Quantum Hamiltonian learning

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Outline

- Learning a quantum system
- Bayesian Inference and Quantum Hamiltonian Learning (QHL)
- Using QHL for magnetic field sensing
- Can we improve our Models?

- Wiebe et al., Hamiltonian Learning and Certification Using Quantum Resources. Phys. Rev. Lett. 112, (2014)
- Wang et al., Experimental quantum Hamiltonian learning - Nature Physics 1, 149 (2017)
- Santagati et al., Magnetic-field-learning using a single electronic spin in diamond… - Phys. Rev. X (2019)
- Gentile et al., Learning models of quantum systems from experiments – arXiv:2002.06169 (2020)
- Flynn et al., Exploring acyclic graphs for the study of quantum systems – manuscript in preparation (2020s)

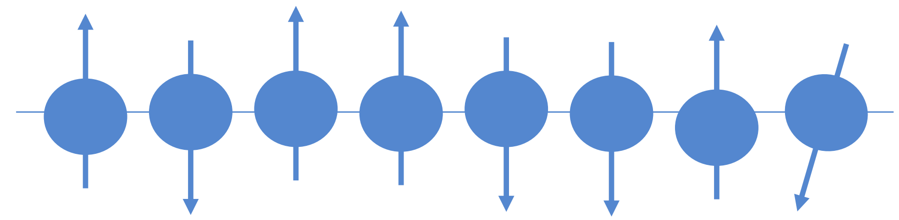
Characterisation of quantum systems:

Characterizing a quantum systems means capturing all the main features that can fully describe its interactions and its dynamics. e.g. by fully learning its Hamiltonian operator

$$H(t)\,|\,\psi(t)\rangle = i\hbar \frac{d}{dt}\,|\,\psi(t)\rangle$$

Why is hard?

Quantum Simulation



Quantum systems are hard to simulate.

If we study a system comprising N particles with two basis states and we want to store their quantum state on a classical computer using an 8 bits (1 byte) resolution.

1 p → 2 bytes $\alpha|\uparrow\rangle + \beta|\downarrow\rangle$

2 p → 6 bytes $\alpha|\uparrow\uparrow\rangle + \beta|\uparrow\downarrow\rangle + \gamma|\downarrow\uparrow\rangle + \delta|\downarrow\downarrow\rangle$

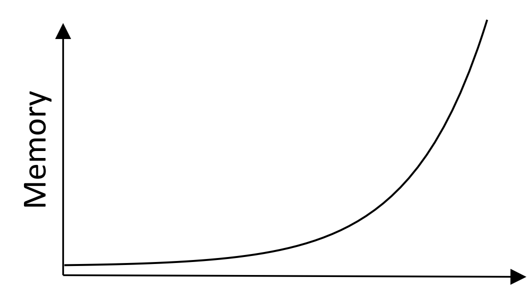
3 p → 14 bytes $\alpha|\uparrow\uparrow\uparrow\rangle + \beta|\uparrow\uparrow\downarrow\rangle + \gamma|\uparrow\downarrow\uparrow\rangle + \ldots$

⋮

200 p → ~ $3.21\ 10^{60}$ bytes

Total memory available on earth today is ~ $5.21\ 10^{21}$ bytes

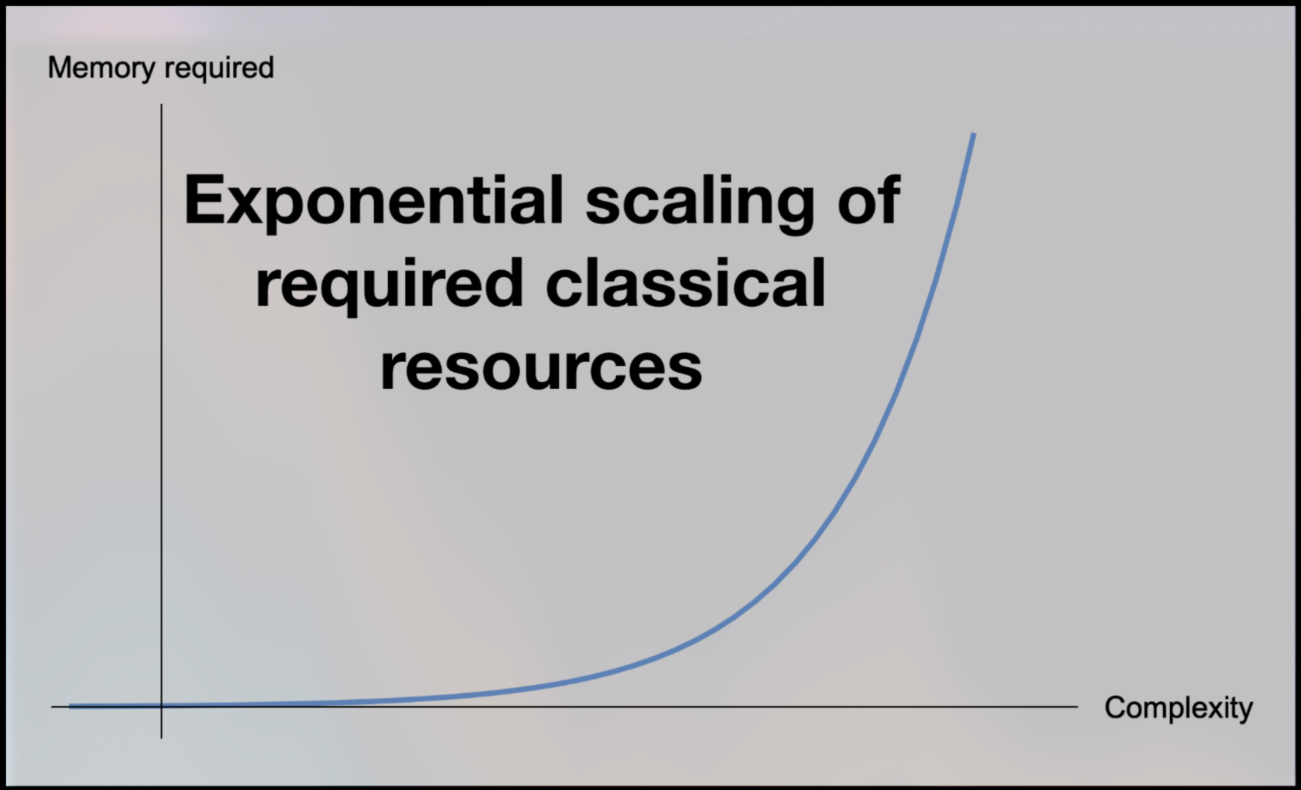
Total number of atoms on earth is ~$133*10^{48}$



$$2^{N+1} - 2$$

The total amount of memory required grows exponentially with the value of N.

Quantum Hamiltonian Learning



The problem becomes rapidly intractable with classical machines.

Quantum simulation is expected to efficiently reproduce the dynamics of quantum systems.

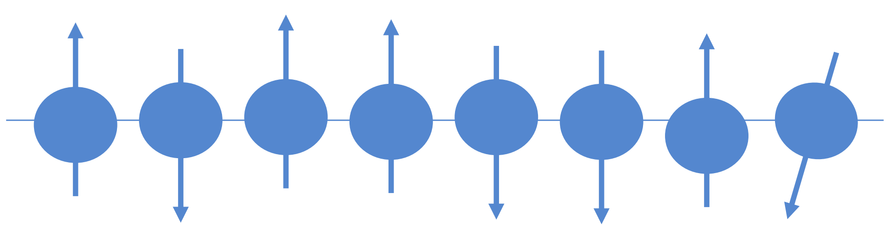
An algorithm that, under relatively weak assumptions, can be used to efficiently infer the Hamiltonian of a large but uncharacterised quantum system using a characterised quantum simulator.

(N. Wiebe, et al. - Phys. Rev. Lett. 112, 190501 (2014))

Quantum Hamiltonian Learning

QHL aims to find efficiently the set of parameters $\vec{\Omega}_0$ which best describe the dynamic of the system.

Quantum System



Model of the system (H)

$$\hat{H}(\vec{\Omega}) = \sum_{i,jG} \Omega_{i,j} \hat{\sigma}_z^i \hat{\sigma}_z^j$$

$\hat{H}(\vec{\Omega})$ with parameters $\vec{\Omega}$

Wiebe et al. Phys. Rev. Lett. 112, 190501 (2014)

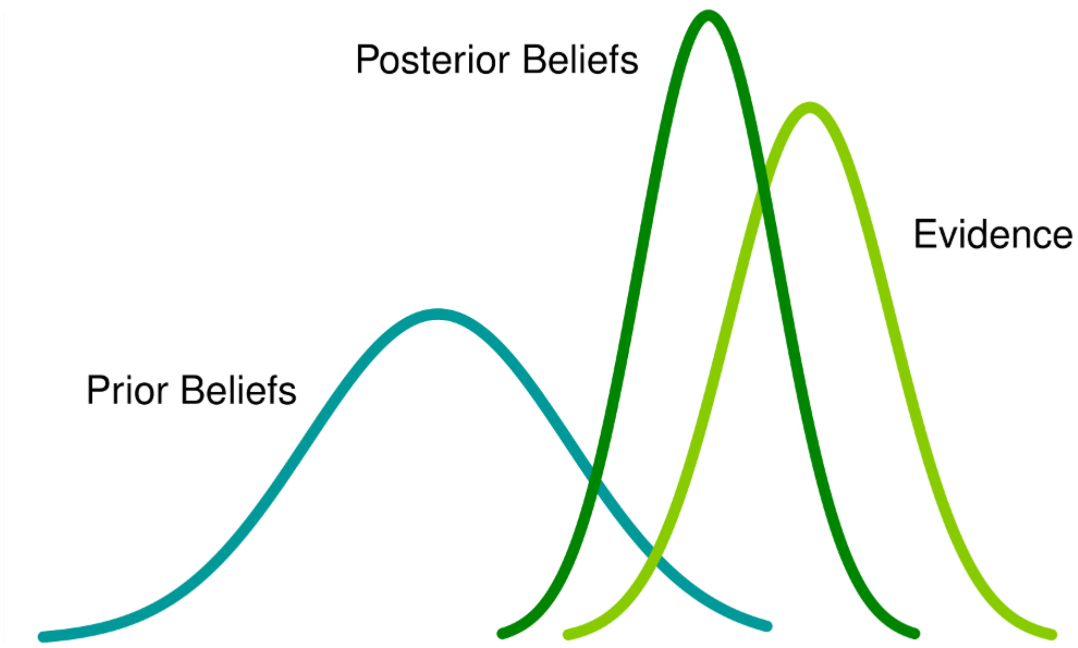
Frequentists vs Bayesian

Probability defined as frequency

$$P(E) = \frac{n_E}{N}$$

Confidence Intervals are not probability distributions

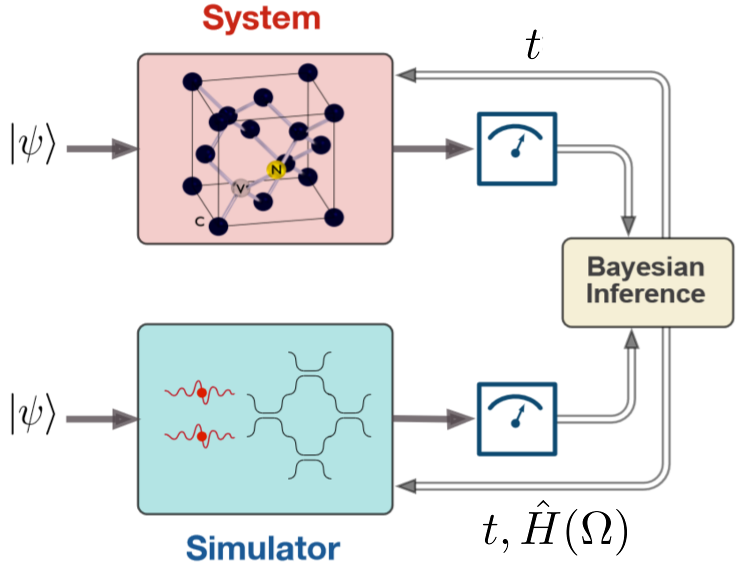
Probability defined as distribution



Confidence Intervals are defined by probability distributions

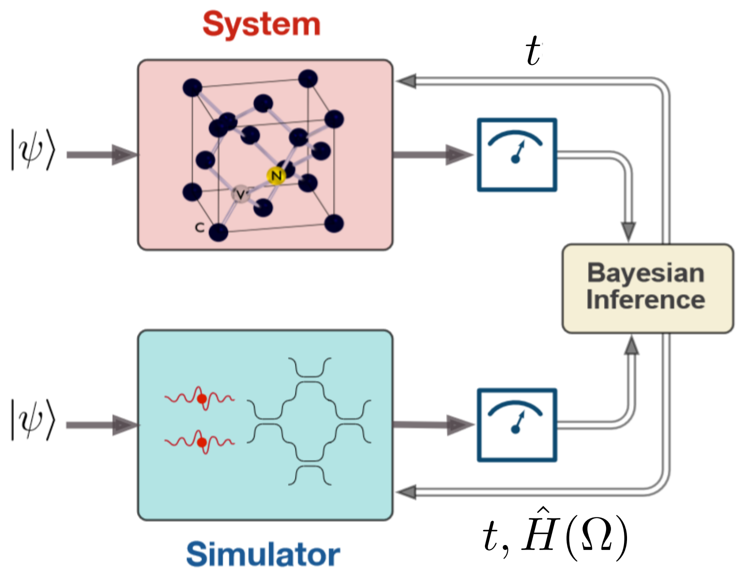
Quantum Likelihood Estimation

Bayesian Inference



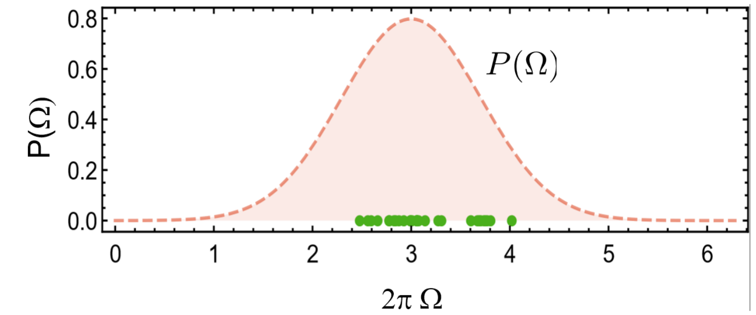
Phys. Rev. Lett. 112, 190501 (2014)
Nature Physics 13, 551-555 (2017)

Quantum Likelihood Estimation

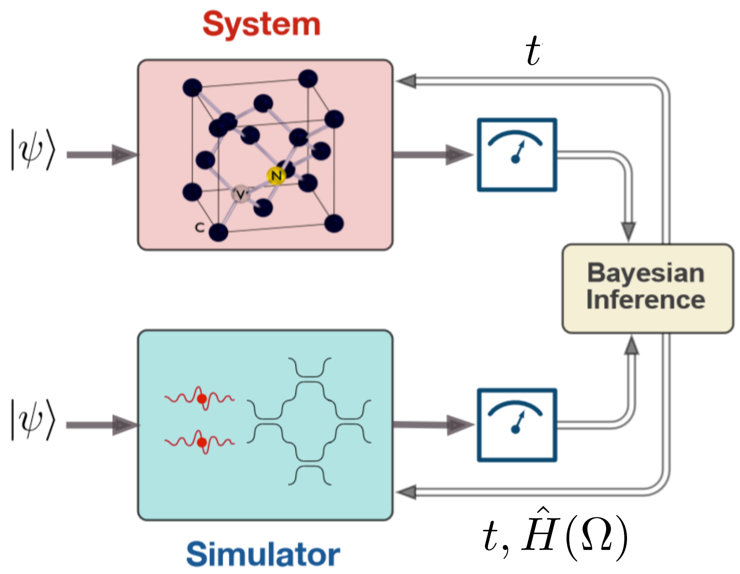


Bayesian Inference

The knowledge of the parameter value is encoded in a **prior distribution** $P(\Omega)$.



Quantum Likelihood Estimation

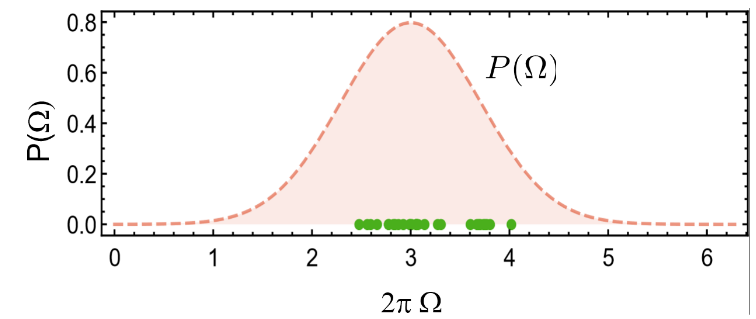


Bayesian Inference

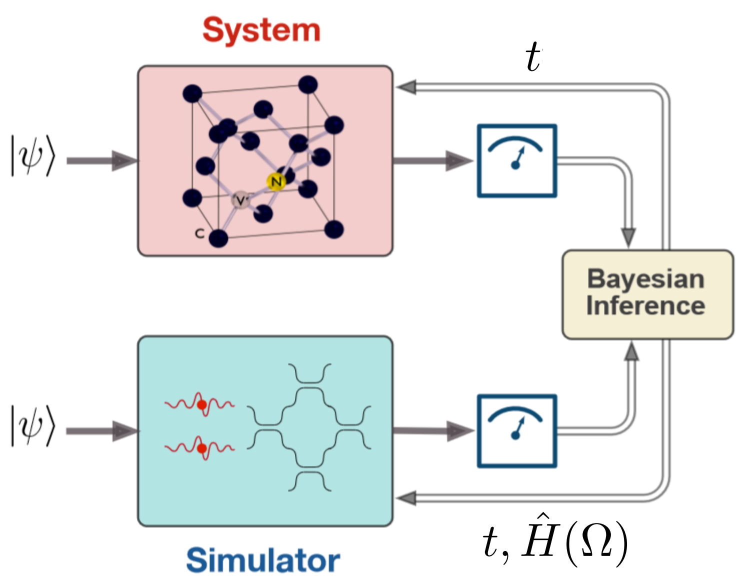
The knowledge of the parameter value is encoded in a **prior distribution** $P(\Omega)$.

Likelihood function:

$$P(E|\Omega,t) = |\langle E|U(\Omega,t)|\psi\rangle|^2$$

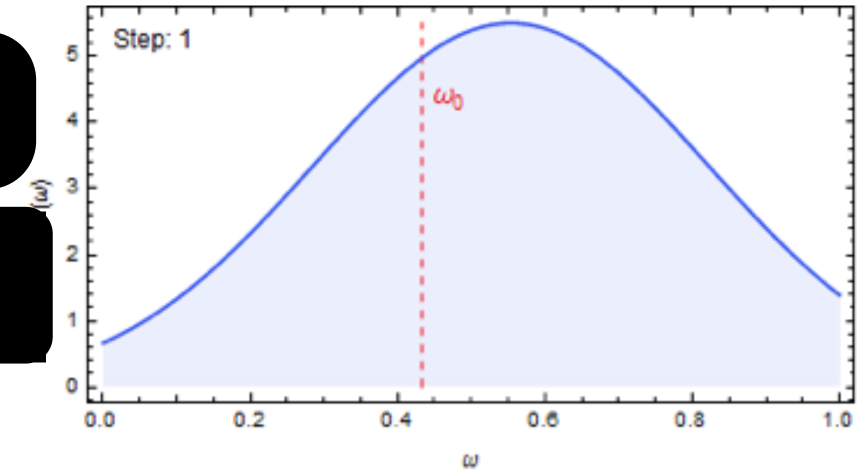


Quantum Likelihood Estimation



Bayesian Inference

The knowledge of the parameter value is encoded in a **prior distribution** $P(\Omega)$.



1. From $P(\Omega)$ choose an experiment (e.g. $t = 1/26\sigma$)

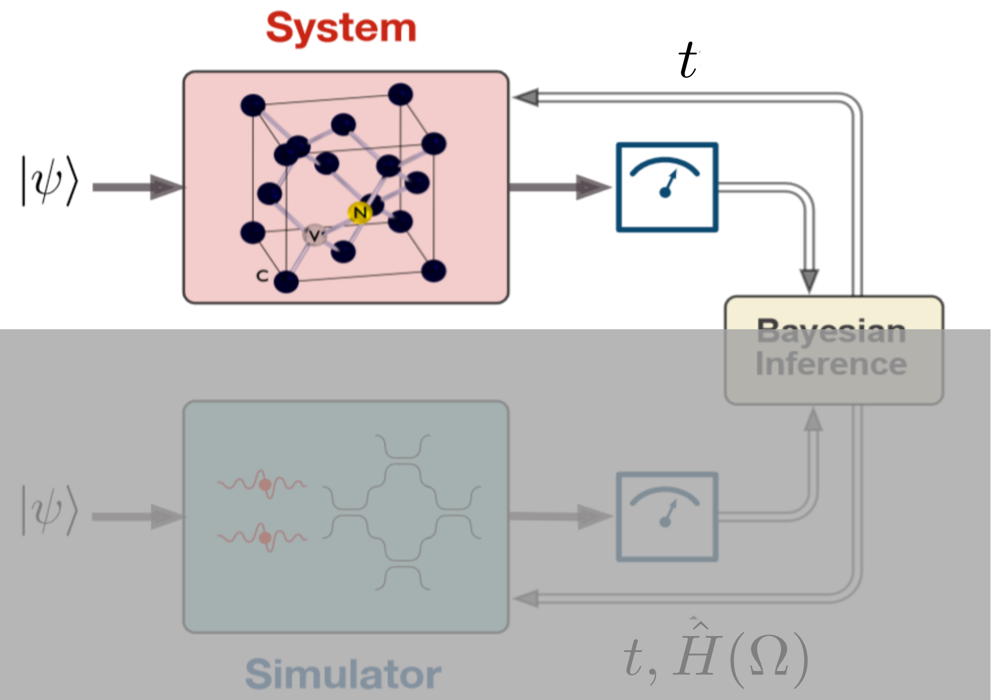
2. Perform experiment on System and obtain outcome E $\{|+\rangle, |-\rangle\}$

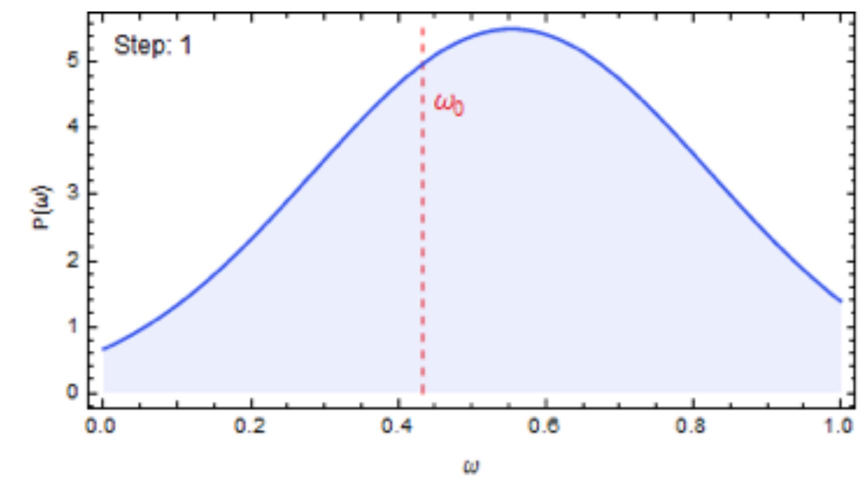
3. Calculate likelihoods $P(E|\Omega;t)$ using quantum simulator

4. Update the prior distribution using Bayes' rule:

$$P'(\Omega) = \frac{P(E|\Omega;t)P(\Omega)}{\int P(E|\Omega;t)P(\Omega)d\Omega}$$

Quantum Likelihood Estimation





1. From $P(\Omega)$ choose an experiment (e.g. $t = 1/2\sigma$

2. Perform experiment on System and obtain outcome E
$\{|+\rangle, |-\rangle\}$

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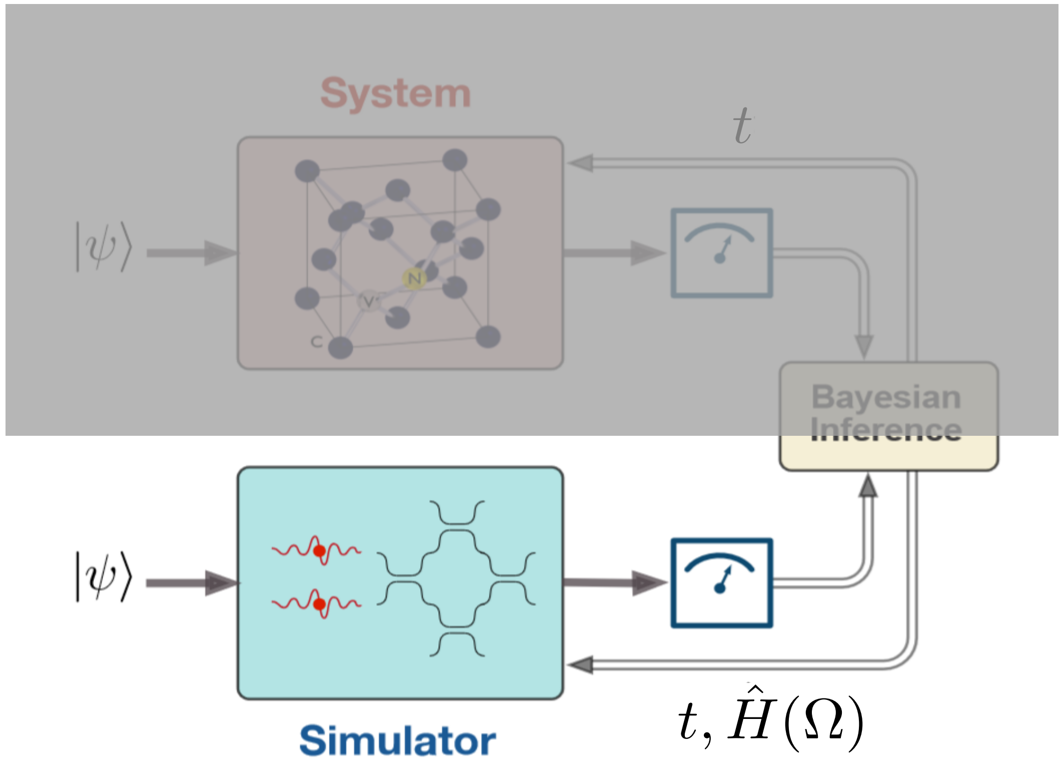
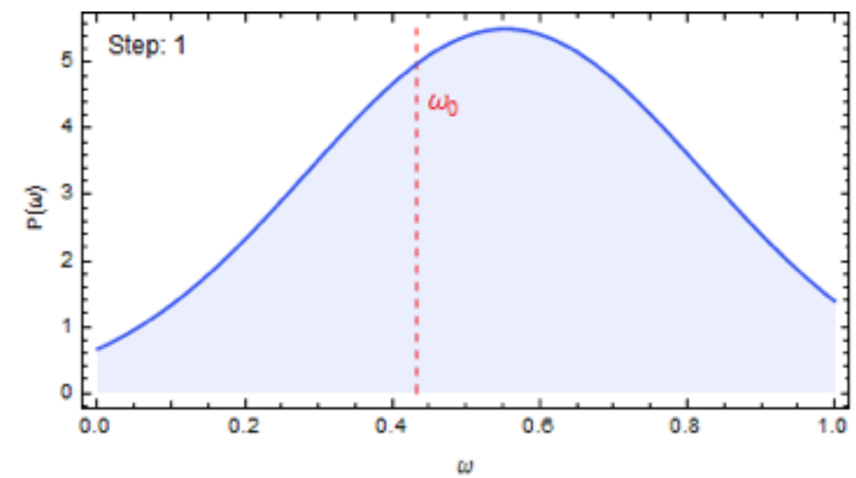
Quantum Likelihood Estimation





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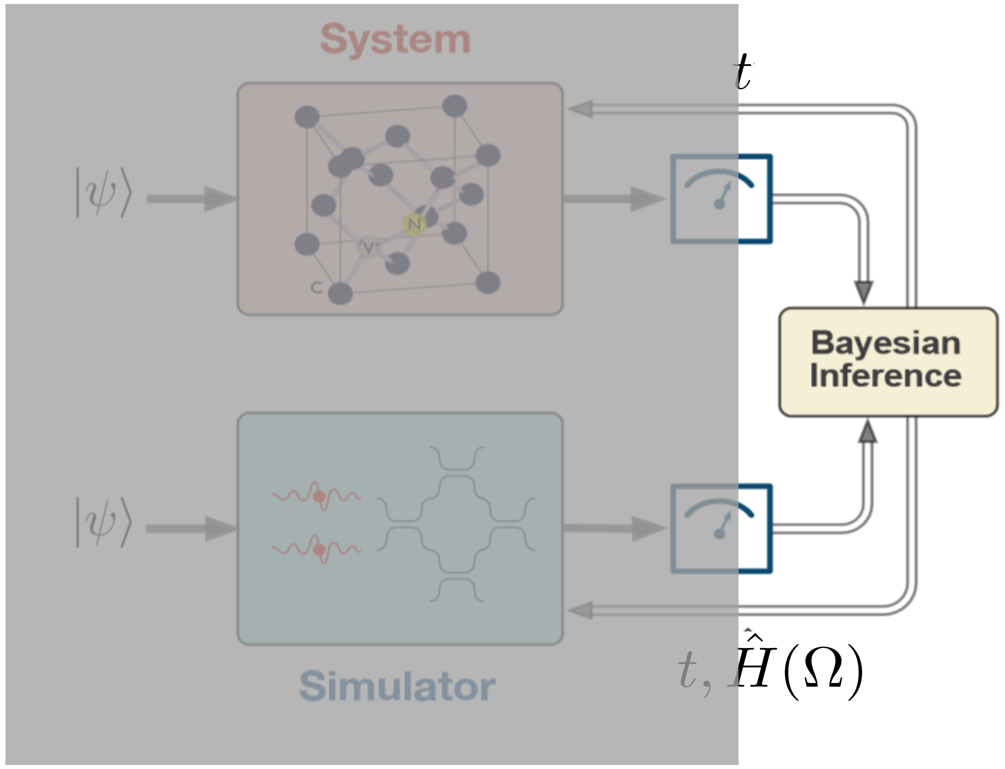
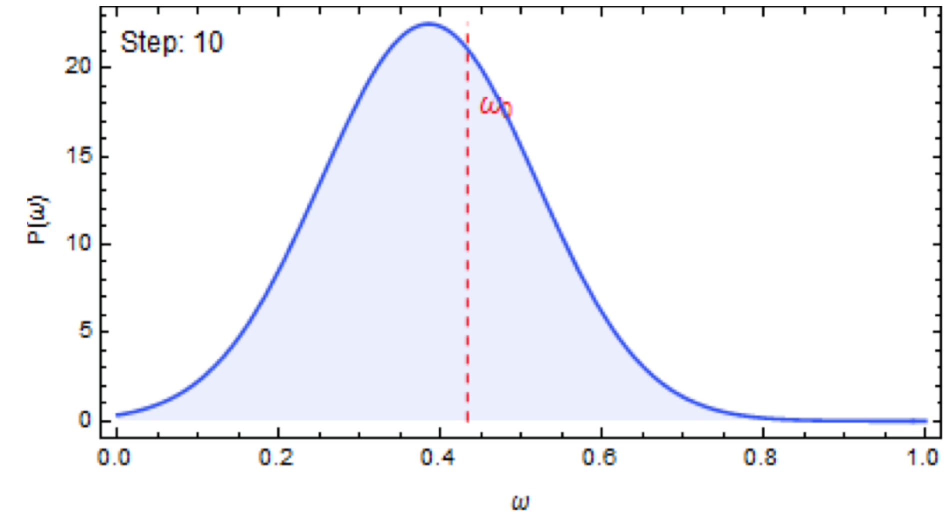
Quantum Likelihood Estimation





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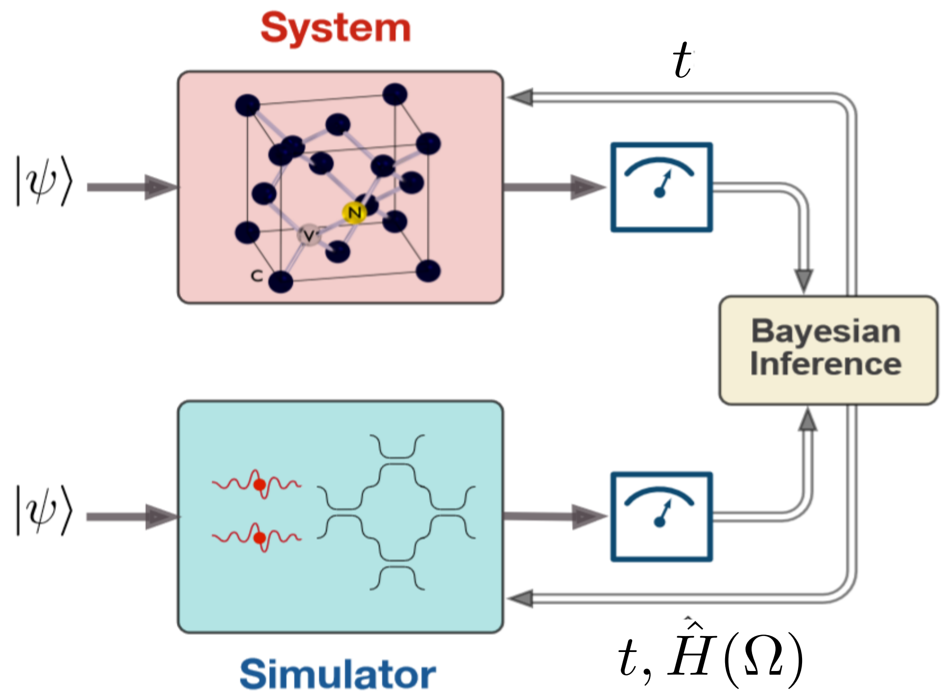
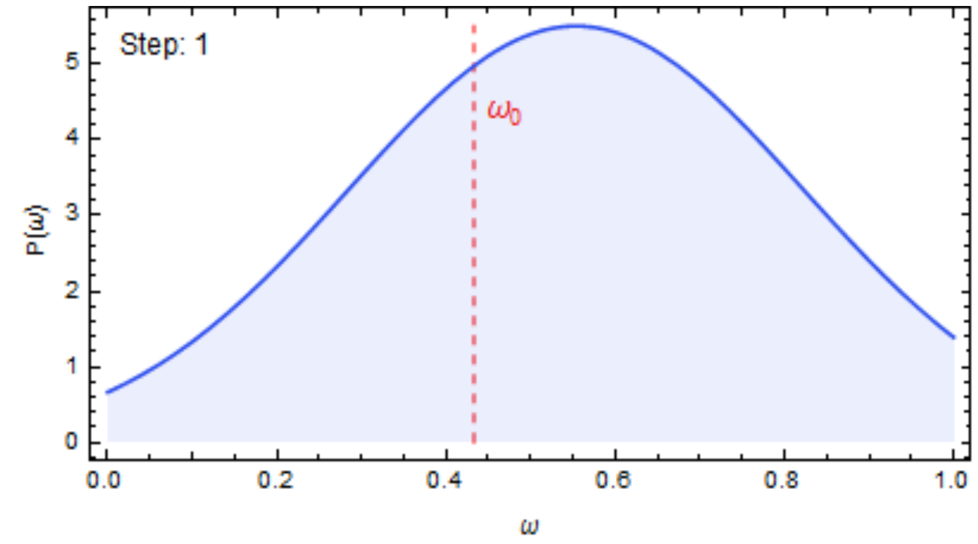
Quantum Likelihood Estimation





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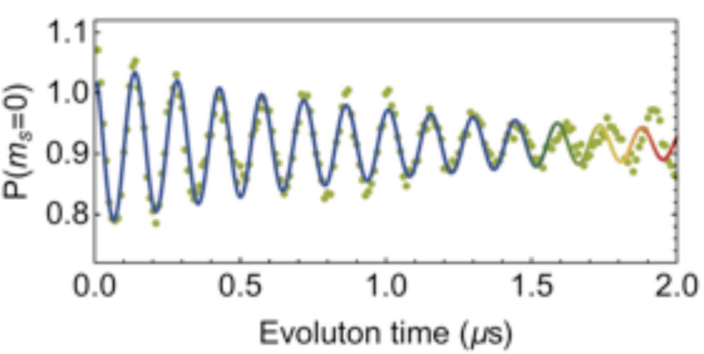
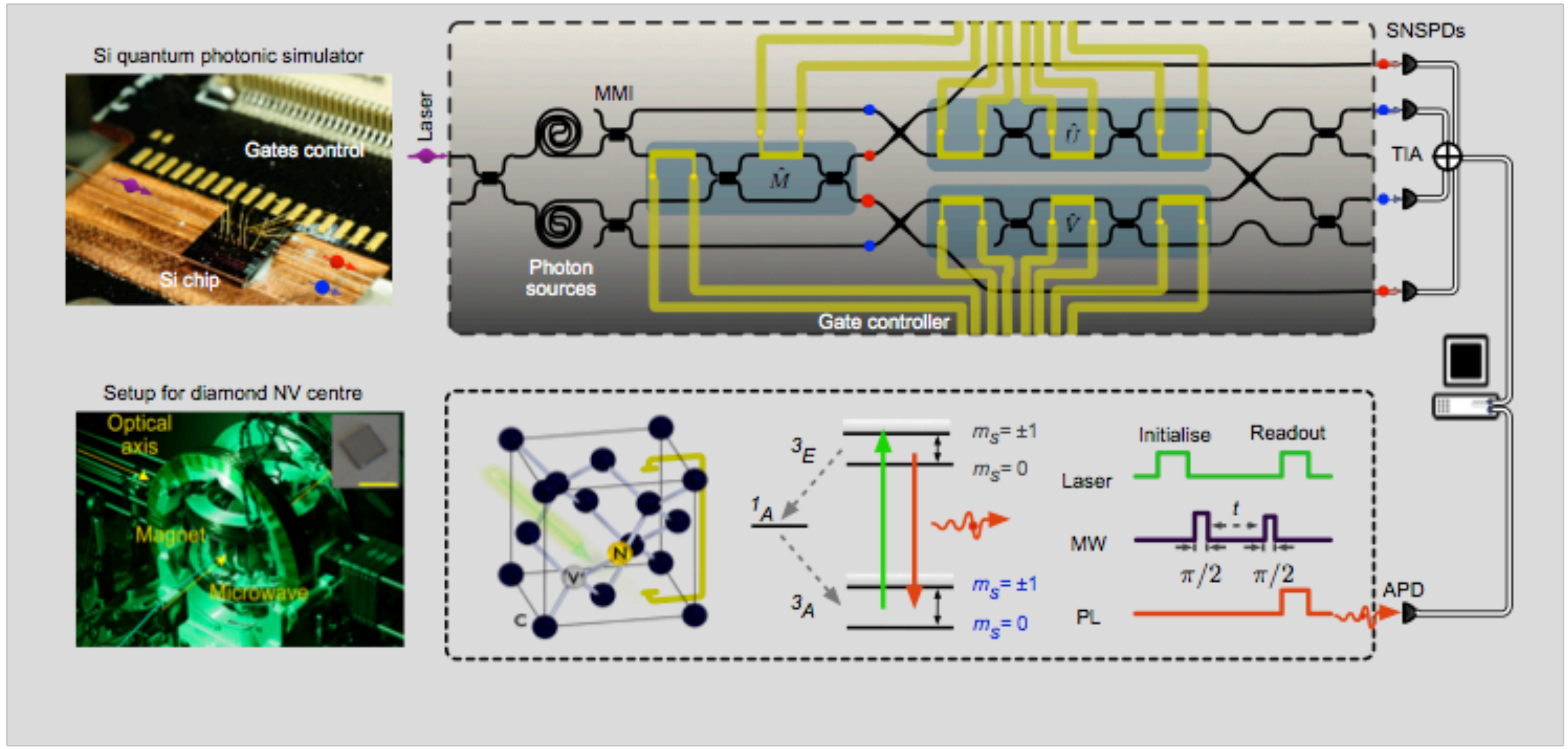
Experimental set-up: Qsim





Hamiltonian:

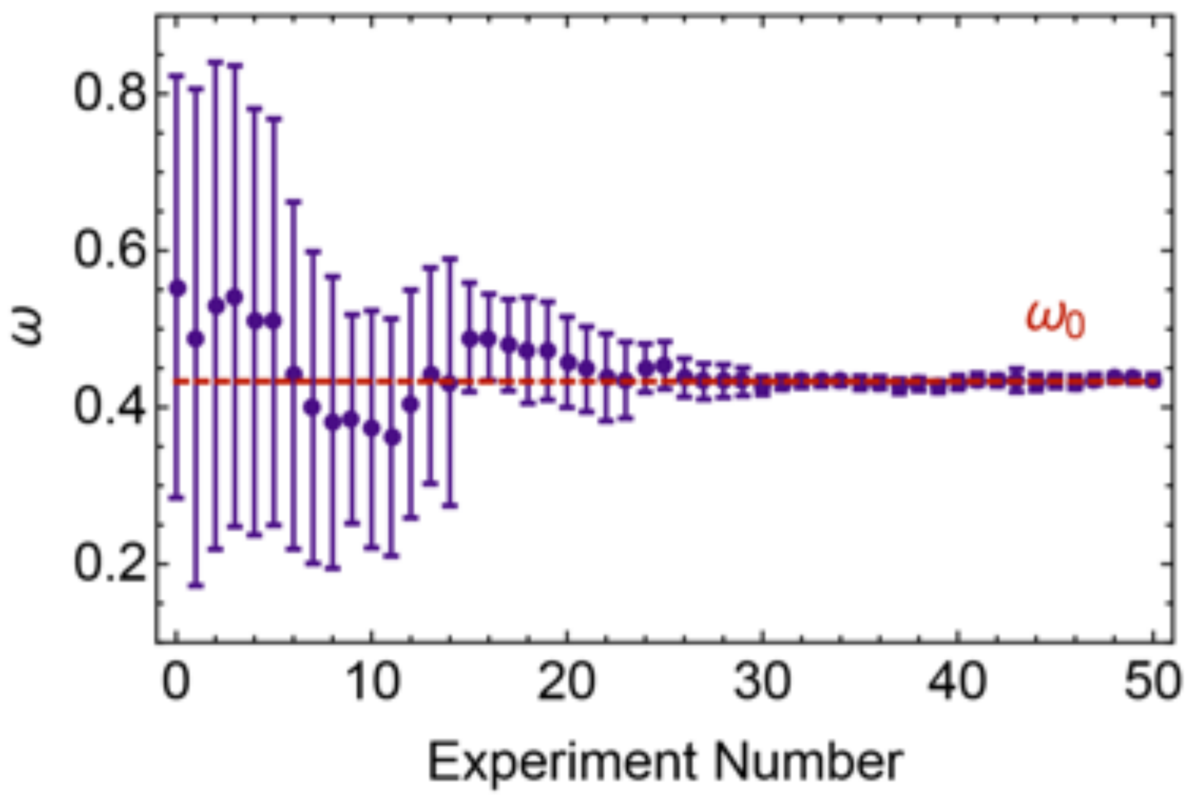
$$\hat{H}(\Omega_0) = \frac{\Omega_0}{2}\hat{\sigma}_z$$

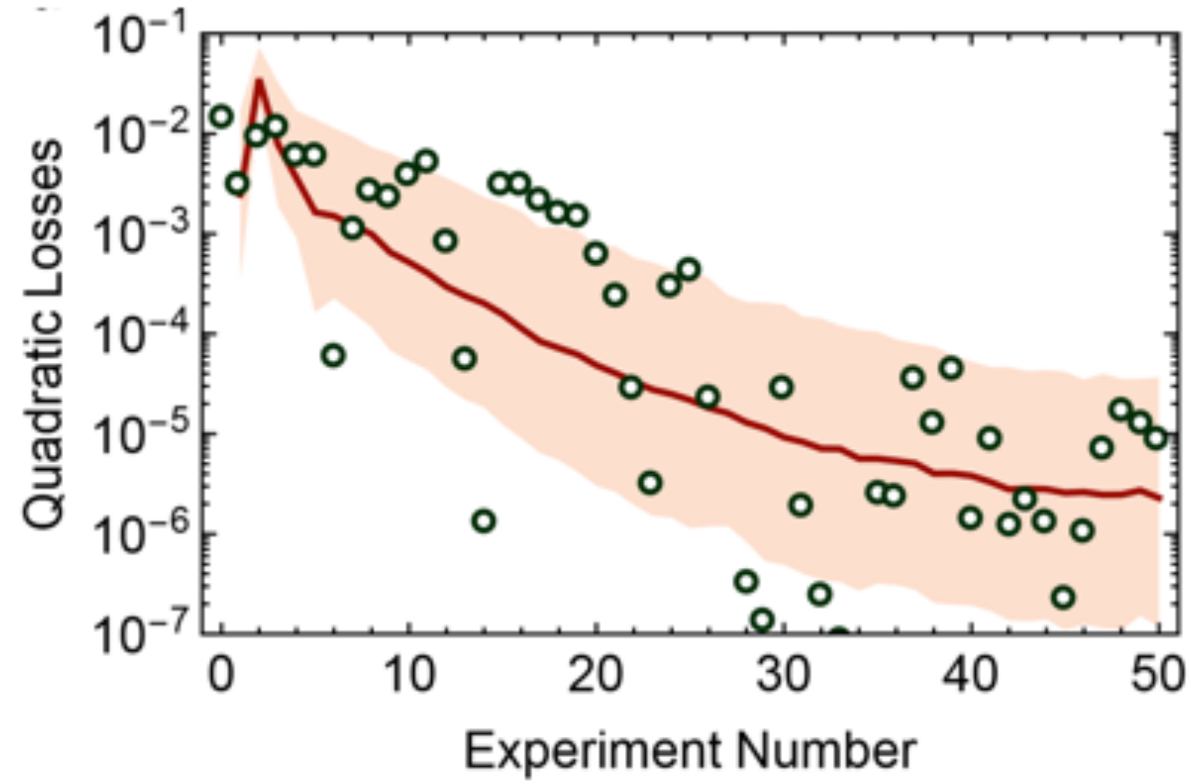
Where we want to estimate the Rabi Frequency

$$\Omega_0$$

Nature Physics 13, 551-555 (2017)

Experimental results





Rescaling

$$\omega = \Omega/\Delta\Omega$$

Rabi frequency inferred with QLE:

$$\Omega_{\rm QLE} = 6.93 \pm 0.09 \text{ MHz}$$

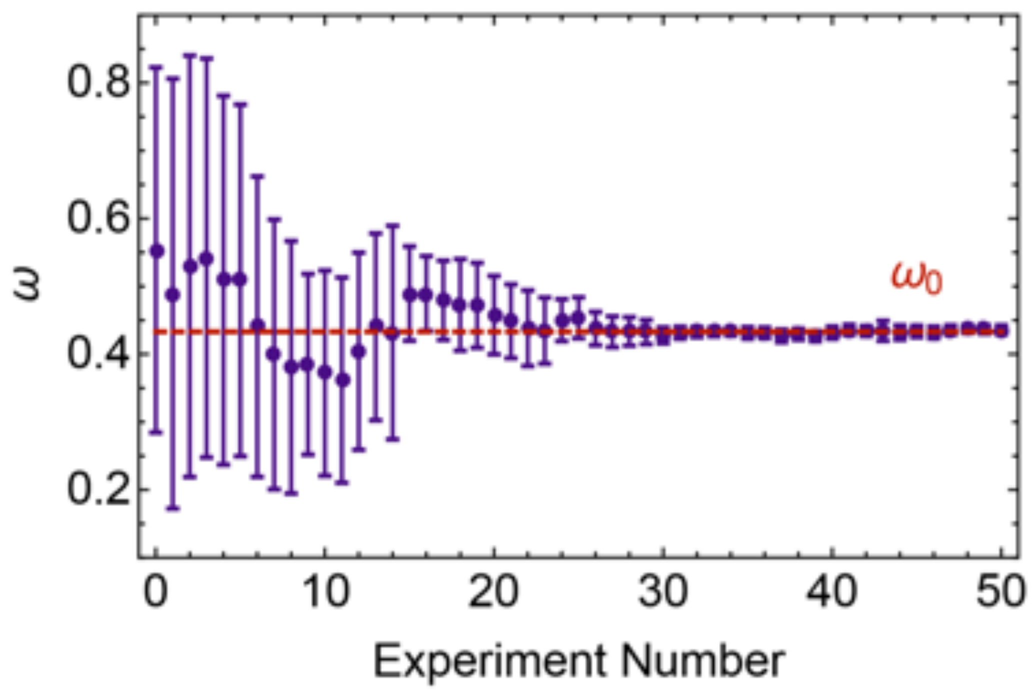
$$\omega_{\rm QLE} = 0.436 \pm 0.006$$

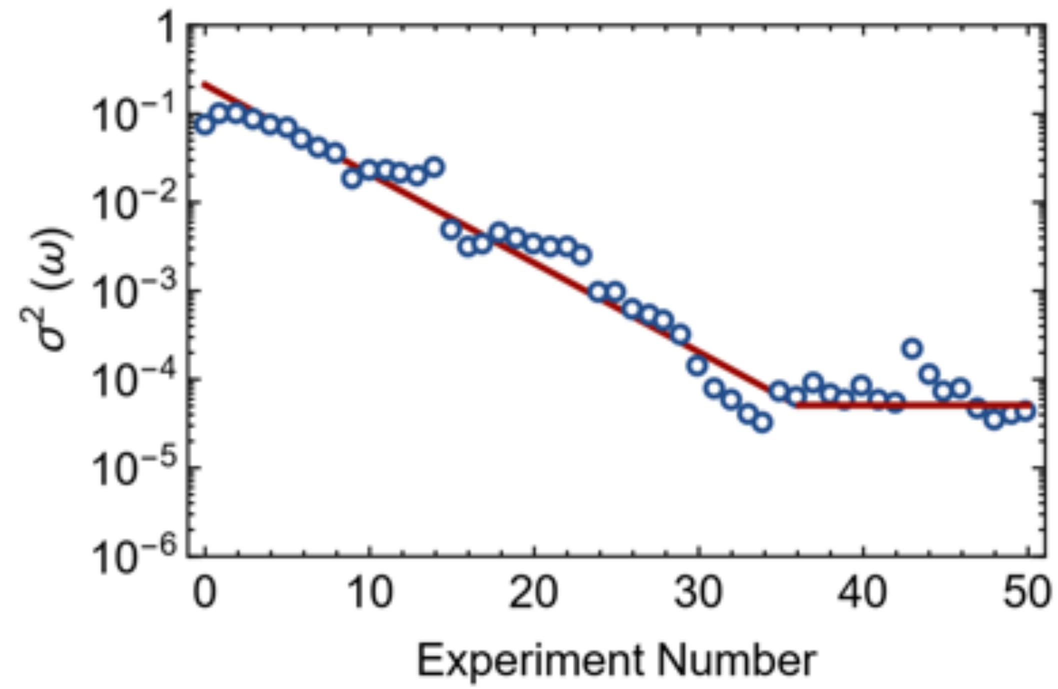
Rabi frequency obtained by fit:

$$\Omega_0 = 6.9$$

$$\omega_0 = 0.433$$

Experimental results: Variance





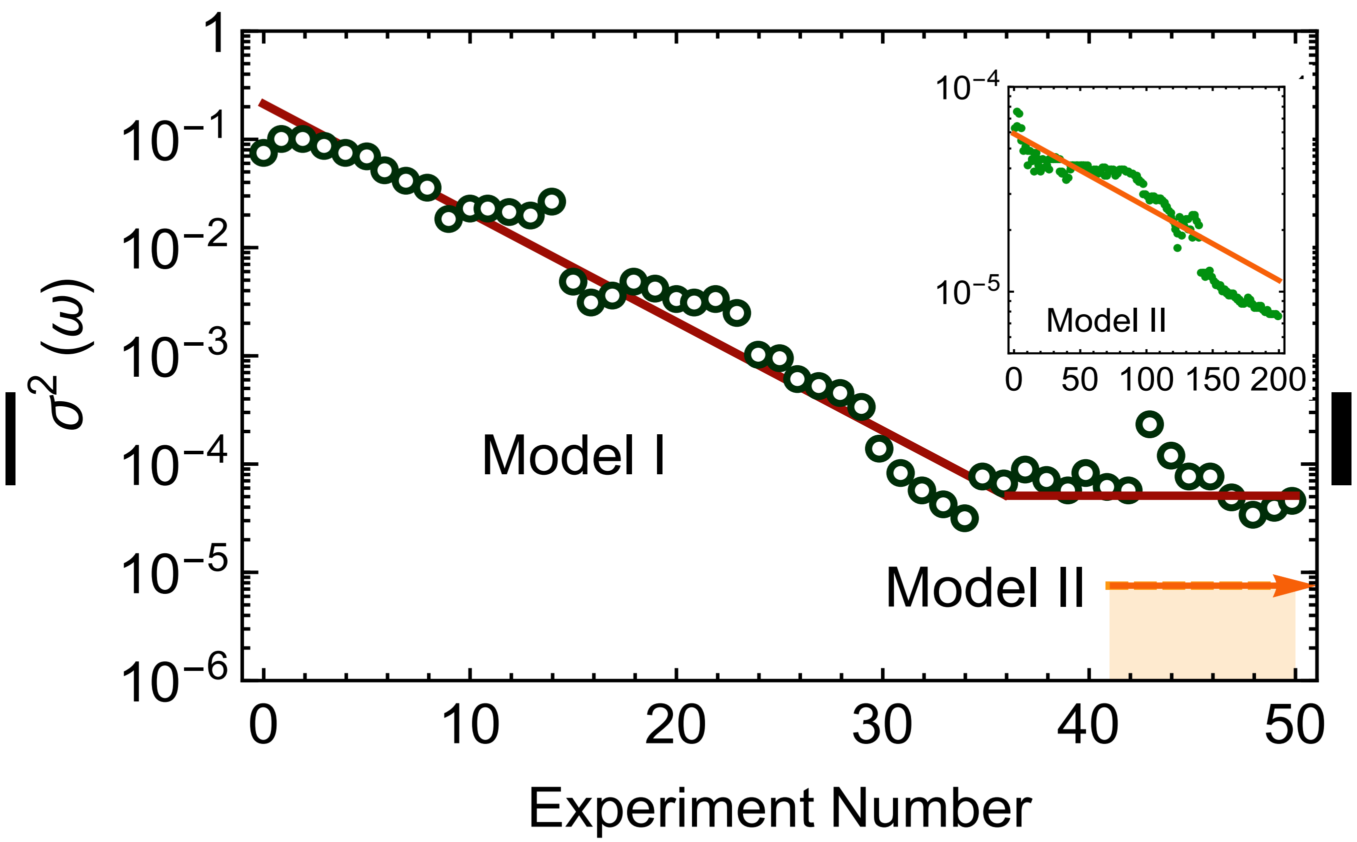
Variance:

$$\sigma^2(\omega) < 4.2 \cdot 10^{-5}$$

SATURATION = Bad quantum simulator or bad model!

Wang et al. Nature Physics 13, 551-555 (2017)

From QHL to comparing models



Model I:

$$\hat{H}(\Omega_0) = \frac{\Omega_0}{2}\hat{\sigma}_z$$

Model II:

$$\hat{H}_{\mathrm{New}}(\Omega, \alpha) = \frac{\Omega + \alpha\, t/2}{2}\hat{\sigma}_z$$

Old Variance with model I:

$$\sigma^2(\omega) < 4.2 \cdot 10^{-5}$$

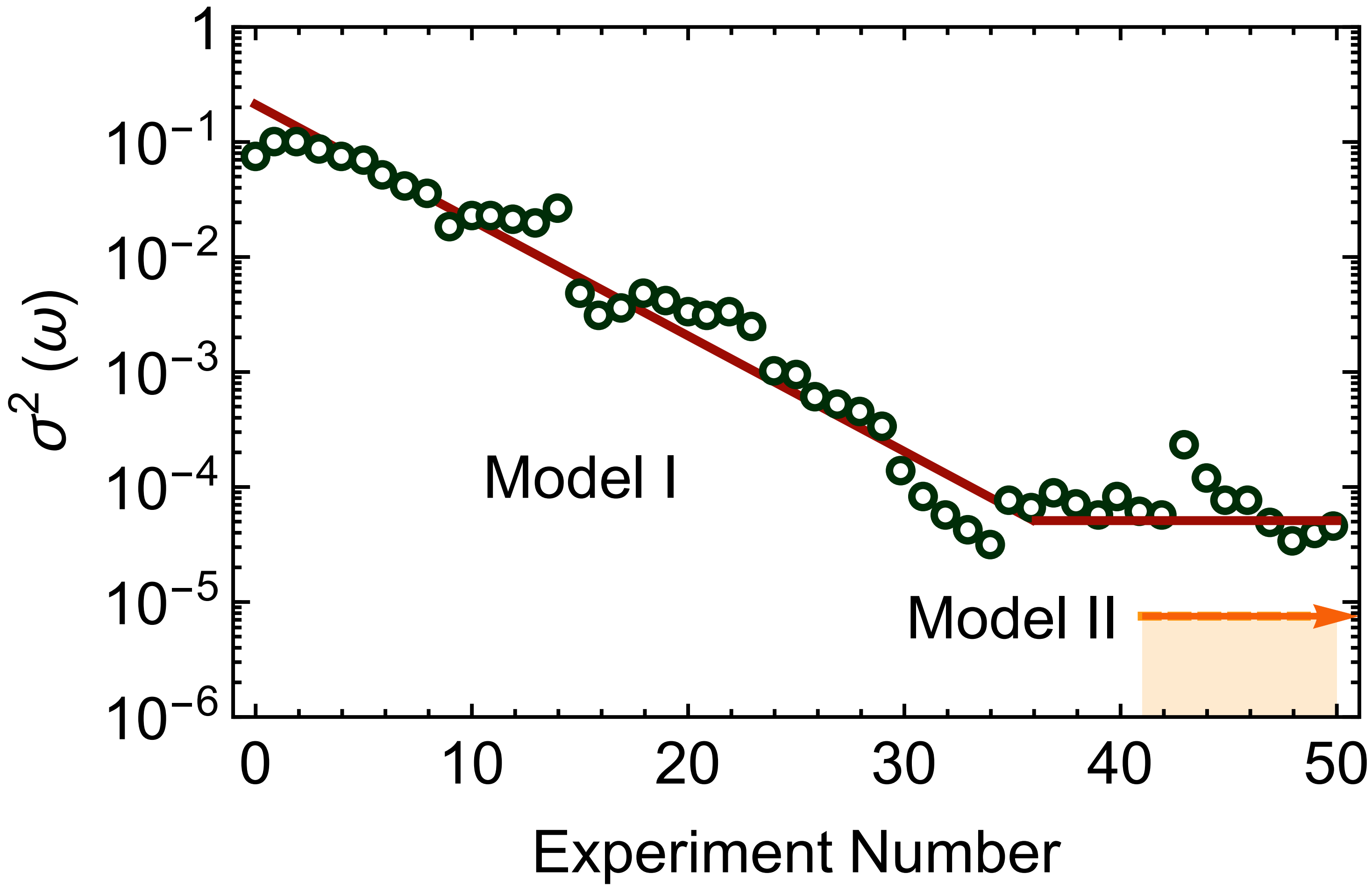
Bayes Factor:

$$R = \frac{\langle P(E|\vec{x})_{\mathrm{II}}\rangle}{\langle P(E|\vec{x})_{\mathrm{I}}\rangle} = 560$$

Updated value (for model II, including chirping) of Norm of covariance matrix:

$$||\Sigma(\omega)||^2 < 7.5 \cdot 10^{-6}$$

Present and Future work: Can we improve models?



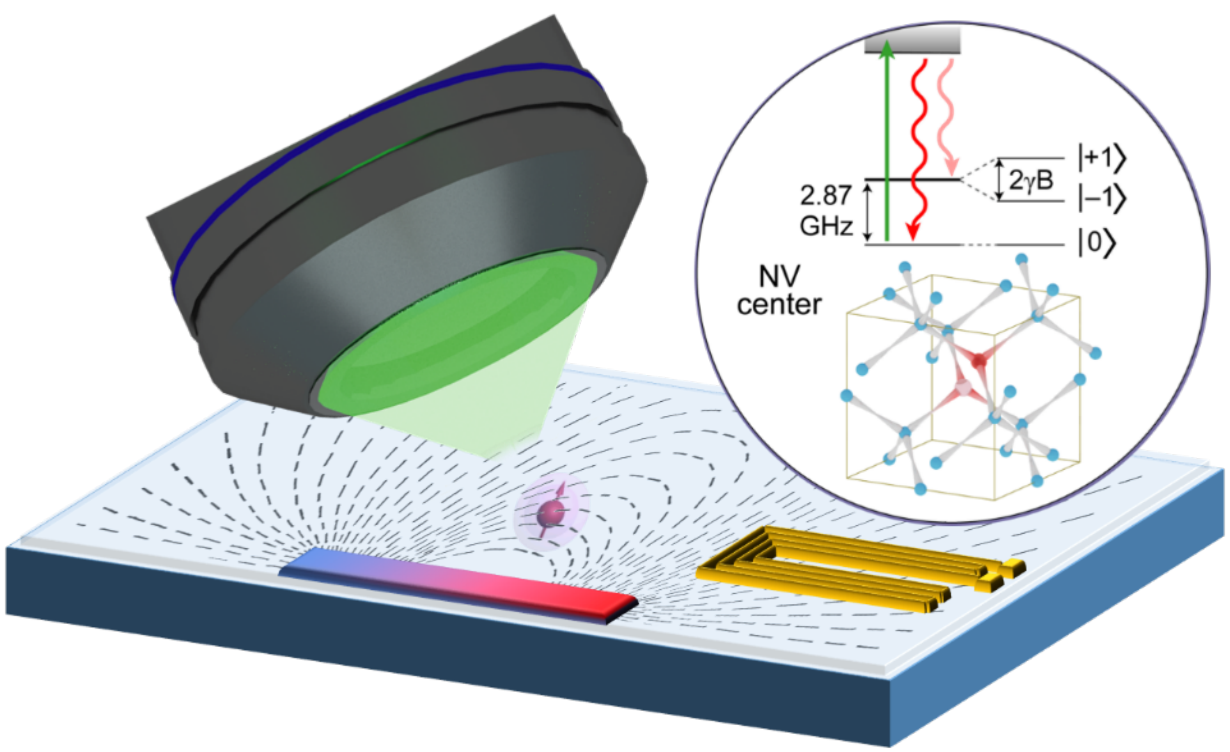
Testing and improving models

Bayes Factor:

$$R = \frac{\langle P(E|\vec{x})_{\mathrm{II}} \rangle}{\langle P(E|\vec{x})_{\mathrm{I}} \rangle} = 560$$

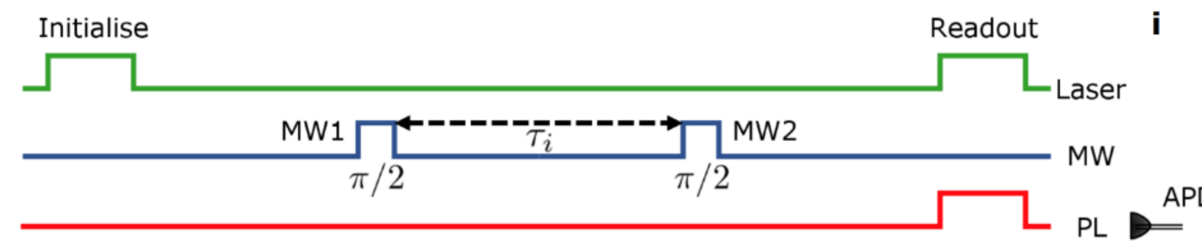
Proof of principle!

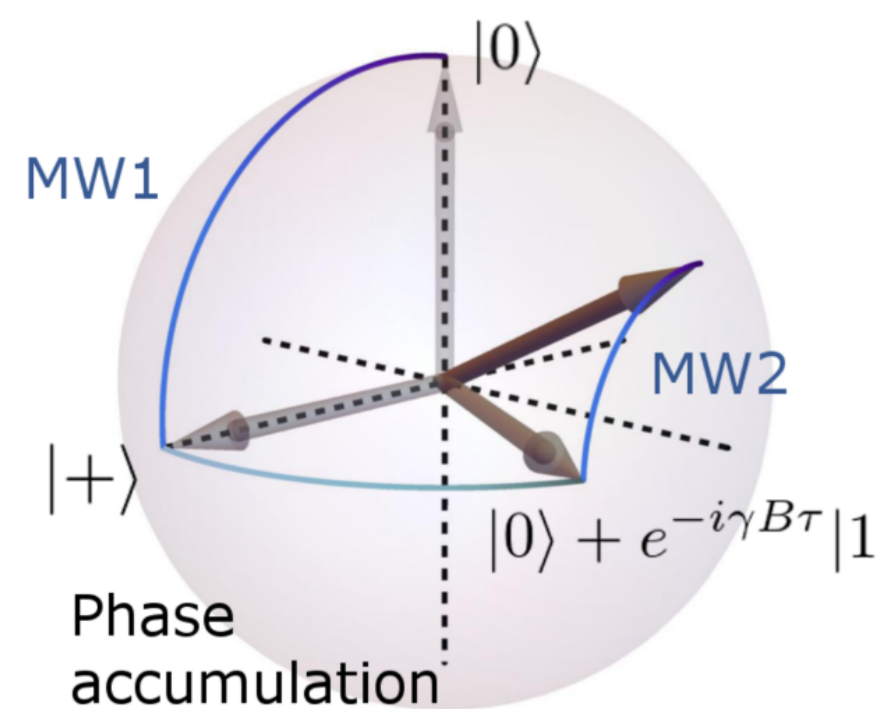
Ramsey Magnetometry

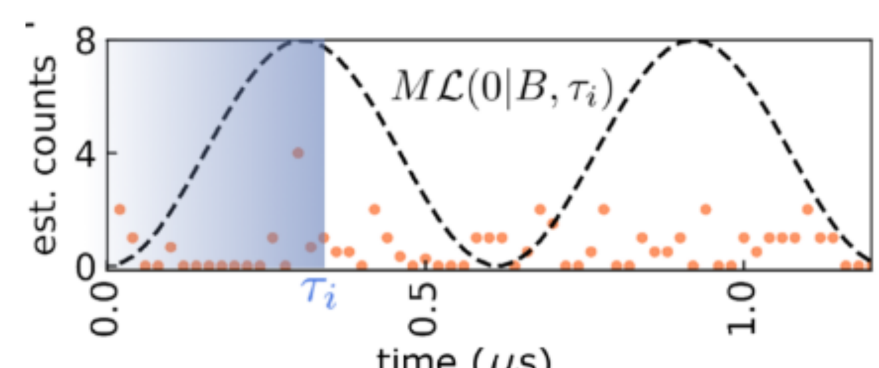


$$\hat{H}(B) = \omega(B)\ \hat{\sigma}_z/2 = \gamma B\ \hat{\sigma}_z/2$$

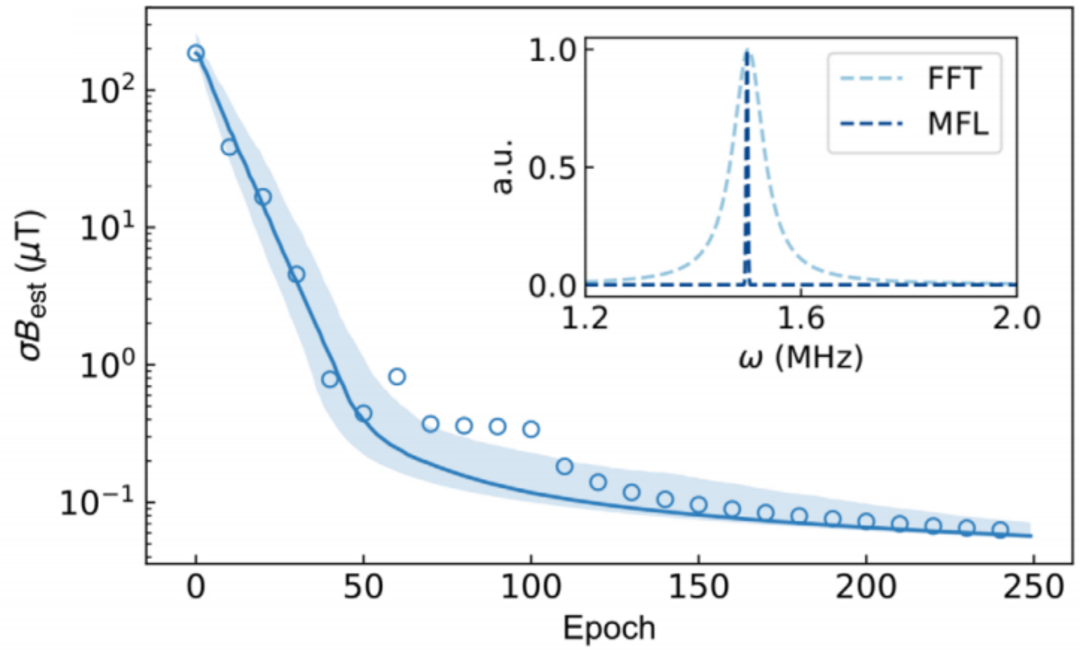
Nano-scale quantum sensors
with high sensitivity



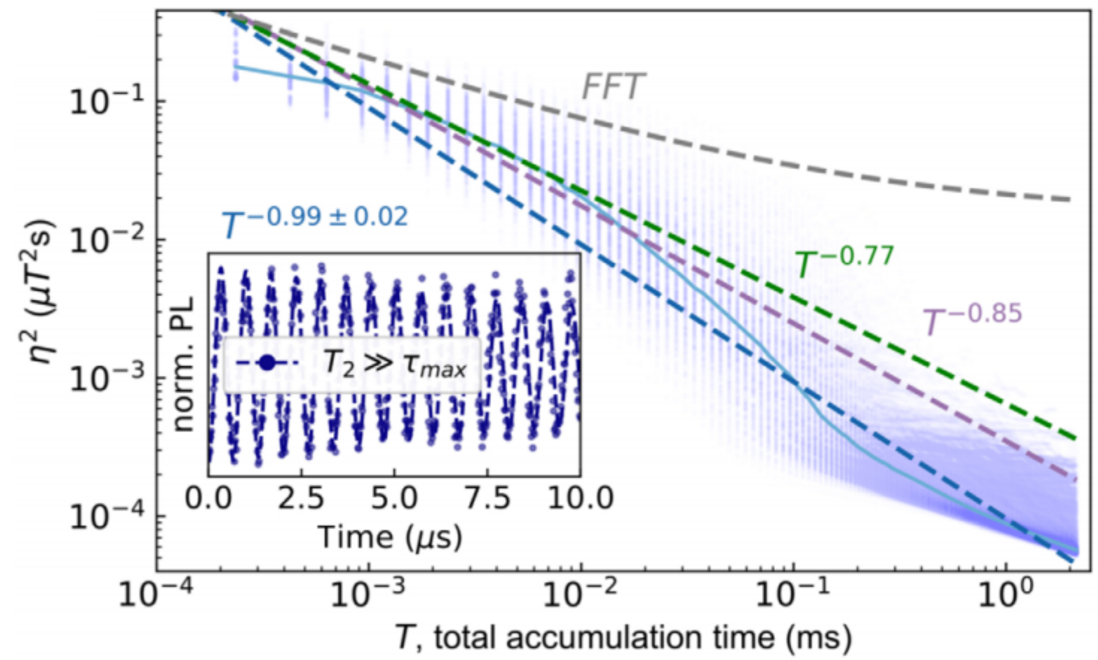




Results



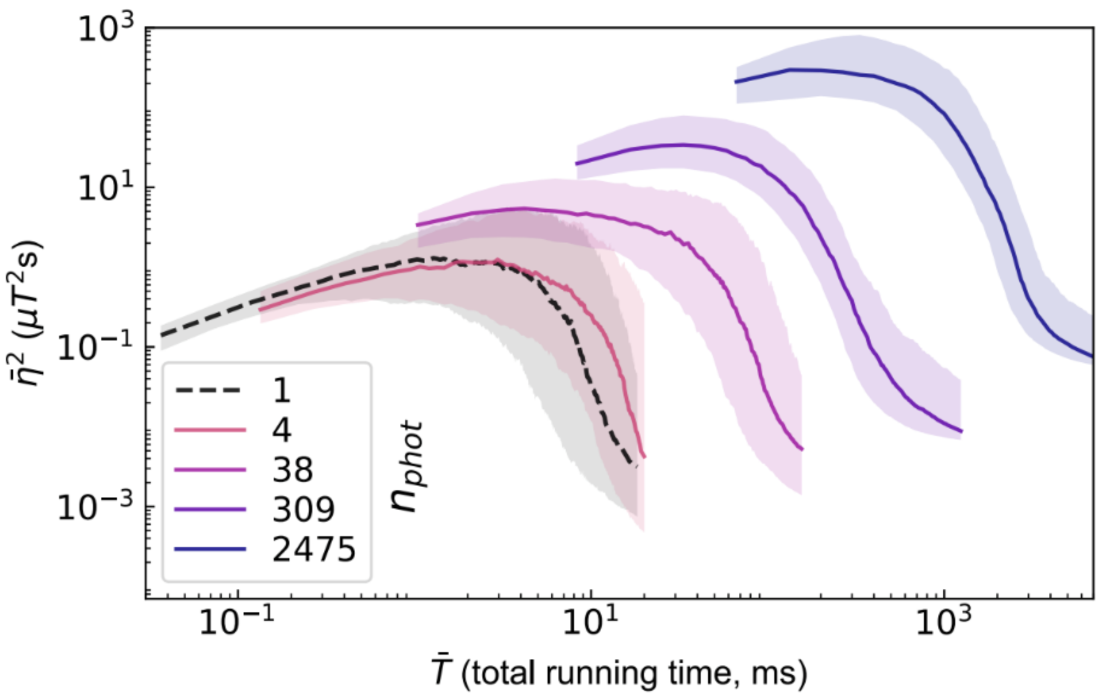
σB_{est} (μT)
10^2
10^1
10^0
10^{-1}
0
50
100
150
200
250
Epoch
1.0
0.5
0.0
a.u.
1.2
1.6
2.0
ω (MHz)
FFT
MFL



η^2 ($\mu T^2 s$)
10^{-1}
10^{-2}
10^{-3}
10^{-4}
FFT
$T^{-0.99 \pm 0.02}$
$T^{-0.77}$
$T^{-0.85}$
norm. PL
$T_2 \gg \tau_{max}$
0.0
2.5
5.0
7.5
10.0
Time (μs)
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^0
T, total accumulation time (ms)

Sensitivity using on average one photon per step at room temperature

Averaging over thousands of protocol runs



When we decrease the number of photons used our sensitivity improves considerably:

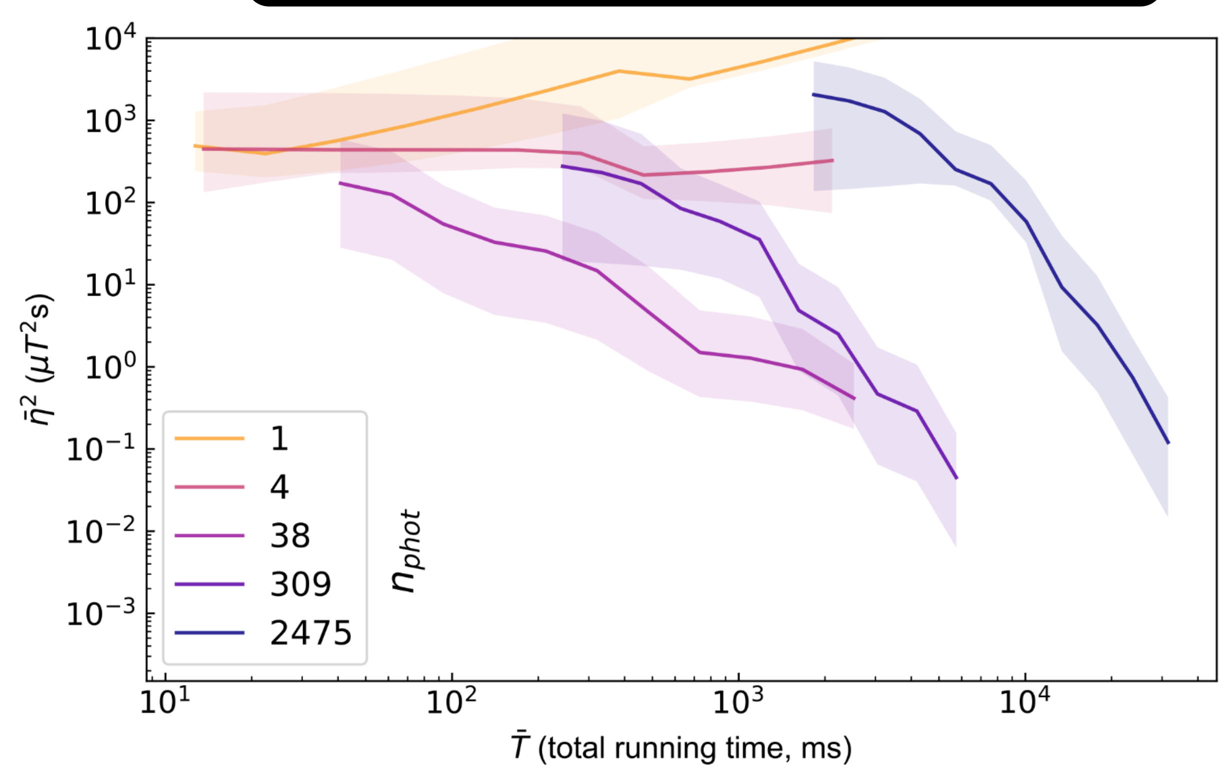
$$\bar{\eta} \approx 60\mathrm{nT\ s}^{1/2}$$

RT sensitivity comparable to cryogenic set-up

When comparing to Quantum Phase Estimation (QPE):

$$\bar{\eta}^2 = \sigma^2(B_{est}) \cdot \bar{T}$$

QPE is 370 times slower
With a worse sensitivity



Santagati et al. Phys. Rev. X 9 (2), 021019 (2019)

Cappellaro. Physical Review A, 85(3):030301 (2012)
Bonato et al. Nature Nanotechnology, 11(3) (2015)

Conclusions

- Statistical inference (QHL) for parameters in H
- Application to quantum sensing
- Exploring models is possible with this approach

Thank You!

Wang et al. Nature Physics 13, 551-555 (2017)

Santagati et al. Phys. Rev. X **9**, 021019 (2019)

Gentile et al. Arxiv (2020)

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