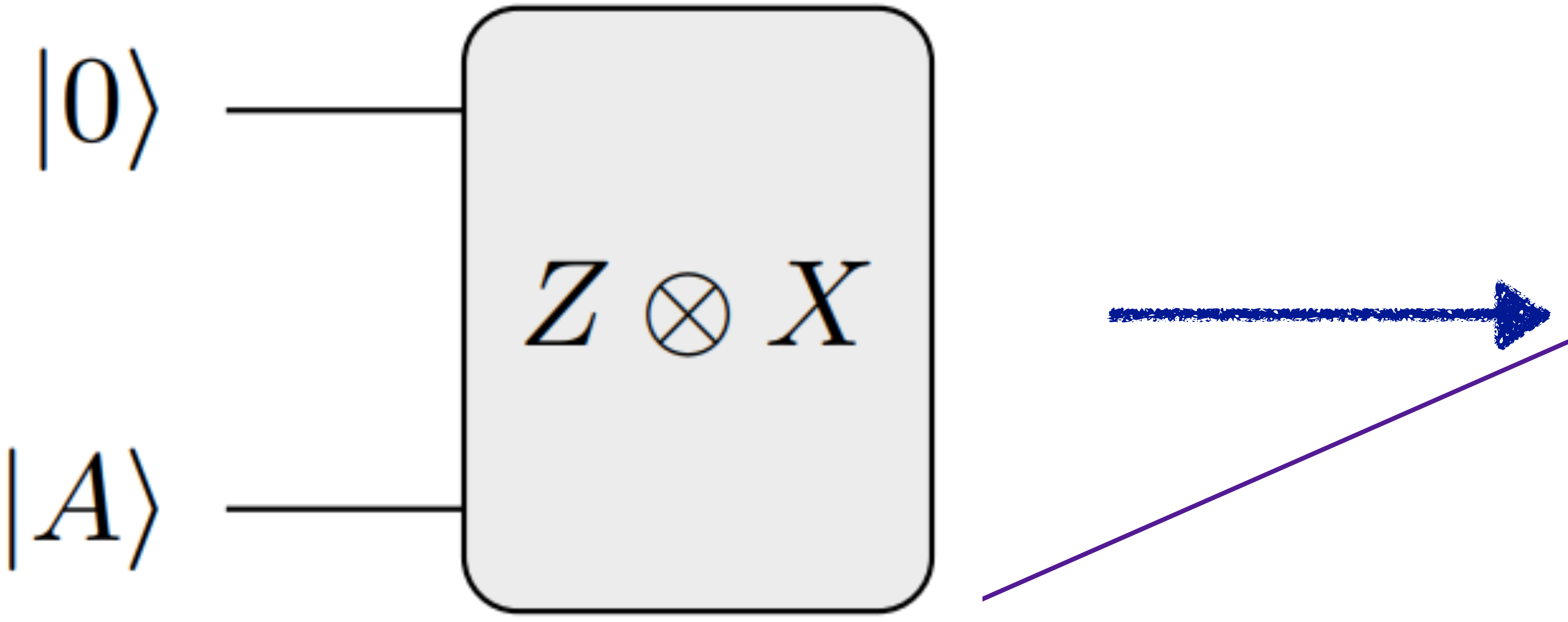


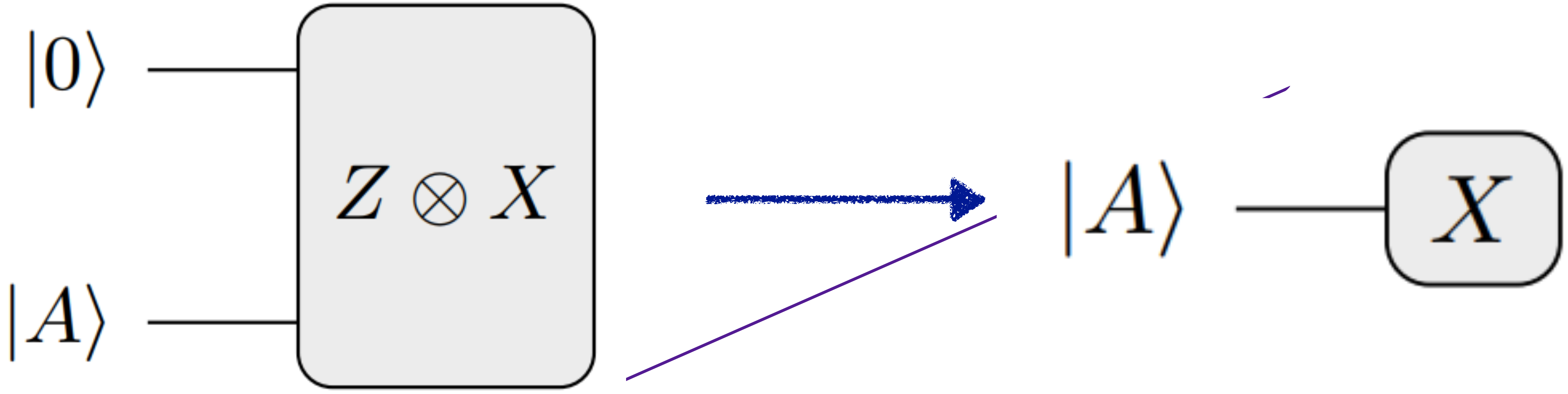
$$V(s) = \frac{Z \otimes I + sX \otimes Z}{\sqrt{2}}$$



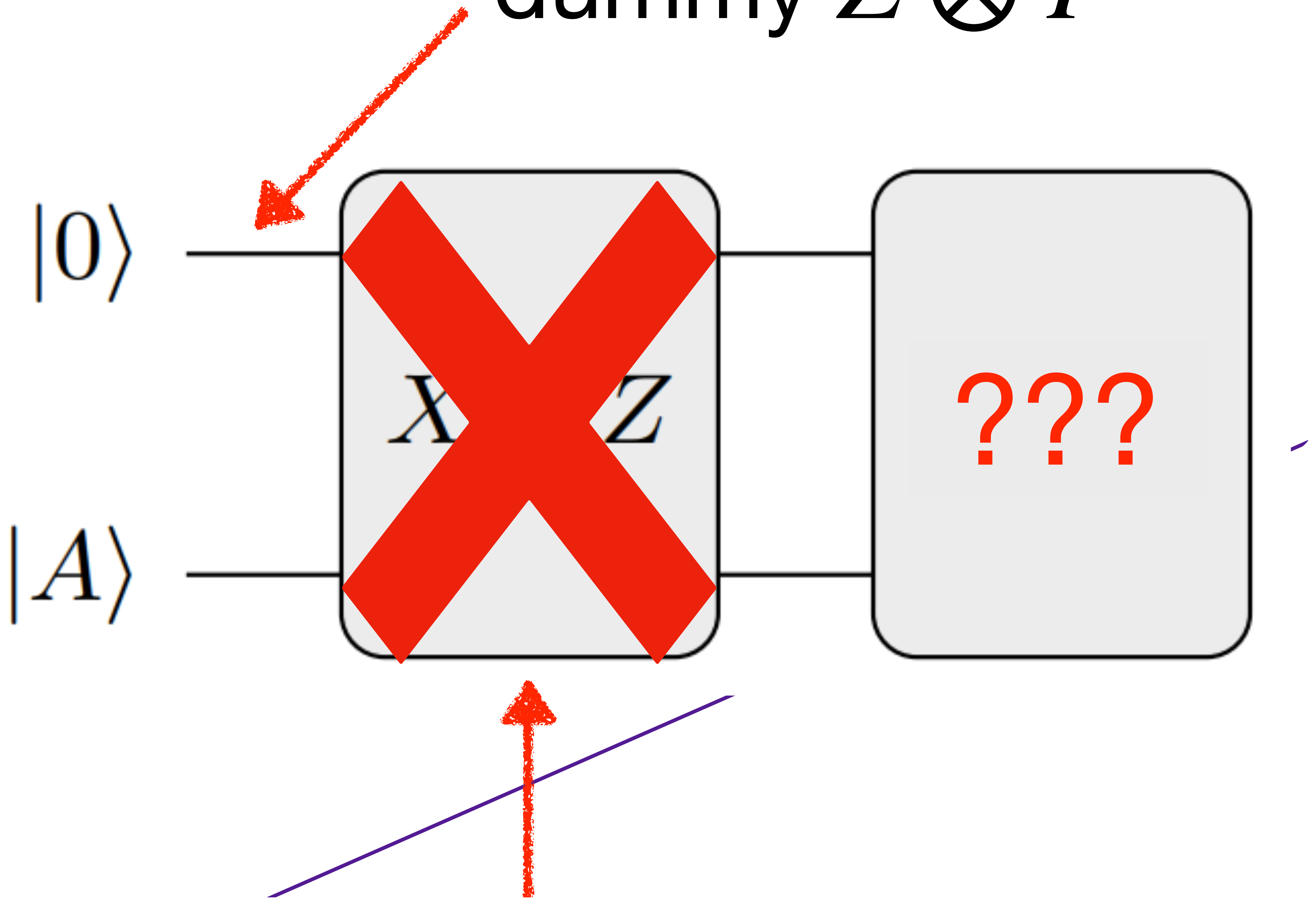
$$V(+1) = (I \otimes H)CX_{12}(H \otimes I)CX_{12}(I \otimes H)$$


$|0\rangle$
$|A\rangle$
$Z \otimes X$

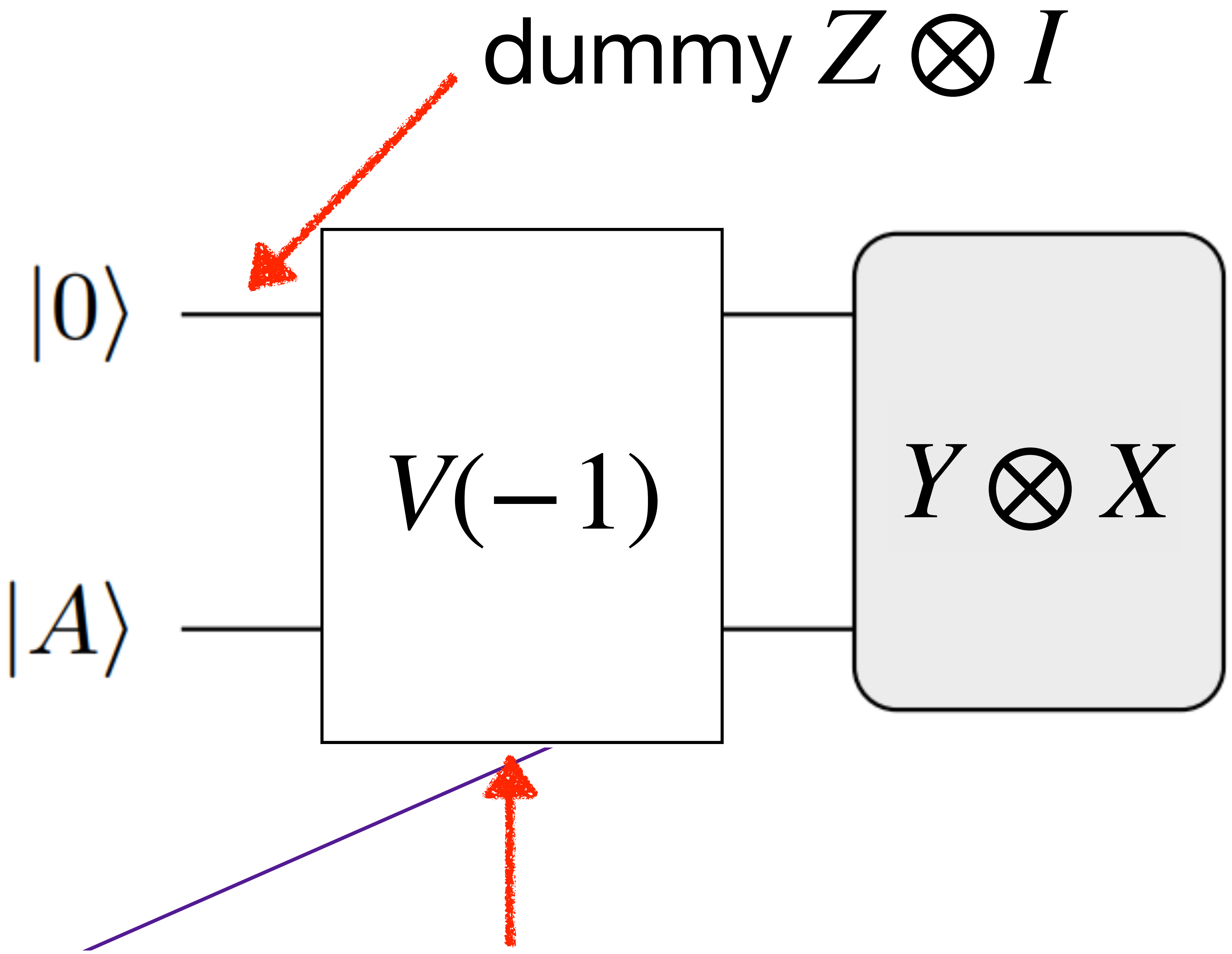

$|0\rangle$
$|A\rangle$
$Z \otimes X$


$|0\rangle$
$|A\rangle$
$Z \otimes X$
$|A\rangle$
X

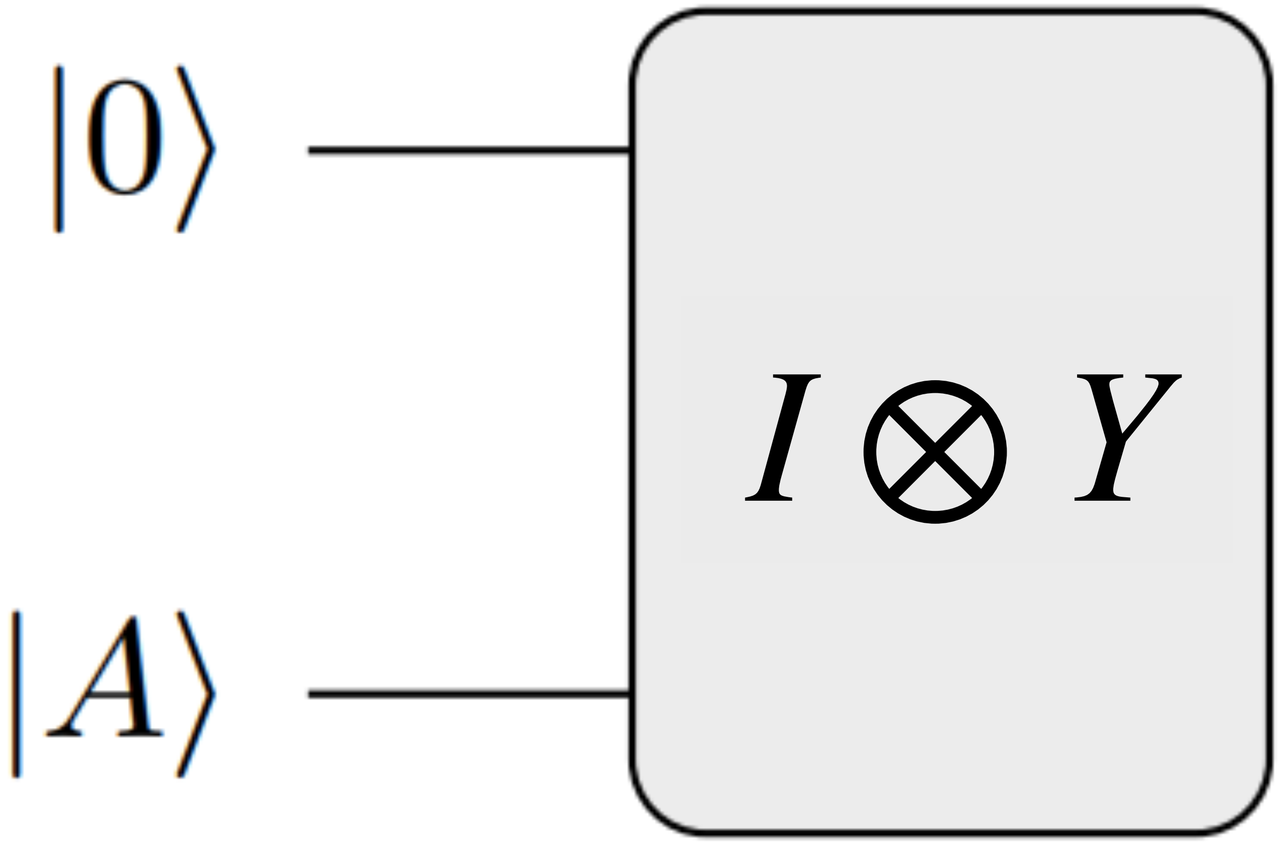
dummy $Z \otimes I$

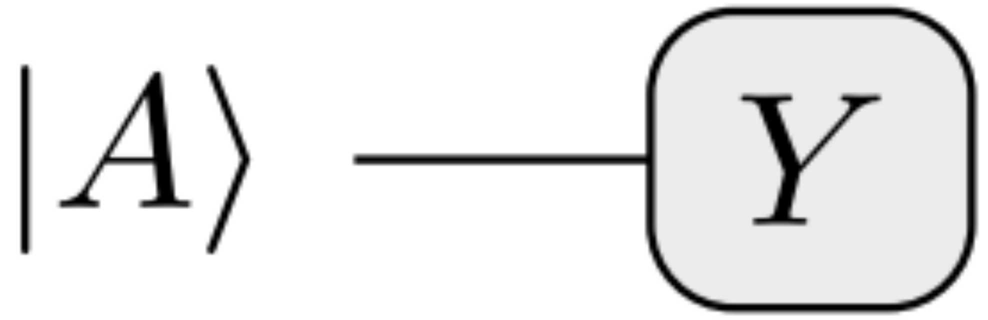


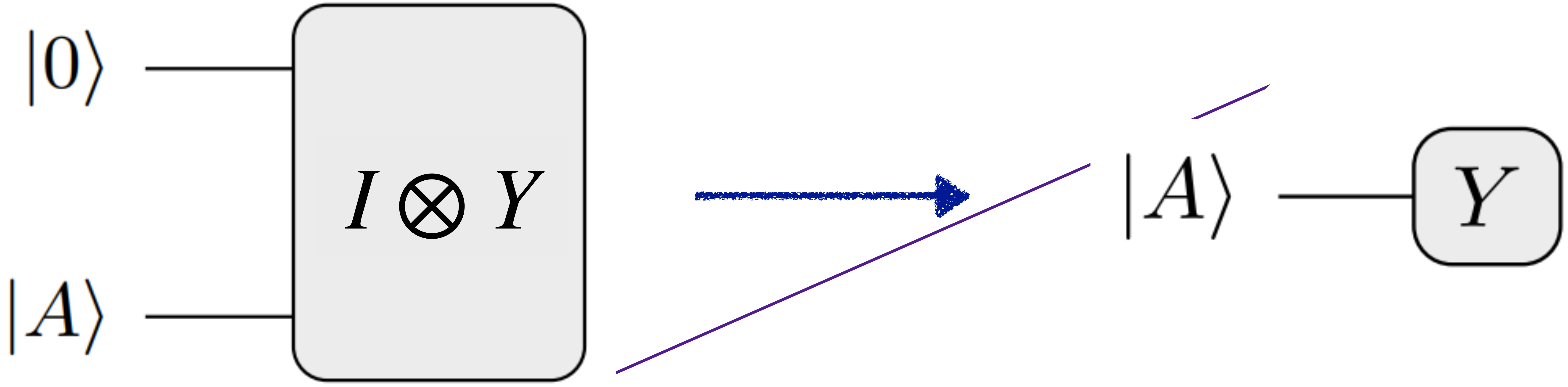
$$V(s) = \frac{Z \otimes I + sX \otimes Z}{\sqrt{2}}$$

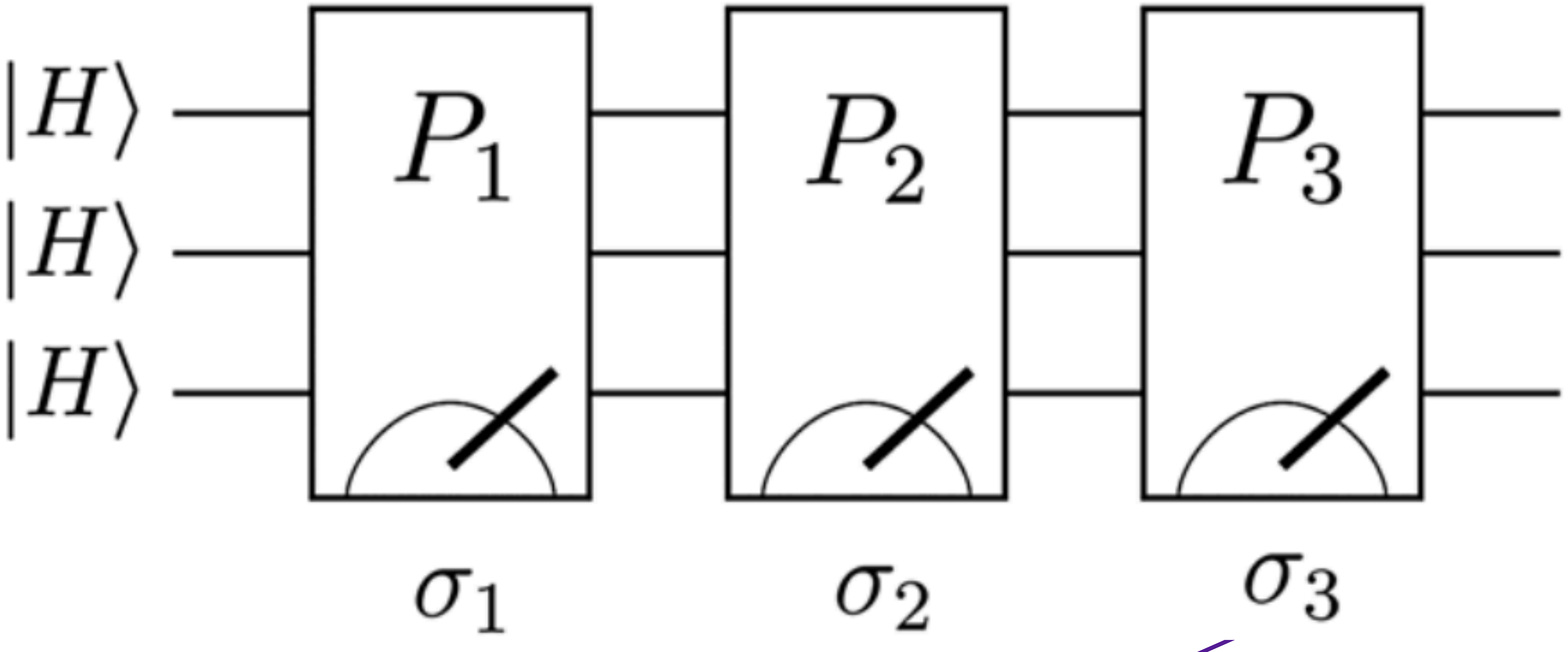


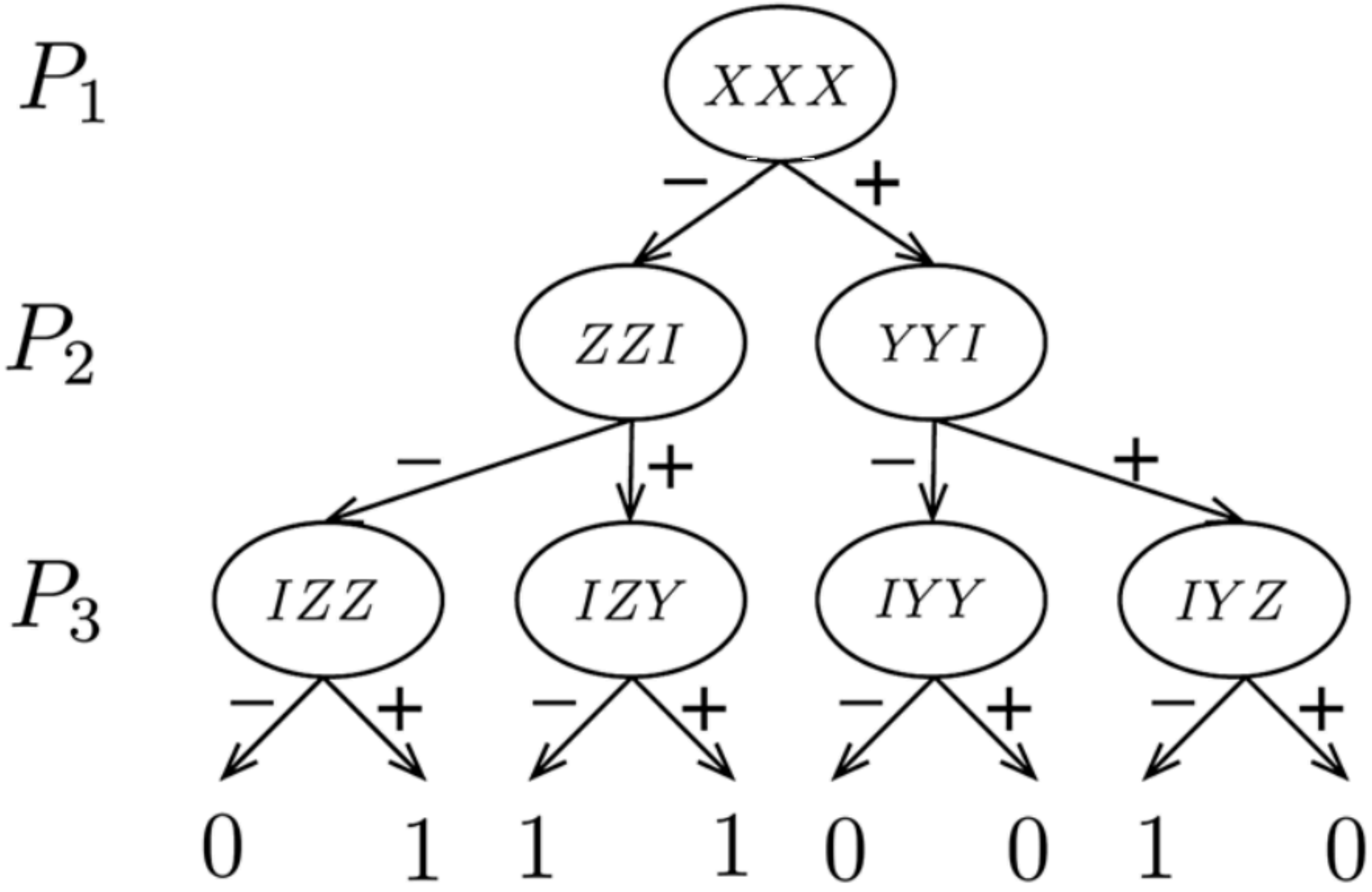
$$V(-1) = (I \otimes H) CX_{12} (ZH \otimes I) CX_{12} (Z \otimes H) .$$


$|0\rangle$
$|A\rangle$
$I \otimes Y$

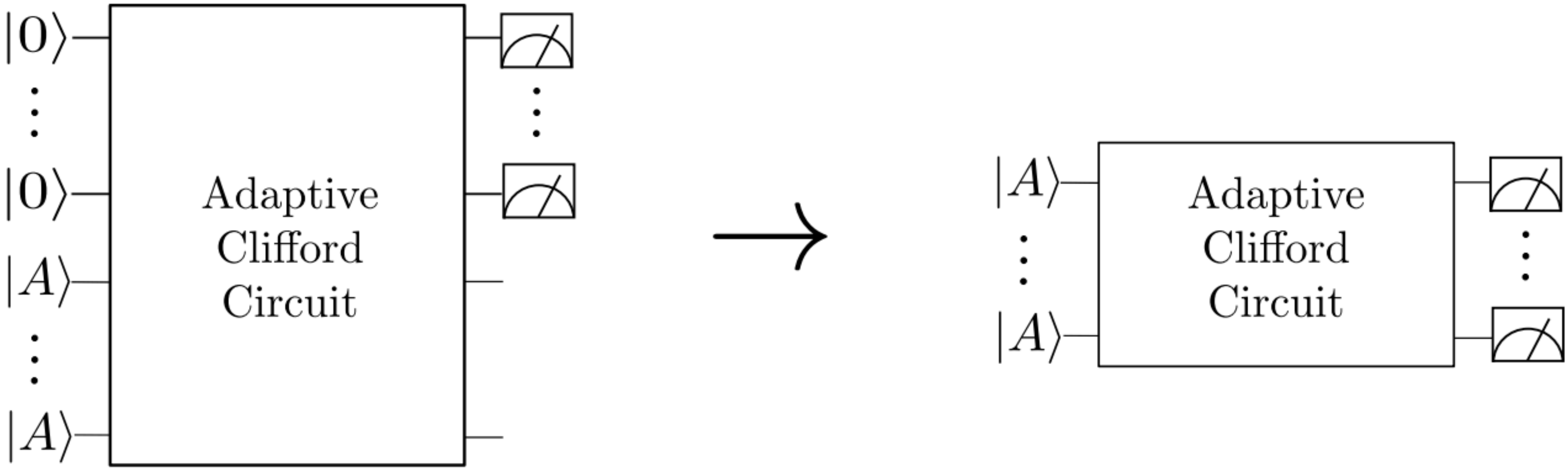

$|A\rangle$
Y


$|0\rangle$
$|A\rangle$
$I \otimes Y$
$|A\rangle$
Y


$|H\rangle$
$|H\rangle$
$|H\rangle$
P_1
P_2
P_3
σ_1
σ_2
σ_3

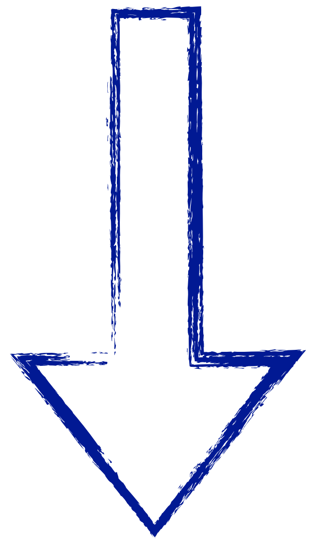

P_1
P_2
P_3
XXX
ZZI
YYI
IZZ
IZY
IYY
IYZ
0 1 1 1 0 0 1 0

Return to the quantum circuit model

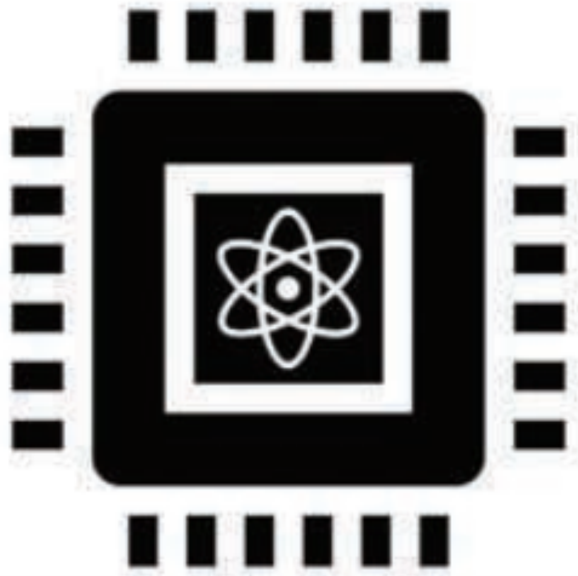


Return to the quantum circuit model

Computation
$n + l$ qubits



n qubits



Concept: [STABILIZER DECOMPOSITIONS]

$$|\psi\rangle = \sum_{i=1}^{\chi} c_i \, |\varphi_i\rangle$$

Concept: [Stabilizer decompositions]

$$|\psi\rangle = \sum_{i=1}^{\chi} c_i \,|\varphi_i\rangle$$

non-stabilizer state

Concept: [STABILIZER DECOMPOSITIONS]

$$|\psi\rangle = \sum_{i=1}^{\chi} c_i |\varphi_i\rangle$$

non-stabilizer state

stabilizer states

Concept: [STABILIZER DECOMPOSITIONS]

stabilizer rank

$$|\psi\rangle = \sum_{i=1}^{\chi} c_i |\varphi_i\rangle$$

non-stabilizer state

stabilizer states

Examples: [SINGLE-QUBIT STATES]

Examples: [SINGLE-QUBIT STATES]

$|0\rangle$

Examples: [Single-qubit states]

$$|0\rangle \;\to\; \chi = 1$$

Examples: [SINGLE-QUBIT STATES]

$$|0\rangle \rightarrow \chi = 1$$

$$\frac{|0\rangle \pm |1\rangle}{\sqrt{2}}$$

Examples: [SINGLE-QUBIT STATES]

$$|0\rangle \rightarrow \chi = 1$$

$$\frac{|0\rangle \pm |1\rangle}{\sqrt{2}} \rightarrow \chi = 1$$

Examples: [Single-qubit states]

Examples: [Single-qubit states]

$$\frac{|0\rangle \pm i\,|1\rangle}{\sqrt{2}}$$

Examples: [SINGLE-QUBIT STATES]

$$\frac{|0\rangle \pm i\,|1\rangle}{\sqrt{2}} \rightarrow \ \chi = 1$$

Examples: [Single-qubit states]

$$\frac{|0\rangle \pm i\,|1\rangle}{\sqrt{2}} \rightarrow \chi = 1$$

$$\frac{|0\rangle + e^{i\pi/4}\,|1\rangle}{\sqrt{2}}$$

Examples: [Single-qubit states]

$$\frac{|0\rangle \pm i\,|1\rangle}{\sqrt{2}} \rightarrow \chi = 1$$

$$\frac{|0\rangle + e^{i\pi/4}\,|1\rangle}{\sqrt{2}} \rightarrow \chi = 2$$

Examples: [2-QUBIT STATES]

Examples: [2-QUBIT STATES]

$$\frac{|00\rangle \pm |11\rangle}{\sqrt{2}}$$

Examples: [2-QUBIT STATES]

$$\frac{|00\rangle \pm |11\rangle}{\sqrt{2}} \rightarrow \chi = 1$$

Examples: [2-QUBIT STATES]

$$\frac{|00\rangle \pm |11\rangle}{\sqrt{2}} \rightarrow \chi = 1$$

$$\frac{|01\rangle \pm |10\rangle}{\sqrt{2}}$$

Examples: [2-QUBIT STATES]

$$\frac{|00\rangle \pm |11\rangle}{\sqrt{2}} \rightarrow \chi = 1$$

$$\frac{|01\rangle \pm |10\rangle}{\sqrt{2}} \rightarrow \chi = 1$$

Examples: [2-QUBIT STATES]

Examples: [2-QUBIT STATES]

$$|A\rangle^{\otimes 2} = \frac{1}{2}\left(|0\rangle + e^{i\pi/4}|1\rangle\right)\left(|0\rangle + e^{i\pi/4}|1\rangle\right)$$

Examples: [2-QUBIT STATES]

$$\left|A\right\rangle^{\otimes 2} = \frac{1}{2}\left(\left|0\right\rangle + e^{i\pi/4}\left|1\right\rangle\right)\left(\left|0\right\rangle + e^{i\pi/4}\left|1\right\rangle\right)$$

$$= \frac{1}{2}\left(\left|00\right\rangle + e^{i\pi/4}\left|01\right\rangle + e^{i\pi/4}\left|10\right\rangle + i\left|11\right\rangle\right)$$

Examples: [2-QUBIT STATES]

$$\left|A\right\rangle^{\otimes 2} = \frac{1}{2}\left(\left|0\right\rangle + e^{i\pi/4}\left|1\right\rangle\right)\left(\left|0\right\rangle + e^{i\pi/4}\left|1\right\rangle\right)$$

$$= \frac{1}{2}\left(\left|00\right\rangle + e^{i\pi/4}\left|01\right\rangle + e^{i\pi/4}\left|10\right\rangle + i\left|11\right\rangle\right)$$

$$\chi = 4$$

Examples: [2-QUBIT STATES]

$$\left|A\right\rangle^{\otimes 2} = \frac{1}{2}\left(\left|0\right\rangle + e^{i\pi/4}\left|1\right\rangle\right)\left(\left|0\right\rangle + e^{i\pi/4}\left|1\right\rangle\right)$$

$$= \frac{1}{2}\left(\left|00\right\rangle + e^{i\pi/4}\left|01\right\rangle + e^{i\pi/4}\left|10\right\rangle + i\left|11\right\rangle\right)$$

$\chi = 4$

Examples: [2-QUBIT STATES]

$$|A\rangle^{\otimes 2} = \frac{1}{2}\left(|0\rangle + e^{i\pi/4}|1\rangle\right)\left(|0\rangle + e^{i\pi/4}|1\rangle\right)$$

$$= \frac{1}{2}\left(|00\rangle + e^{i\pi/4}|01\rangle + e^{i\pi/4}|10\rangle + i|11\rangle\right)$$

$\chi = 4$ $\chi = 2$

Examples: [2-QUBIT STATES]

$$|A\rangle^{\otimes 2} = \frac{1}{2}\left(|0\rangle + e^{i\pi/4}|1\rangle\right)\left(|0\rangle + e^{i\pi/4}|1\rangle\right)$$

$$= \frac{1}{2}\left(|00\rangle + e^{i\pi/4}|01\rangle + e^{i\pi/4}|10\rangle + i|11\rangle\right)$$

$\chi = 4$ ✗

$\chi = 2$ ✓

Examples: [2-QUBIT STATES]

Examples: [2-QUBIT STATES]

$$|A\rangle^{\otimes 2} = \frac{1}{2}\left(|00\rangle + e^{i\pi/4}|01\rangle + e^{i\pi/4}|10\rangle + i|11\rangle\right)$$

Examples: [2-QUBIT STATES]

$$|A\rangle^{\otimes 2} = \frac{1}{2}\Big(|00\rangle + e^{i\pi/4}|01\rangle + e^{i\pi/4}|10\rangle + i|11\rangle\Big)$$

$$= \frac{|00\rangle + i|11\rangle}{\sqrt{2}} + e^{i\pi/4}\frac{|01\rangle + |10\rangle}{\sqrt{2}}$$

Examples: [2-QUBIT STATES]

$$|A\rangle^{\otimes 2} = \frac{1}{2}\left(|00\rangle + e^{i\pi/4}|01\rangle + e^{i\pi/4}|10\rangle + i|11\rangle\right)$$

$$= \frac{|00\rangle + i|11\rangle}{\sqrt{2}} + e^{i\pi/4}\frac{|01\rangle + |10\rangle}{\sqrt{2}}$$

Examples: [2-QUBIT STATES]

$$|A\rangle^{\otimes 2} = \frac{1}{2}\left(|00\rangle + e^{i\pi/4}|01\rangle + e^{i\pi/4}|10\rangle + i|11\rangle\right)$$

$$= \frac{|00\rangle + i|11\rangle}{\sqrt{2}} + e^{i\pi/4}\frac{|01\rangle + |10\rangle}{\sqrt{2}}$$

Examples: [2-QUBIT STATES]

$$|A\rangle^{\otimes 2} = \frac{1}{2}\left(|00\rangle + e^{i\pi/4}|01\rangle + e^{i\pi/4}|10\rangle + i|11\rangle\right)$$

$$= \frac{|00\rangle + i|11\rangle}{\sqrt{2}} + e^{i\pi/4}\frac{|01\rangle + |10\rangle}{\sqrt{2}} \quad \chi = 2$$

stabilizer rank of $|A\rangle^{\otimes n}$ $\Longleftrightarrow$

n	χ_n
1	2
2	2
3	3
4	4
5	6
6	7

Concept: [STABILIZER DECOMPOSITIONS]

stabilizer rank

$$|\psi\rangle = \sum_{i=1}^{\chi} c_i \, |\varphi_i\rangle$$

non-stabilizer state

stabilizer states

Concept: [STABILIZER DECOMPOSITIONS]

$$|\psi\rangle\langle\psi| = \sum_{i=1}^{\chi'} c_i' |\varphi_i\rangle\langle\varphi_i|$$

Concept: [Stabilizer decompositions]

$$|\psi\rangle\langle\psi| = \sum_{i=1}^{\chi'} c_i' |\varphi_i\rangle\langle\varphi_i|$$

non-stabilizer state

Concept: [STABILIZER DECOMPOSITIONS]

$$\left|\psi\right\rangle\left\langle\psi\right| = \sum_{i=1}^{\chi'} c_i' \left|\varphi_i\right\rangle\left\langle\varphi_i\right|$$

non-stabilizer state

stabilizer states

Concept: [Stabilizer decompositions]

stabilizer rank

$$\left|\psi\right\rangle\left\langle\psi\right| = \sum_{i=1}^{\chi'} c_i' \left|\varphi_i\right\rangle\left\langle\varphi_i\right|$$

non-stabilizer state

stabilizer states

Concept: [STABILIZER DECOMPOSITIONS]

stabilizer rank

real coefficients

$$|\psi\rangle\langle\psi| = \sum_{i=1}^{\chi'} c_i' |\varphi_i\rangle\langle\varphi_i|$$

non-stabilizer state

stabilizer states

Examples: [Single-qubit states]

Examples: [SINGLE-QUBIT STATES]

$$|A\rangle = \frac{|0\rangle + e^{i\pi/4}|1\rangle}{\sqrt{2}}$$

Examples: [SINGLE-QUBIT STATES]

$$|A\rangle = \frac{|0\rangle + e^{i\pi/4}|1\rangle}{\sqrt{2}} \quad \rightarrow \quad \chi = 2$$

Examples: [Single-qubit states]

$$|A\rangle = \frac{|0\rangle + e^{i\pi/4}|1\rangle}{\sqrt{2}} \quad \rightarrow \quad \chi = 2$$

$$|A\rangle\langle A| = \frac{1}{2}|+\rangle\langle+| + \frac{1-\sqrt{2}}{2}|-\rangle\langle-| + \frac{\sqrt{2}}{2}\left|+_y\right\rangle\left\langle+_y\right|$$

Examples: [Single-qubit states]

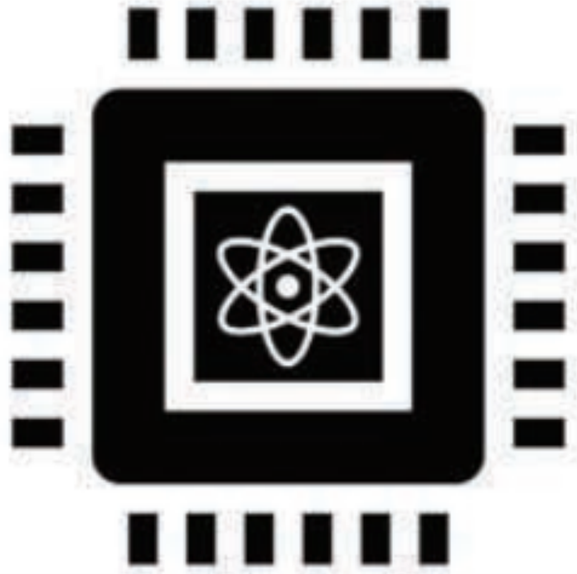
$$|A\rangle = \frac{|0\rangle + e^{i\pi/4}|1\rangle}{\sqrt{2}} \quad \rightarrow \quad \chi = 2$$

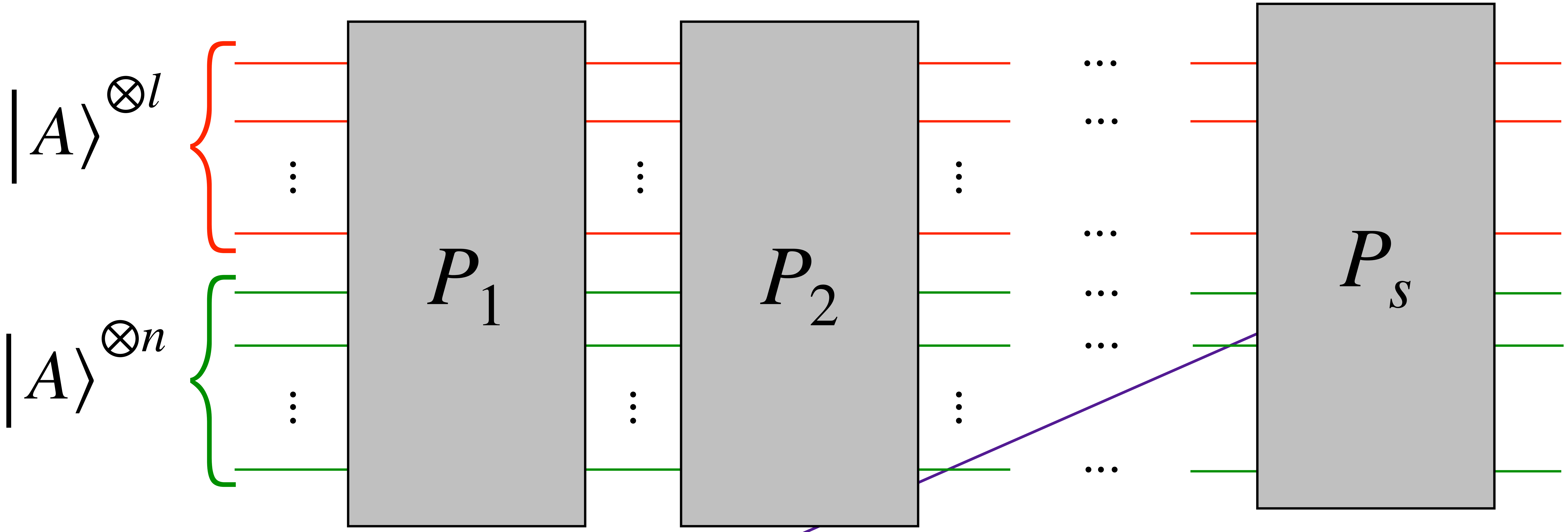
$$|A\rangle\langle A| = \frac{1}{2}|+\rangle\langle +| + \frac{1-\sqrt{2}}{2}|-\rangle\langle -| + \frac{\sqrt{2}}{2}|+_y\rangle\langle +_y|$$

$$\chi' = 3$$

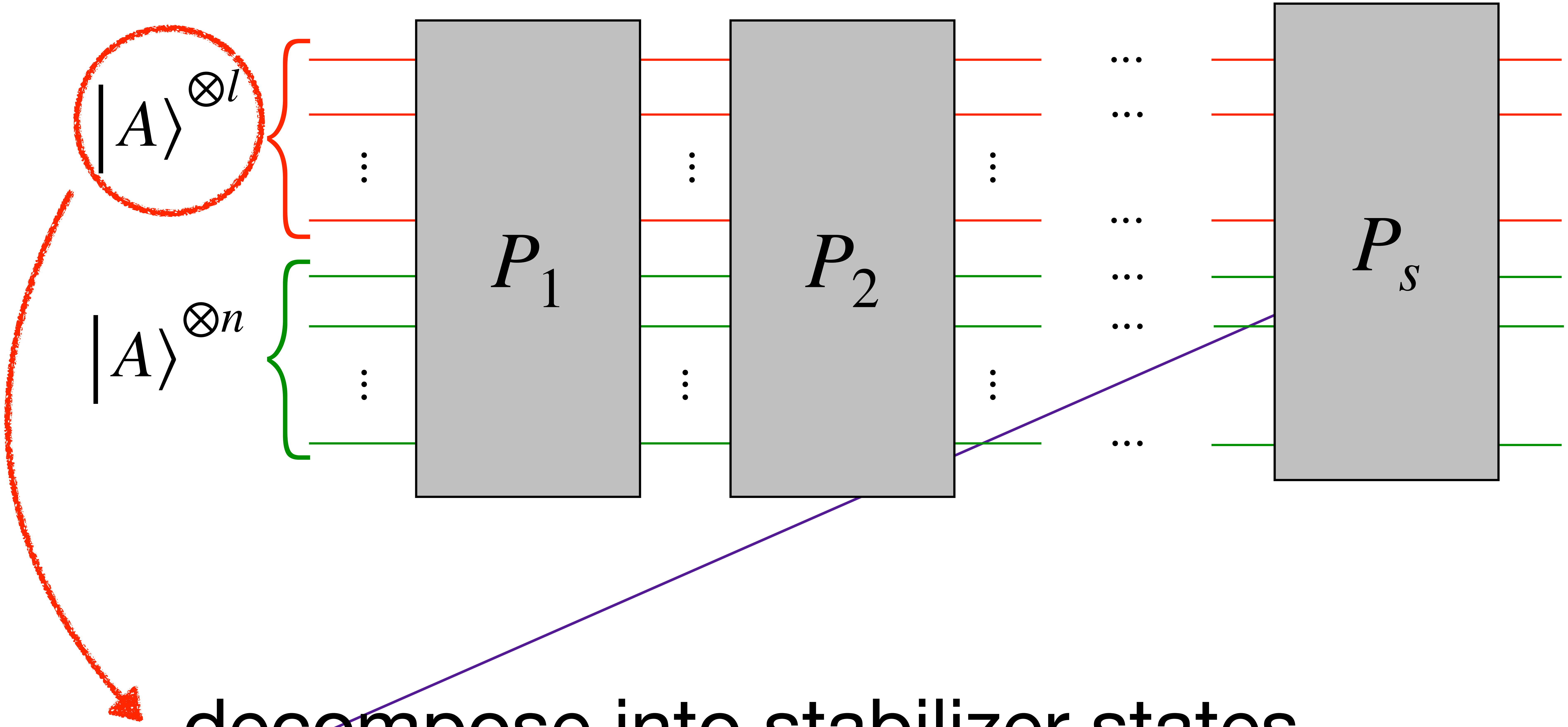
Computation
$n + l$ qubits

n qubits

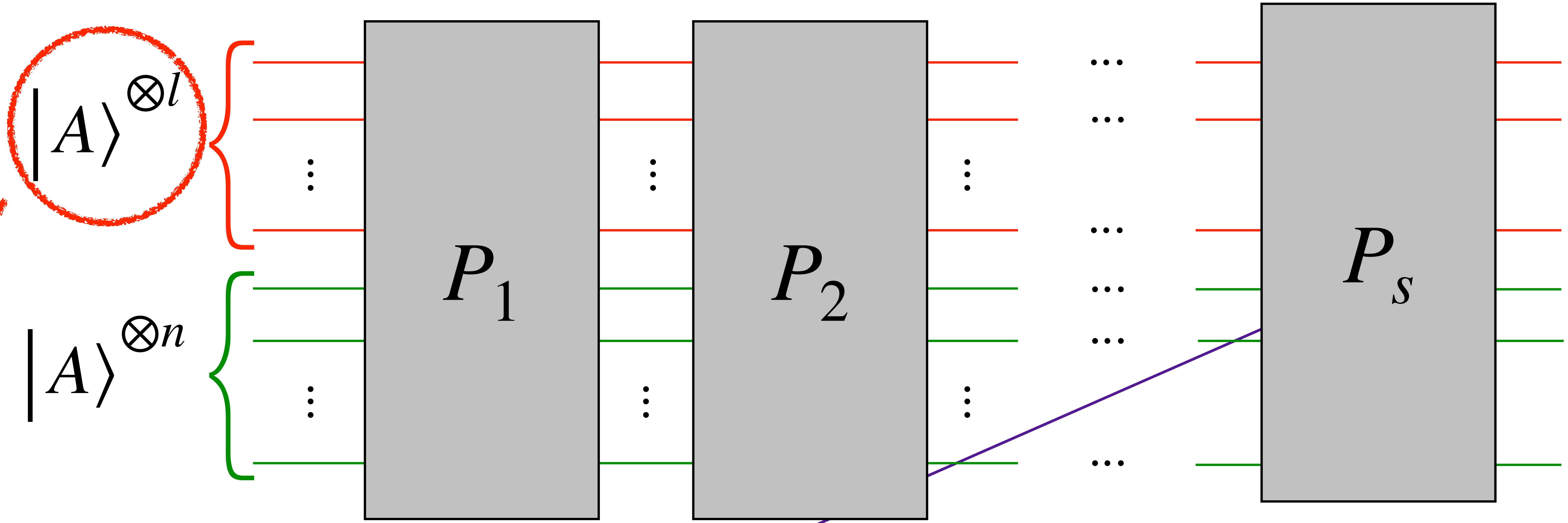




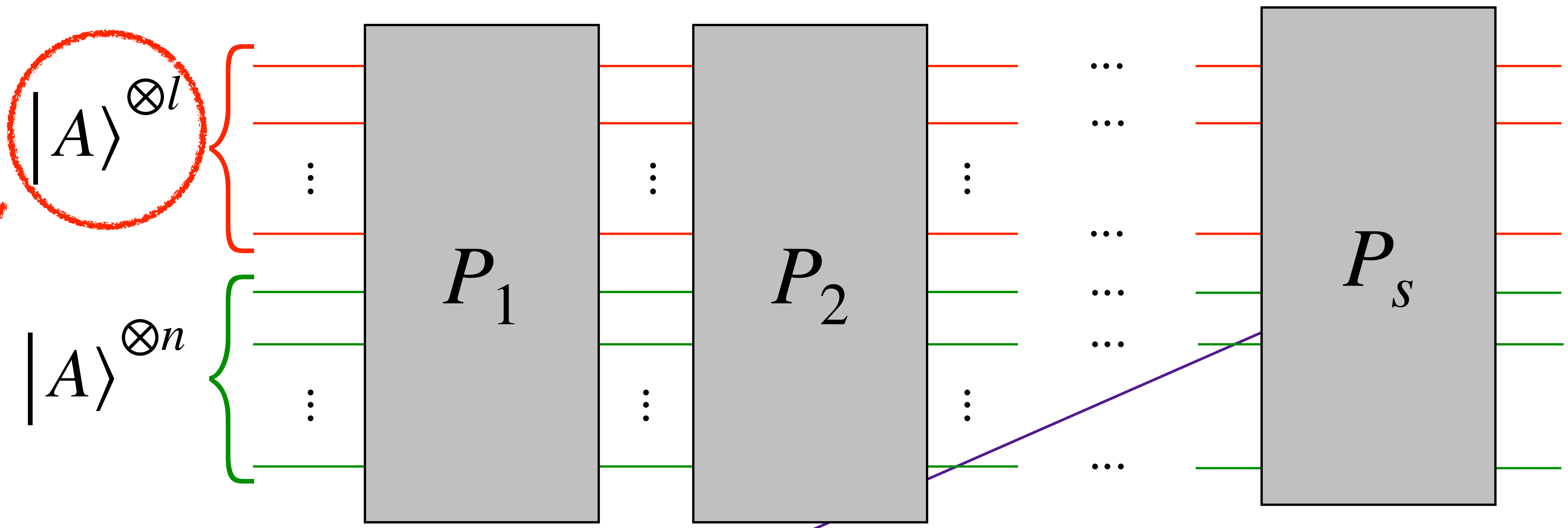
$|A\rangle^{\otimes l}$
$|A\rangle^{\otimes n}$
P_1
P_2
P_s



decompose into stabilizer states

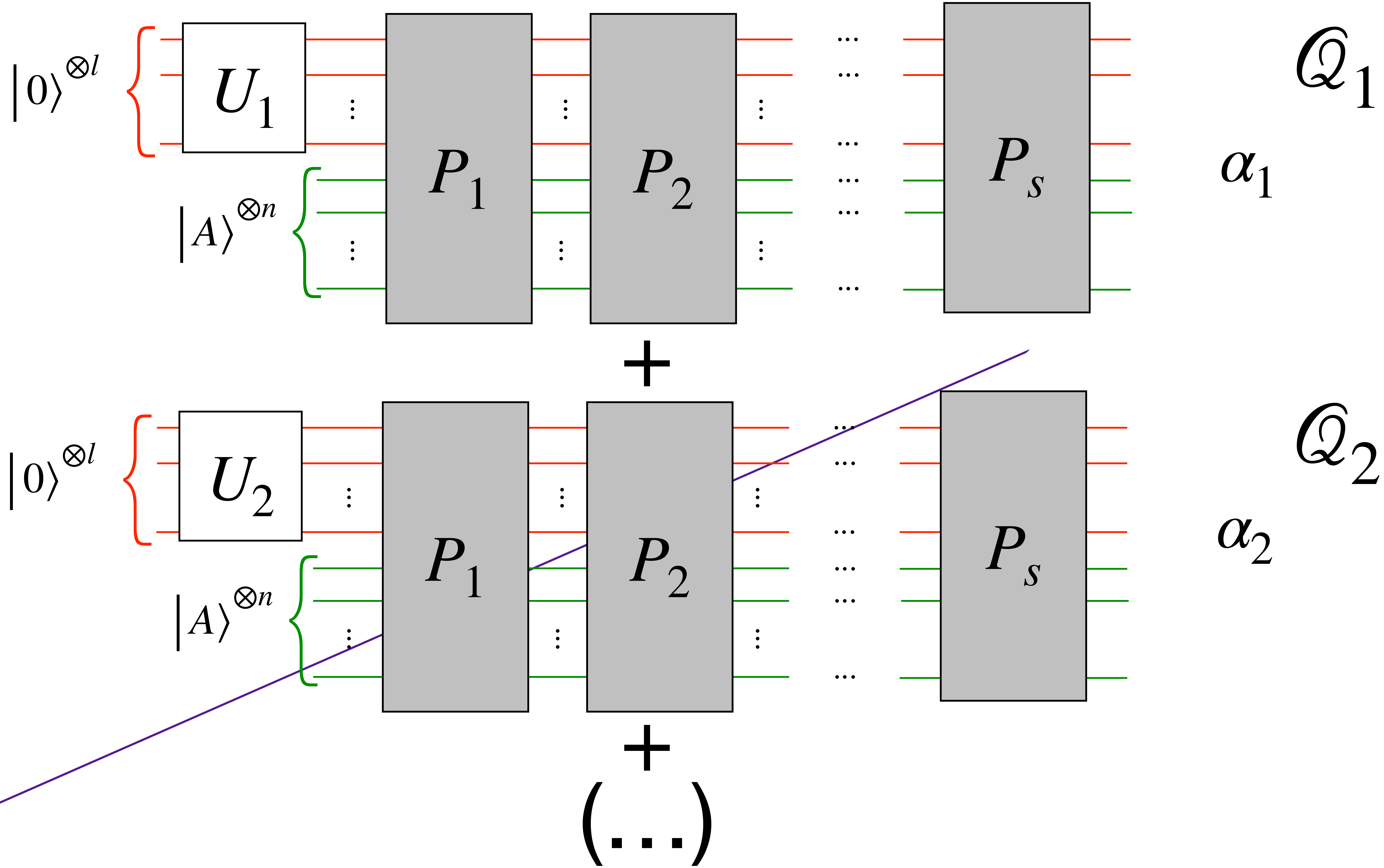


$$|A\rangle\langle A|^{\otimes l} = \sum_{i=1}^{\chi} \alpha_i \, |\varphi_i\rangle\langle\varphi_i|$$



$$|A\rangle\langle A|^{\otimes l} = \sum_{i=1}^{\chi} \alpha_i |\varphi_i\rangle\langle\varphi_i|;$$

$$|\varphi_i\rangle = U_i |0\rangle^{\otimes l}$$



$|0\rangle^{\otimes l}$
U_1
$|A\rangle^{\otimes n}$
P_1
P_2
P_s
$\mathcal{Q}_1$
α_1
+
$|0\rangle^{\otimes l}$
U_2
$|A\rangle^{\otimes n}$
P_1
P_2
P_s
$\mathcal{Q}_2$
α_2
+
(...)

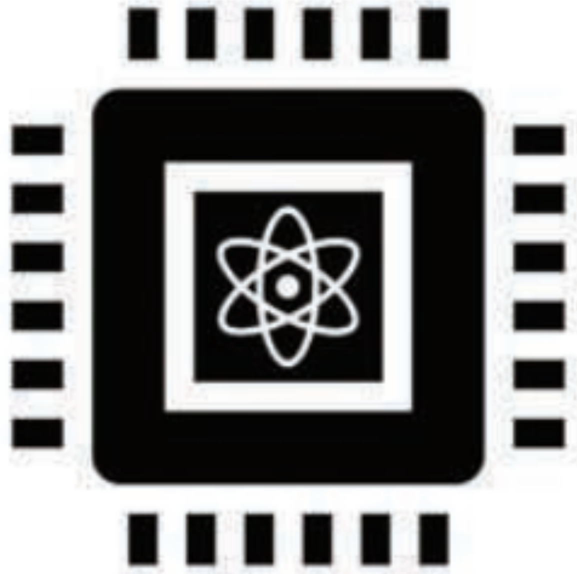
$$\text{Linearity} \Rightarrow p(\mathcal{Q}) = \sum_{i=1}^{\chi} p(\mathcal{Q}_i)$$

Theorem: A PBC on $n + l$ qubits can be simulated by $\chi = 2^{\mathcal{O}(l)}$ PBCs on n qubits, and a classical processing that takes time $2^{\mathcal{O}(l)} poly(n)$.

Computation
$n + l$ qubits

↓

n qubits



Thank you for your attention!