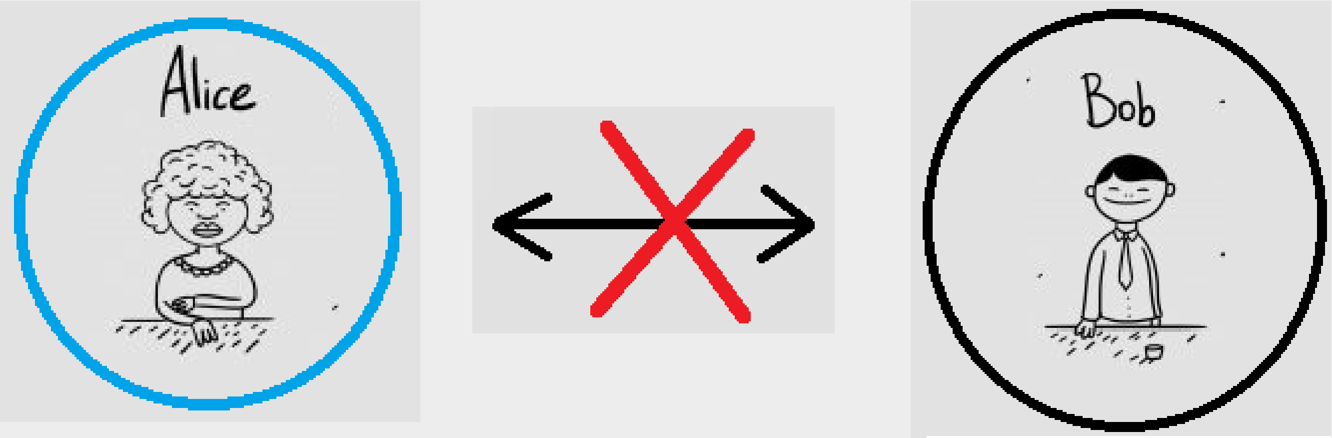
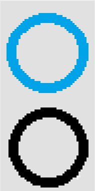
From EPR to Bell, with a description on correlation polytopes.

Can Quantum-Mecanical Description of Physical Realty be Complete?

- A complete theory should in Einstein,Podolsky and Rosen’s description be:
  - **Local**:
    - No “spooky” action at a distance should be allowed. So, it should be possible with the use of “hidden” variables($\boldsymbol{\gamma}$) to write two locally separated measurements as independent:

$$P(ab|xy,\gamma) = P(a|x,\gamma) * \mathrm{P}(\mathrm{b}|\mathrm{y},\gamma)$$




Alice local light cone

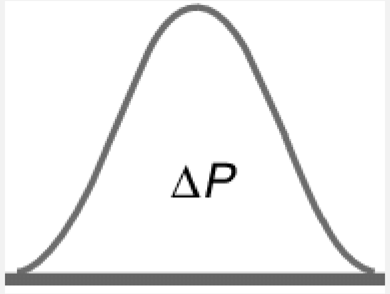
Bob’s local light cone

Einstein et al., 1935

Can Quantum-Mecanical Description of Physical Realty be Complete?

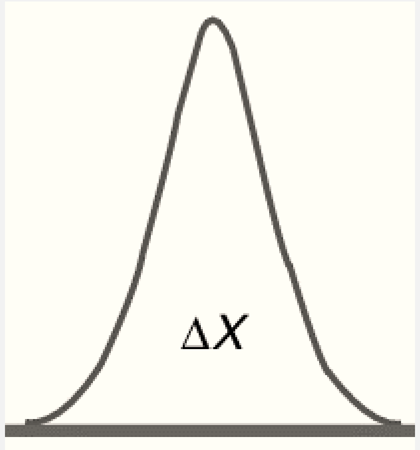
Realism:

Every physical propriety should have a definite value independent of its observation, simultaneous to every other propriety.



Momentum Distribution

$$\Delta P \bullet \Delta X \geq \hbar$$

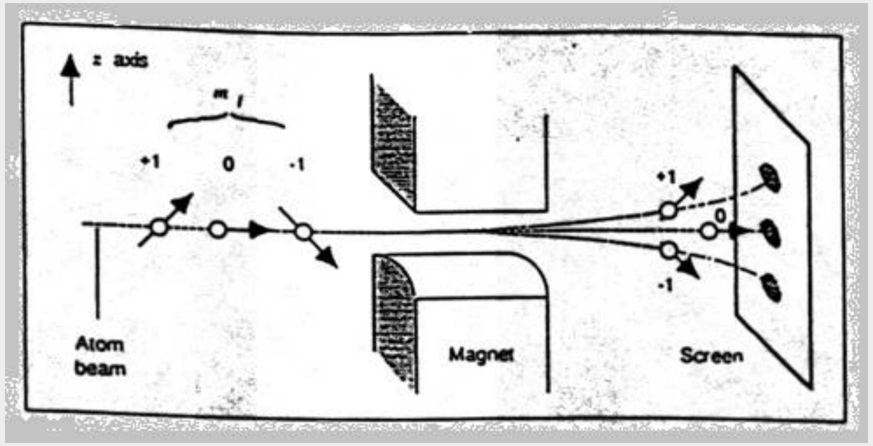


Position Distribution

Quantum Mechanics violates this propriety by containing observables that do not commute, so the wave-function can not describe all proprieties independently of the measurement performed.

Bell provided a testbed for the question

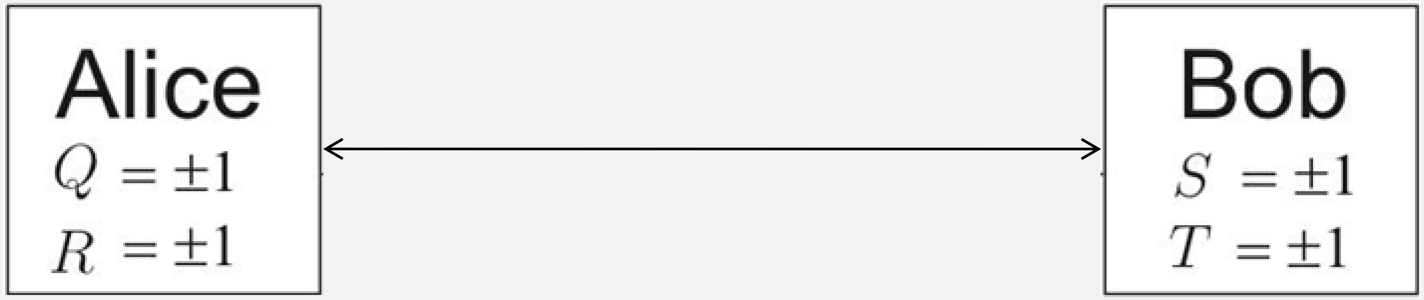
- John Bell idealized an experiment on two particles going through two Stern Gerlach magnets within a local hidden variable theory (with any number of hidden variables).



- Creating the theoretical understanding that could project an experiment that could bring inside to the EPR questions.

Note: He proved that quantum mechanics would disrespect the local limit with entangled particles.

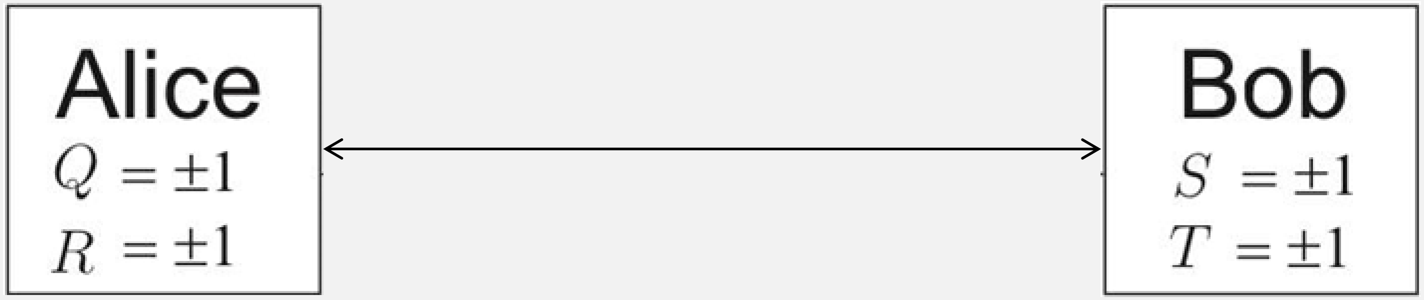
Example: CHSH Inequality



QS + RS + RT – QT

Nielsen & Chuang, 2010, Clauser et al., 1969, Cirelson, 1980

Example: CHSH Inequality



$$QS + RS + RT - QT = (Q + R)S + (R - Q)T$$

Nielsen & Chuang, 2010, Clauser et al., 1969, Cirelson, 1980

Example: CHSH Inequality

QS + RS + RT – QT = (Q + R)S + (R - Q)T

From the description of these measurements, the value can only be equal to:

QS + RS + RT – QT =±2

Translating this expression to the expected values maintains the same boundary:

E(QS)+E(RS)+E(RT)-E(QT) ≤ 2

Nielsen & Chuang, 2010, Clauser et al., 1969, Cirelson, 1980

The quantum violation of CHSH inequality

To prove it we will use a Bell state (maximally entangled state):

$$|\varphi\rangle = \frac{1}{2}|00\rangle + \frac{1}{2}|11\rangle \quad \leftrightarrow \quad \rho = |\varphi\rangle\langle\varphi| = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$$

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Now we want to select the correct observables with the highest expected values as not every choice disrespects CHSH inequality:

$$< A > = tr(\rho A)$$

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Now we want to select the correct observables with the highest expected values as not every choice disrespects CHSH inequality:

$$< A >= tr(\rho A)$$

A possible way to find is to consider the possible expected value of two observables by the angle in between them:

$$Obs_1 = \sigma_x \otimes \sigma_\theta = \begin{pmatrix} 0 & 0 & -\sin(\theta) & \cos(\theta) \\ 0 & 0 & \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) & 0 & 0 \\ \cos(\theta) & \sin(\theta) & 0 & 0 \end{pmatrix}$$

The quantum violation of CHSH inequality

Knowing that the expected value between two rotated spin observables give:

$$tr(\rho A) = tr\left[\frac{1}{2}\begin{pmatrix} 0 & 0 & -\sin(\theta) & \cos(\theta) \\ 0 & 0 & \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) & 0 & 0 \\ \cos(\theta) & \sin(\theta) & 0 & 0 \end{pmatrix}\right] = \cos(\theta)$$

The quantum violation of CHSH inequality

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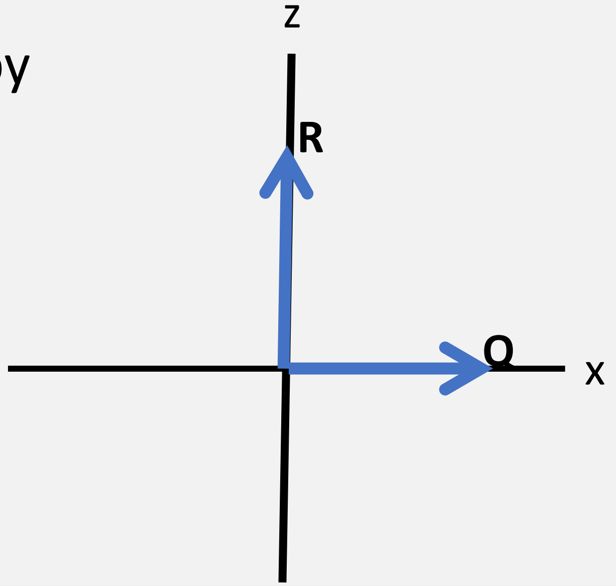
$$tr(\rho A) = tr\left[\frac{1}{2}\begin{pmatrix} 0 & 0 & -\sin(\theta) & \cos(\theta) \\ 0 & 0 & \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) & 0 & 0 \\ \cos(\theta) & \sin(\theta) & 0 & 0 \end{pmatrix}\right] = \cos(\theta)$$

We are able to attempt to violate the expected value predicted by LHV:

<QS> + <RS> + <RT> – <QT> $\leq$ 2

$$\cos(\theta_1) + \cos(\theta_2) + \cos(\theta_3) - \cos(\theta_4)$$



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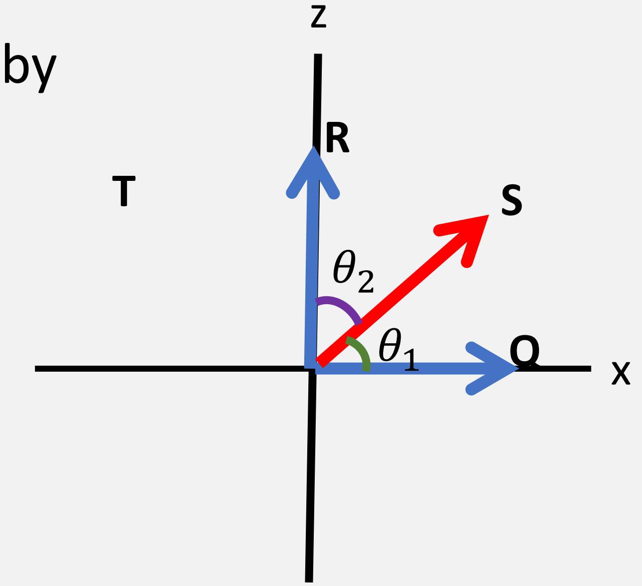
We are able to attempt to violate the expected value predicted by LHV:

$$\langle QS\rangle + \langle RS\rangle + \langle RT\rangle - \langle QT\rangle \leq 2$$

$$\cos(\theta_1) + \cos(\theta_2) + \cos(\theta_3) - \cos(\theta_4)$$

$$\cos(45º) + \cos(45º) + \cos(\theta_3) - \cos(\theta_4) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + ? + ?$$



The quantum violation of CHSH inequality

Knowing that the expected value between two rotated spin observables give:

$$tr(\rho A) = tr\left[\frac{1}{2}\begin{pmatrix} 0 & 0 & -\sin(\theta) & \cos(\theta) \\ 0 & 0 & \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) & 0 & 0 \\ \cos(\theta) & \sin(\theta) & 0 & 0 \end{pmatrix}\right] = \cos(\theta)$$

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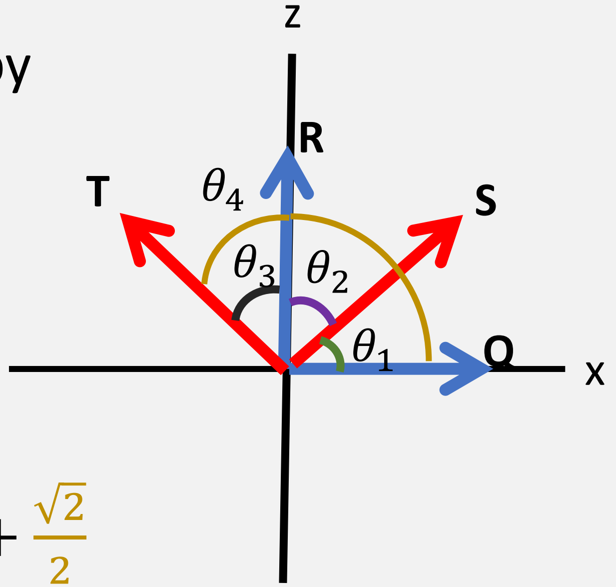
$$\cos(\theta_1) + \cos(\theta_2) + \cos(\theta_3) - \cos(\theta_4)$$

$$\cos(45º) + \cos(45º) + \cos(45º) - \cos(125º) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}$$

$$= \mathbf{2\sqrt{2}}$$



Complete propriety table

a_1	a_2	b_1	b_2	$a_1 b_1$	$a_1 b_2$	$a_2 b_1$	$a_2 b_2$
$+1$		-1		-1			
	-1		-1				$+1$
$+1$		$+1$		$+1$			
-1			-1		$+1$		
	$+1$		$+1$			$+1$	
-1			$+1$		-1		
$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$
				$< a_1 b_1 >$	$< a_1\ b_2 >$	$< a_2 b_1 >$	$< a_2\ b_2 >$
				$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$

$\pm 1\ - Experimental\ Values$

Complete propriety table (Bell Sudoku)

a_1	a_2	b_1	b_2	a_1b_1	a_1b_2	a_2b_1	a_2b_2
$+1$	$+1$	-1	-1	-1	-1	-1	-1
?	-1	?	-1	?	?	?	$+1$
$+1$	?	$+1$	?	$+1$	?	?	?
-1	?	?	-1	?	$+1$	?	?
?	$+1$	?	$+1$	?	?	$+1$	?
-1	?	?	$+1$	?	-1	?	?
…	…	…	…	…	…	…	…
				$< a_1b_1 >$	$< a_1\, b_2 >$	$< a_2b_1 >$	$< a_2\, b_2 >$
				$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$

$\pm 1\ - Experimental\ Values$

$\pm 1\ - Predictions\ for\ the\ non-measured\ values$

Complete propriety table (Bell Sudoku)

a_1	a_2	b_1	b_2	a_1b_1	a_1b_2	a_2b_1	a_2b_2
$+1$	$+1$	-1	-1	-1	-1	-1	-1
?	-1	?	-1	?	?	?	$+1$
$+1$	?	$+1$	?	$+1$	?	?	?
-1	?	?	-1	?	$+1$	?	?
?	$+1$	?	$+1$	?	?	$+1$	?
-1	?	?	$+1$	?	-1	?	?
…	…	…	…	…	…	…	…
				$< a_1b_1 >$	$< a_1\, b_2 >$	$< a_2b_1 >$	$< a_2\, b_2 >$
				$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
				$\mathbf{\frac{1}{2}}$	$\mathbf{\frac{1}{2}}$	$\mathbf{\frac{1}{2}}$	$\mathbf{\frac{1}{2}}$

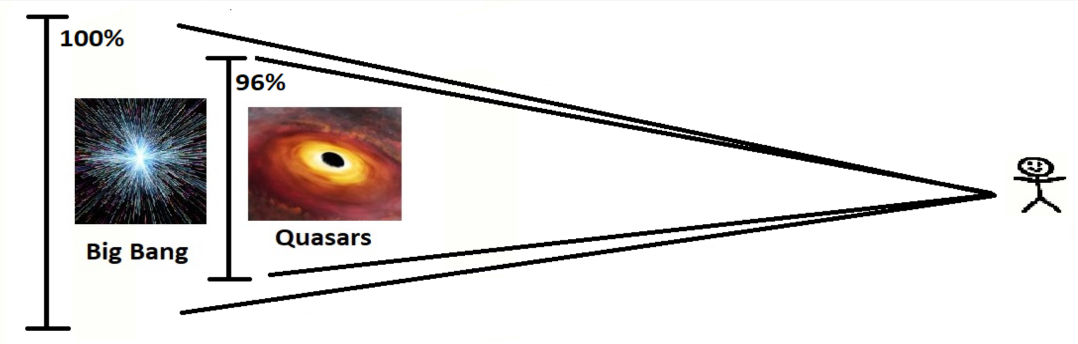
$\pm 1\; - Experimental\; Values$

$\pm 1\; - Predictions\; for\; the\; non - measured\; values$

Experimental Verification

- In 2015 an experiment broad new evidence to the correctness of QM:
  - It ruled out some loopholes:
    - **The locality loophole -** the particles do "talk" to each other -> solved with spatial separation.
    - **Freedom-of-choice -** relates to Bell's requirement of a random choice of measurement settings -> this is closed by generating locally and randomly the measurement settings.
    - **Fair-sampling loophole -** a subspace of the measurement results may disrespect the Bell inequality within local realism -> with and adequate efficiency detection this loophole can be closed too.

Curiosity: To extend the invalidity of the freedom-of-choice in 2018 Bell-inequality was violated using light from High-Redshift Quasars which pushes back 7,8x10^9 years (approx. 96% of the space-time volume of the past light cone.)



Giustina et al., 2015, Rauch et al., 2018, Aspect et al., 1982

Correlations in a wider picture (with Correlation Polytopes)

Every “behavior” or “correlation” can be described as a point:

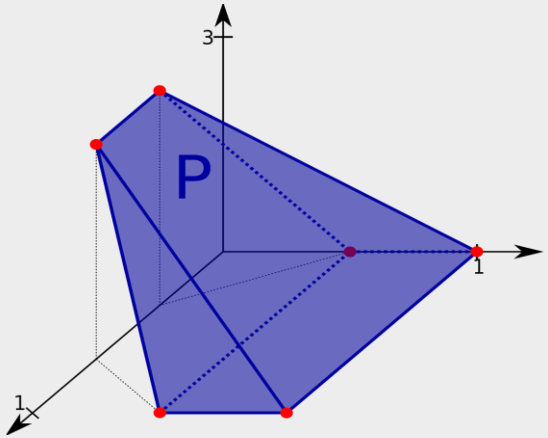
$$\boldsymbol{p} = \{p(ab|xy)\}\text{, a,b } outcomes\ and\ x, y\ measurements,$$

$$\boldsymbol{p}\ \in P\ (Probabilty\ space) \in R^{\Delta^2 m^2}, \Delta = \dim(outcomes)\ and\ m = \dim(measurments)$$

To describe the complete set of points that are allowed by **Local** “correlations” $\boldsymbol{L}$, a geometric object called (convex) **Polytope** can be used. It encloses all allowed “correlations” with boundaries as the CHSC inequalities.

A **Polytope** is a n-dimensional space enclosed by a finite number of hyperplanes of the form:

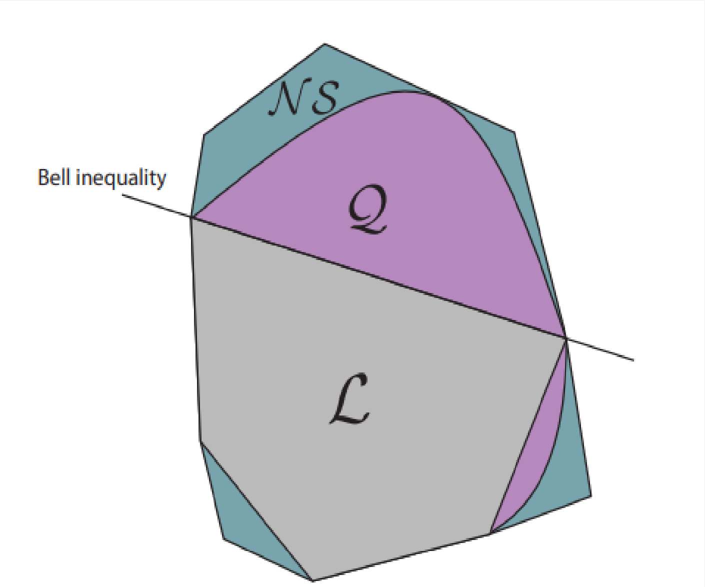
$$\sum_{i=1}^{n} a_i * x_i < b$$



Brunner et al., 2014

Local, Quantum and No-signaling

- The local set $\boldsymbol{L}$ and the No-signaling set $\boldsymbol{NS}$ are polytopes.
- The quantum set $\boldsymbol{Q}$ is convex but not a polytope.



$\boldsymbol{L} \in \boldsymbol{Q} \in \boldsymbol{NS}$

$Dim(\boldsymbol{L}) = Dim(\boldsymbol{Q}) = Dim(\boldsymbol{NS})$

Note:

- The No-signaling theorem derives from the axioms of general relativity. It generates the set off all correlations allowed by general relativity (**NS**).
- $\boldsymbol{NS}$ contains stronger correlations than the ones in **Q**.
- A vertex on the polytope describes a deterministic strategy.

Facet are boundaries of dimension $\boldsymbol{d-1}$ and are the more advantageous boundaries because they provide a minimal representation of the set $\boldsymbol{L}$.

Brunner et al., 2014

No-signaling theorem

- The axiom can be expressed by the following expression:

$$\sum_{b=1}^{\Delta} P(ab|xy) = \sum_{b=1}^{\Delta} P(ab|xy') \;\; for\ all\ a, x, y, y'$$

- Meaning that Bob can not communicate to Alice any kind of information by the measurement setting selected.

- Quantum mechanics does not violate this axiom. The proof can be performed by applying a measurement operator on Bob's subsystem and tracing his influence out, obtaining the density matrix of Alice:

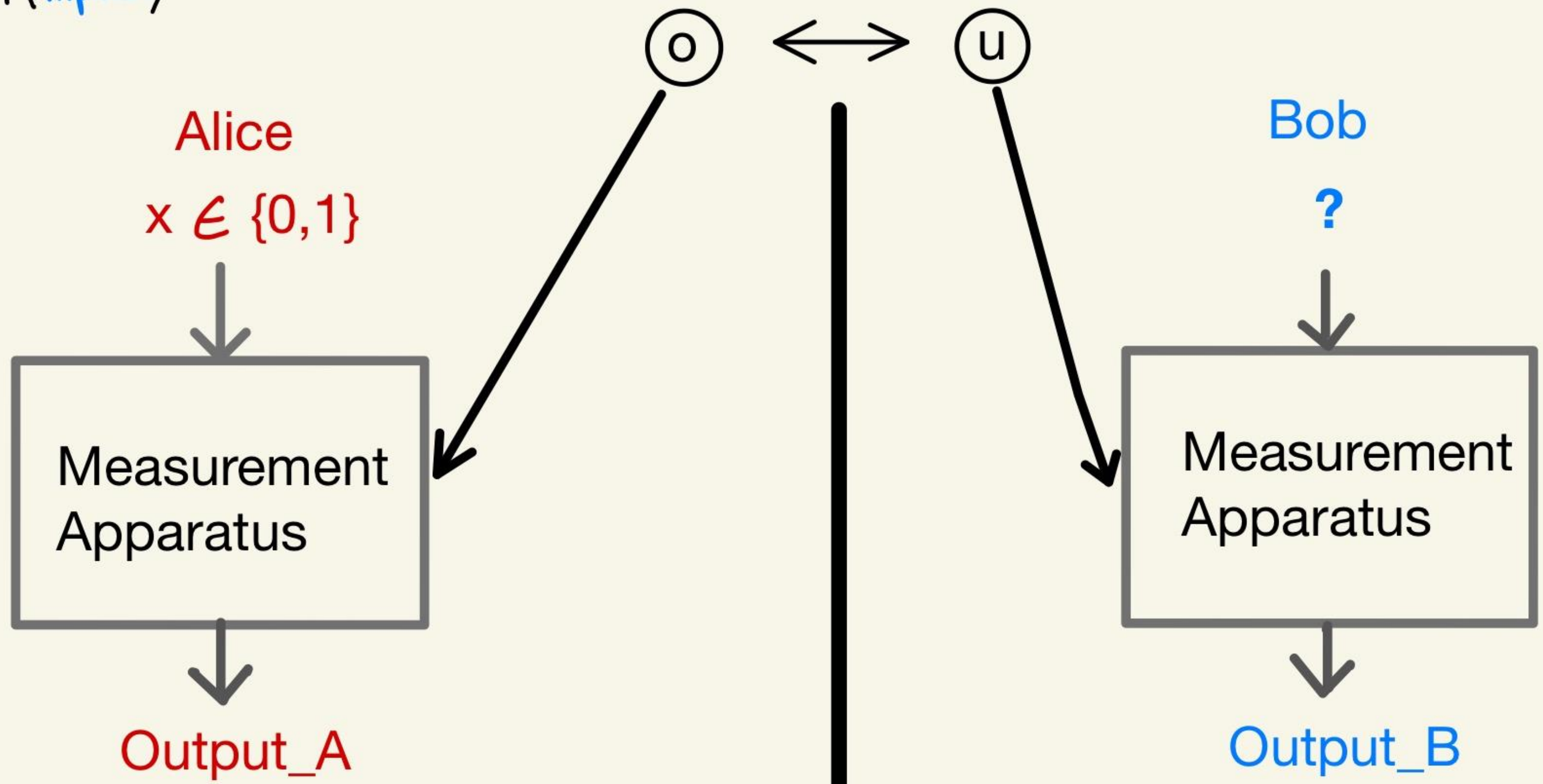
$$tr_{H_A}\big(P(\sigma)\big) = tr_{H_A}(\textstyle\sum_k \big(V_k \otimes I_{H_A}\big)^* \sigma (V_k \otimes I_{H_A})$$

$$= tr_{H_A}(\textstyle\sum_i \sum_k V_k^* T_i V_k \otimes S_i)$$

$$= \; \ldots$$

$$= tr_{H_A}(\sigma)$$

Probability of Alice knowing Bob's measurement before receiving the particle- $P(\text{Input\_B})$



$$\boxed{P(\text{Input\_B} \mid \text{Output\_A}, \text{Input\_A}) = P(\text{Input\_B})}$$

Popescu-Rohrlich boxes

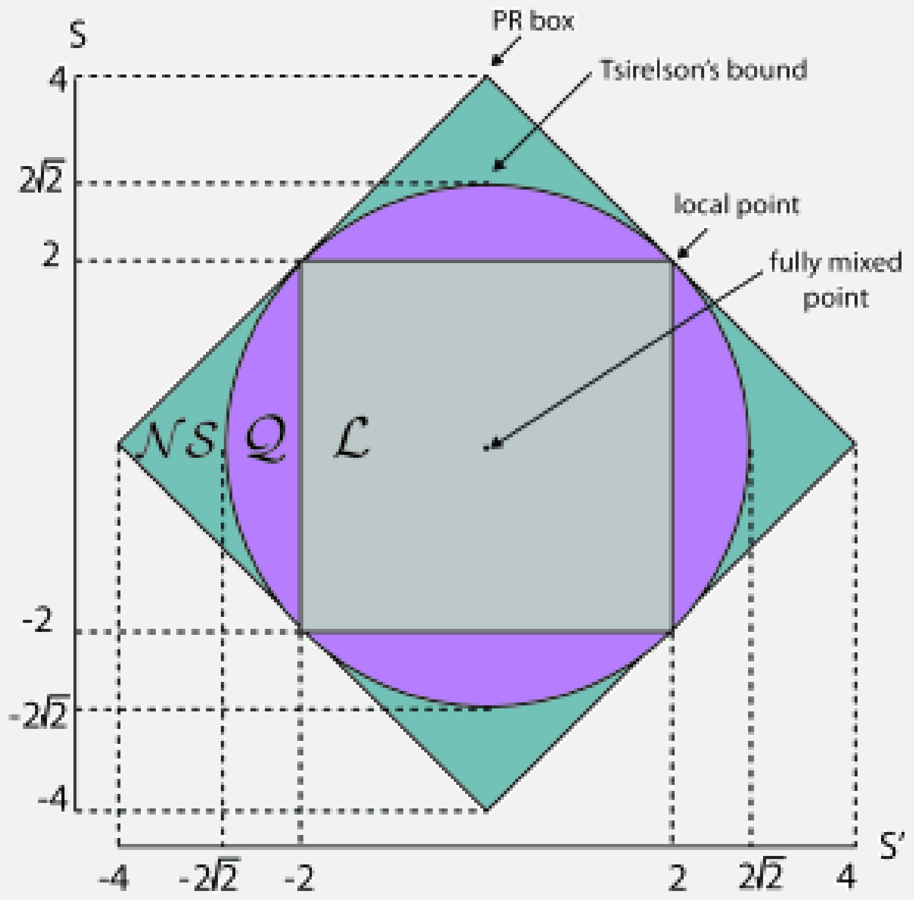
- They are maximally nonlocal for the class of two-input, two-output boxes. (Representing vertexes on the set $\boldsymbol{NS/L}$)

- PR-boxes were introduced as a candidate to be a unit of nonlocality.

- And in the biparted case, they serve as a measure of correlation:

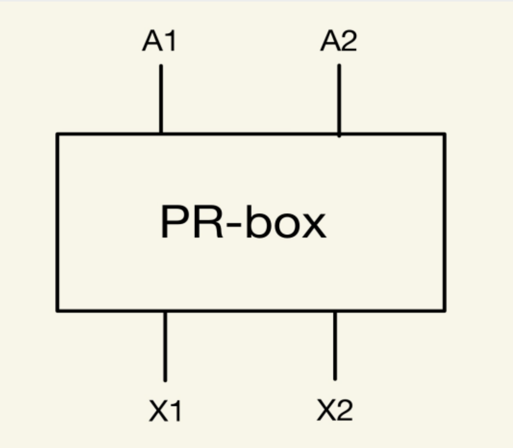
$$P_\varepsilon = \alpha * PR + (1-\alpha)\overline{PR},$$

$$\overline{PR} - \text{called the anti PR} - \text{box works as element generating noise}$$



(Brunner et al., 2014)

Propescu-Rohrlich boxes



- PR-boxes are described by the follwing correlations:

$$P(a_1 a_2|x_1\ x_2) = \begin{cases} \frac{1}{2}:\ a_1 + a_2 = x_1 x_2 mod\ 2 \\ 0: otherwise \end{cases}$$

- These boxes solve important communication problems trivially.

- However, they fail to describe nonlocality present in multipartite systems, in Barrett & Pironio, 2005.

	$a_1 = 0$ $a_2 = 0$	$a_1 = \mathbf{1}$ $a_2 = 0$	$a_1 = 0$ $a_2 = \mathbf{1}$	$a_1 = \mathbf{1}$ $a_2 = \mathbf{1}$
$x_1 = 0$ $x_2 = 0$	$\frac{1}{2}$	0	0	$\frac{1}{2}$
$x_1 = \mathbf{1}$ $x_2 = 0$	$\frac{1}{2}$	0	0	$\frac{1}{2}$
$x_1 = 0$ $x_2 = \mathbf{1}$	$\frac{1}{2}$	0	0	$\frac{1}{2}$
$x_1 = \mathbf{1}$ $x_2 = \mathbf{1}$	0	$\frac{1}{2}$	$\frac{1}{2}$	0

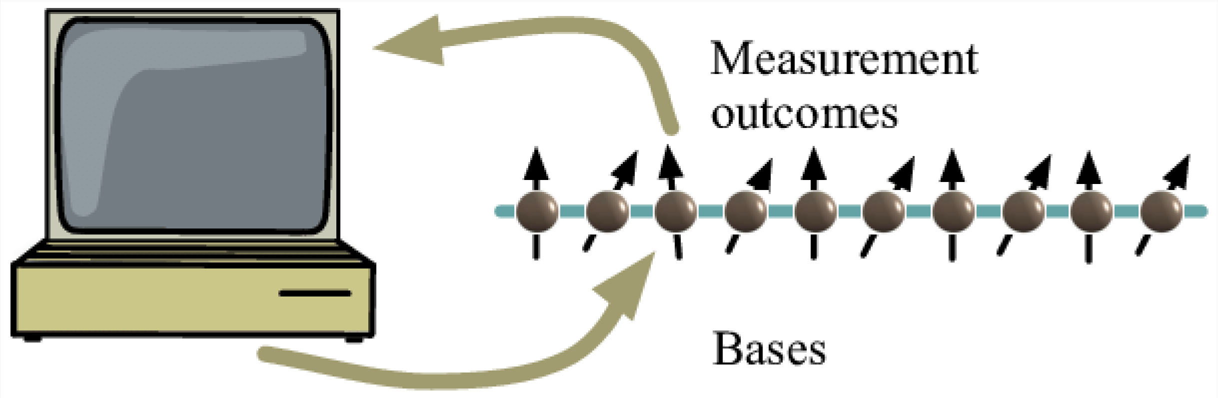
Barrett & Pironio, 2005

Non-locality in Quantum computations

- In different Quantum computing schemes, nonlocality is seen as a resource for computational advantage:
  - For the circuit model, entangled states are required for quantum algorithms with a computational advantage.
  - But the Measurement-based quantum computing scheme requires a strongly entangled resource state to perform universal quantum computations.
    - Bell inequalities are related to computational questions.

Measurement-based quantum computing

- This quantum computation model is based on adaptively performing measurements on a cluster state (highly entangled resource state).



Linear Boolean Functions(LBF)

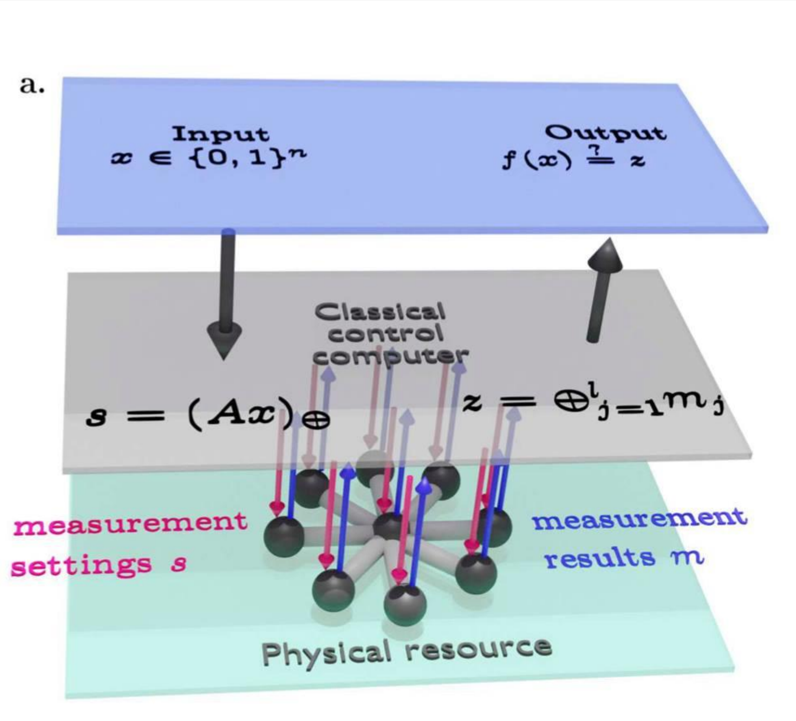
$$f: \{0,1\}^n \rightarrow \{0,1\}$$
$$f(c_1\, x_1 \oplus c_2\, x_2 \oplus \cdots \oplus c_n\, x_n)$$

Note: It is very interesting that the classical computer only requires to compute linear Boolean functions, to choose the subsequente measurement bases.

Non-adaptative MBQC

- Consider now that all measurements have to be selected beforehand.

$$LBF(x_1, x_2, \cdots, x_n) \to (M_1, M_2, \cdots, M_k)$$
$$\to LBF(m_1, m_2, \cdots, m_k) \to \{0,1\}$$



- Any linear Boolean function can be computed with the auxiliary classical device. The **non-linear Boolean functions (NBF)** are the ones of interest.

Note: In this model, universal quantum computations are lost, but it would be interesting to understand if entangled resources create some advantage for NBF !!!

(Demirel et al., 2020)

Non-linear Boolean function on NMBQC

It was shown that the probability of computing correctly and non-linear Boolean function depends on the following equations:

$$P\big(z = f(x)\big) = \frac{1}{2}(1 + \beta)$$

Where $\boldsymbol{\beta}$ is descibed as:

$$\beta = \sum_{x} p(x)(-1)^{f(x)}E(x)$$

This second expression has the same form as a **Bell inequality** and works in the same way, **classical** and **quantum** resources have different limits.

(Demirel et al., 2020)

Limits for the NAND operator

The $\boldsymbol{\beta}$ expression for the NAND operator does obtain exactly the same form as the CHSH inequality normalized to 1.

A	B	Q
0	0	1
0	1	1
1	0	1
1	1	0

NAND

$$\boldsymbol{\beta}_{NAND} = \frac{1}{4}E(x_1 = 0, x_2 = 0) + \frac{1}{4}E(x_1 = 1, x_2 = 0) + \frac{1}{4}E(x_1 = 0, x_2 = 1) - \frac{1}{4}E(x_1 = 1, x_2 = 1)$$

The classical limit is the predicted previously, the quantum for two entangled particles also.

$$\beta \leq \begin{cases} 0{,}5, classical \\ 0{,}71, quantum \end{cases} \quad \rightarrow \quad P\big(z = f(x)\big) \leq \begin{cases} 0{,}75, classical \\ 0{,}85, quantum \end{cases}$$

But, in this computational scheme, it is possible to use greater quantum resources.

GHZ states as resource state

If we use an entangled resource state with higher dimensionality, for example, a triparted GHZ state $(|GHZ^{(l)}\rangle = |0^l\rangle + |1^l\rangle, l = 3)$

$$\beta = 1$$

It is possible to violate the CHSH inequality with probability 1.

$$P\big(z = f(x)\big) = \frac{1}{2}(1 + \beta) = \frac{1}{2}(1 + 1) = 1$$

Meaning that with a GHZ state it is possible to implement **deterministically** a **NAND operator**. This means that the NMBQC scheme can compute any **classical** computation!

(Demirel et al., 2020)

How to implemente it now?

The complementary linear computer decides which measurements are performed on the GHZ state

$$s_1 = x_1, \;\; s_2 = x_2, \;\; s_3 = x_1 \oplus x_2 \oplus 1$$

Associating with each binary value a measurment setting:

$$s_k = 0 \;\rightarrow \sigma_x \,, s_k = 1 \;\rightarrow \sigma_y;$$

Performing the selected measurement on the GHZ gives:

$$\begin{array}{c} x_1{=}0 \; x_2 = 0 \\ x_1{=}1 \; x_2 = 0 \\ x_1{=}0 \; x_2 = 1 \\ x_1{=}1 \; x_2 = 1 \end{array} \quad \rightarrow \quad \begin{array}{l} \sigma_x \otimes \sigma_x \otimes \sigma_y = -|GHZ\rangle \\ \sigma_y \otimes \sigma_x \otimes \sigma_x = -|GHZ\rangle \\ \sigma_x \otimes \sigma_y \otimes \sigma_x = -|GHZ\rangle \\ \sigma_y \otimes \sigma_y \otimes \sigma_y = +|GHZ\rangle \end{array}$$

A	B	Q
0	0	1
0	1	1
1	0	1
1	1	0

NAND

$$\boxed{m_1 \oplus m_1 \oplus m_1 = NAND(x_1, x_2)}$$

(Anders & Browne, 2009)

Conclusions

- Non-locality is a very broad and interesting topic, which has an important contribution to quantum communications and computations.

- Measurement-based quantum computing uses non-local resources, understanding how they are used could help us understand what gives quantum computer an advantage over classical ones.

Questions

Std: **Has any violation of the quantum limit ever been observed?**

Ernesto: No, there is no observation of such stronger correlations.

Std: **Why is quantum mechanics not more nonlocal?**

Ernesto: Some axioms were proposed to answer to this question (Information causality (Allcock et al., 2009)(Popescu, 2014) for instance would eliminate the ability to reduce communication problems to an absurd.) But no one could recover the quantum boundary.

Std: **Why is Bell nonlocality used as na exemple of contextuality?**

Ernesto: It does work with a context, but every measurement set is allowed. So it is a very special exemple.

Std: **Are hidden variable theories still at the table?**

Ernesto: Yes, but they have to be nonlocal. Bohmian mechanics (Goldstein, 2017) is an example of a nonlocal hidden variable theory that predicts the same results as QM.

Std- Student that should know better

References

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