

Witness assisted variational eigensolver



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Jarrold McClean



Andrea Gentile

Me



David Tew

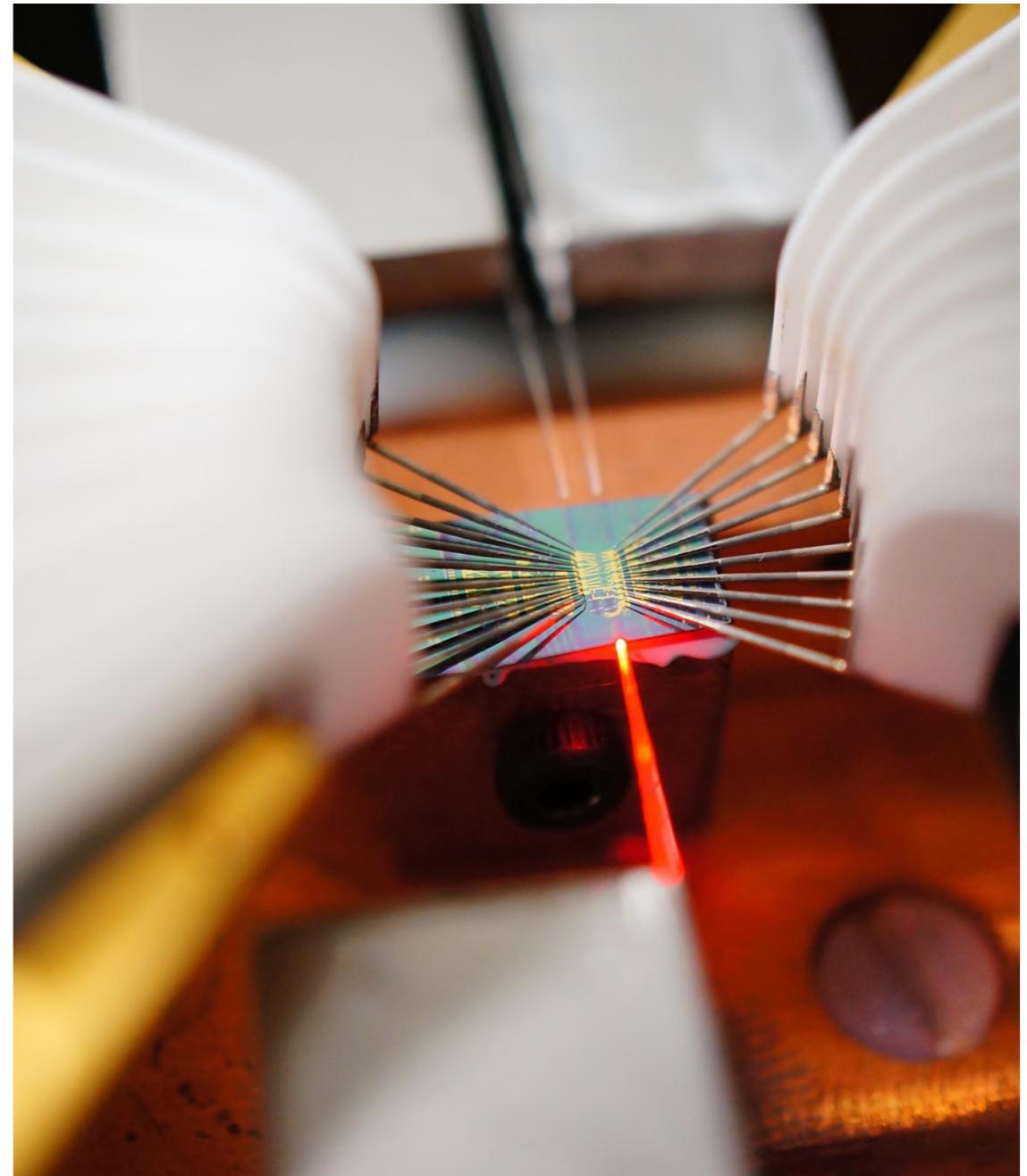


Nathan Wiebe



Content

- Variational quantum eigensolvers
- Experimental implementations
- WAVES
- Experimental and simulation results
- (Optional) A bit of photonic quantum hardware



Variational quantum eigensolver

Given \hat{H} we want to find $|\psi\rangle$ s.t. $\hat{H}|\psi\rangle = \lambda|\psi\rangle$.

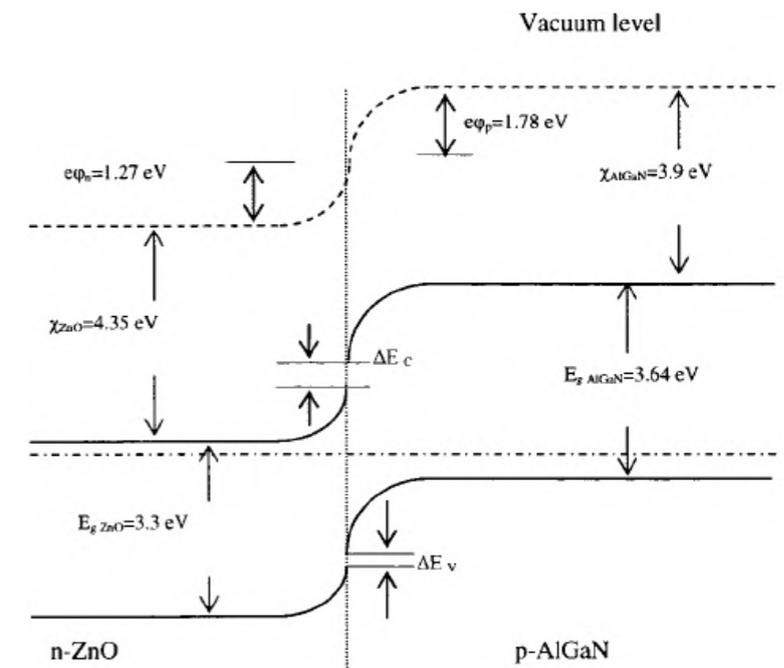
High precision estimation of λ

Agriculture -> 100-200 qubits -> Design catalysts for efficient conversion of Ni to fertilizer

Carbon Capture -> 100-200 qubits -> Catalysts to extract carbon dioxide from air with less energy

Battery design -> 10^3 qubits

Drug discovery -> Molecular docking

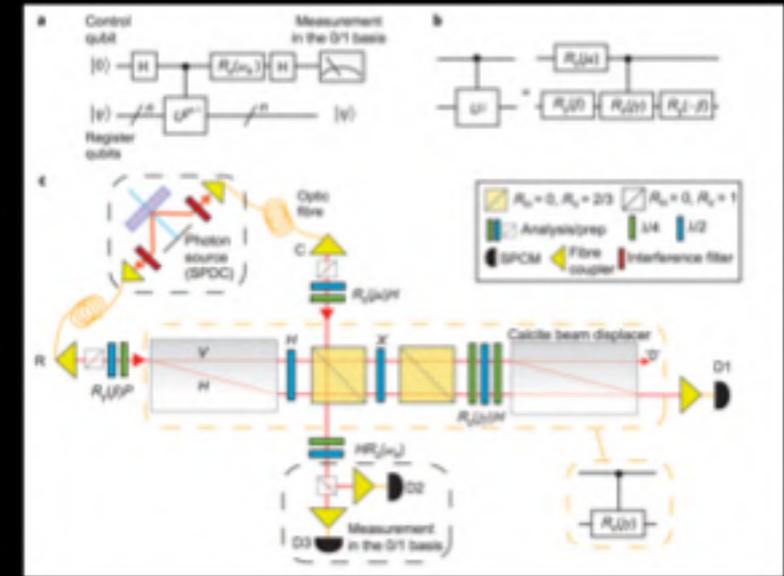
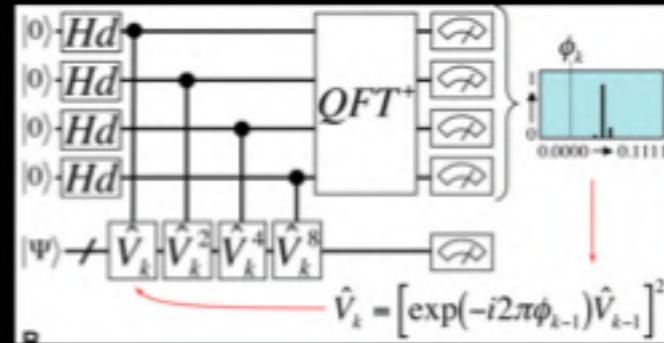


Feynman, R. P. Int. J. Theor. Phys. 21, 467 (1982).
Lloyd, S. Science 273, 1073–1078 (1996).
Aspuru-Guzik, A. et. al. Science 309, 1704 (2005).

Canonical methods

Adiabatic state preparation

PEA

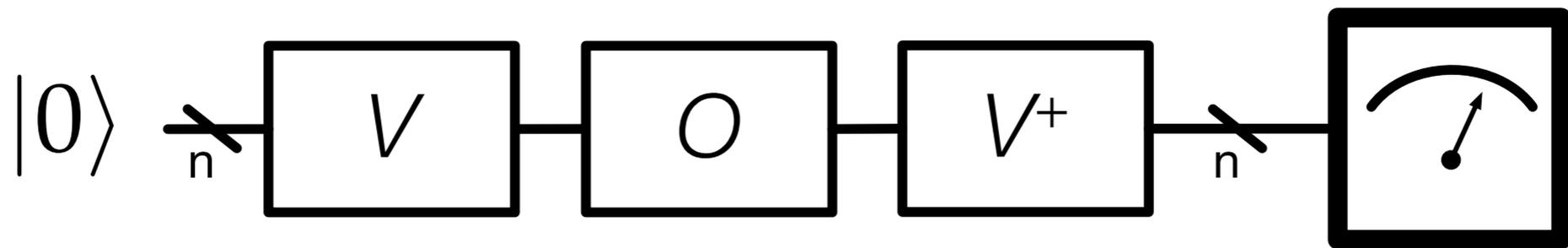


A. Aspuru-Guzik, Love, A. D. Dutoi Science (2005)

Lanyon et al. Nat Chem. (2010)

Measuring expectation values

$$\langle O \rangle_{|\Psi\rangle} = \frac{\langle \Psi | O | \Psi \rangle}{\langle \Psi | \Psi \rangle}$$



Scalable if $O = \sum_{\alpha} h_{\alpha} O_{\alpha}$ With a poly number of O_{α}

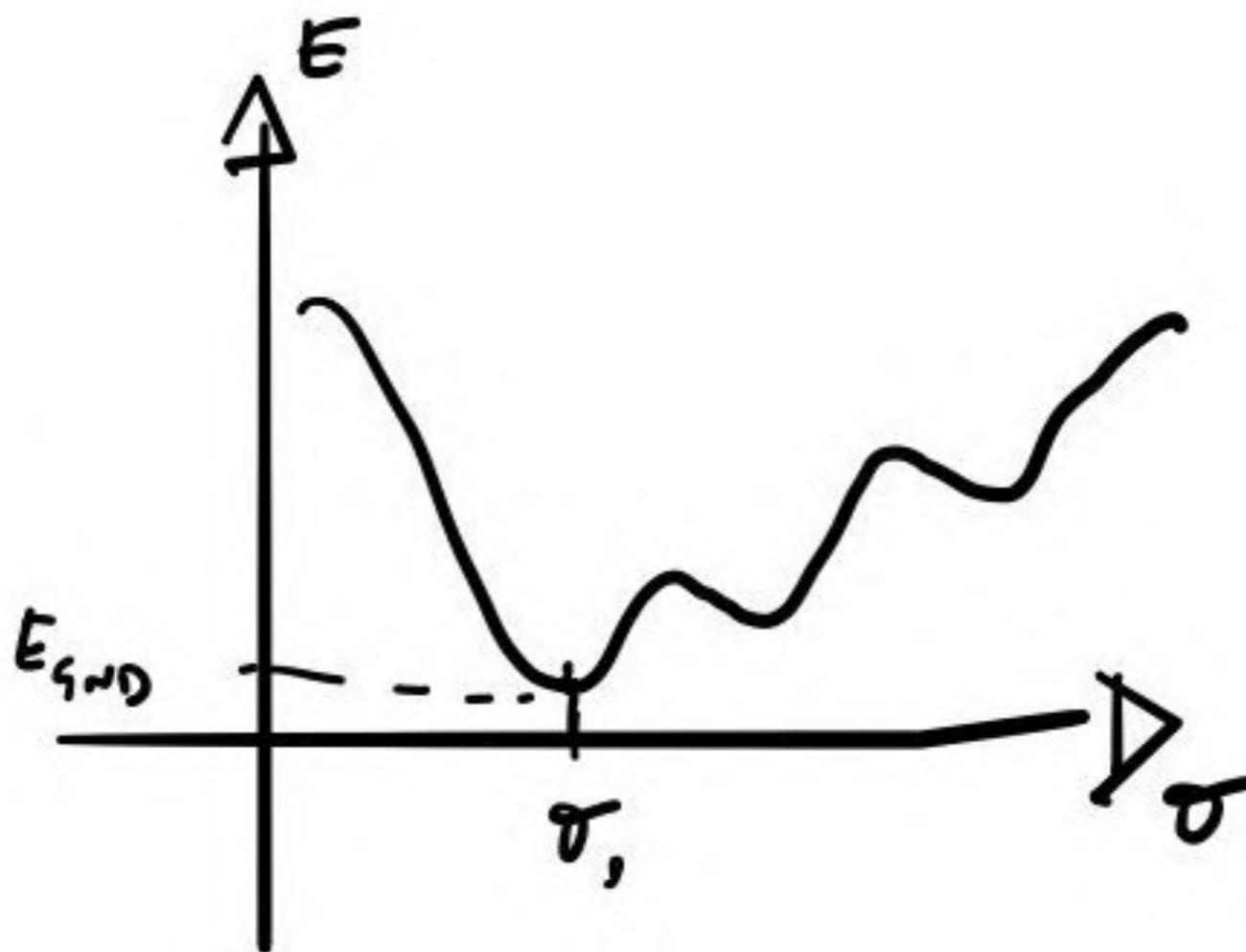
Variational Principle

$$\langle \mathcal{H} \rangle(\vec{\theta}) = \langle \psi(\vec{\theta}) | \mathcal{H} | \psi(\vec{\theta}) \rangle \geq E_{\text{gnd}}.$$

In **VQE**, we want to find the set of parameters that minimize the expectation value of the energy.

If our state preparation routine is general enough and the minimization protocol good enough we should find our ground state

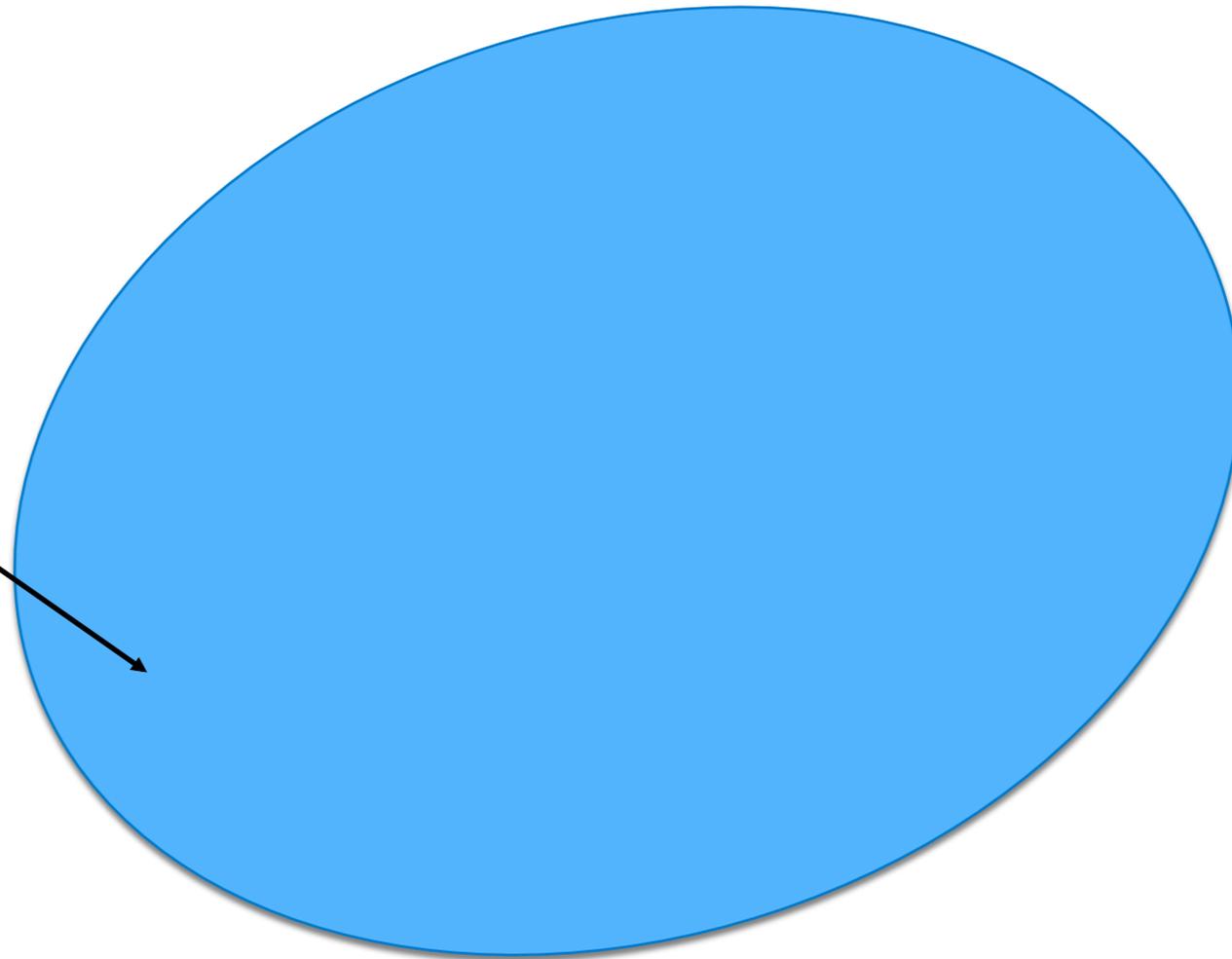
VQE



Variational Principle

$$\langle \mathcal{H} \rangle(\vec{\theta}) = \langle \psi(\vec{\theta}) | \mathcal{H} | \psi(\vec{\theta}) \rangle \geq E_{\text{gnd}}.$$

Hilbert space
of n qubits
(Exponential)

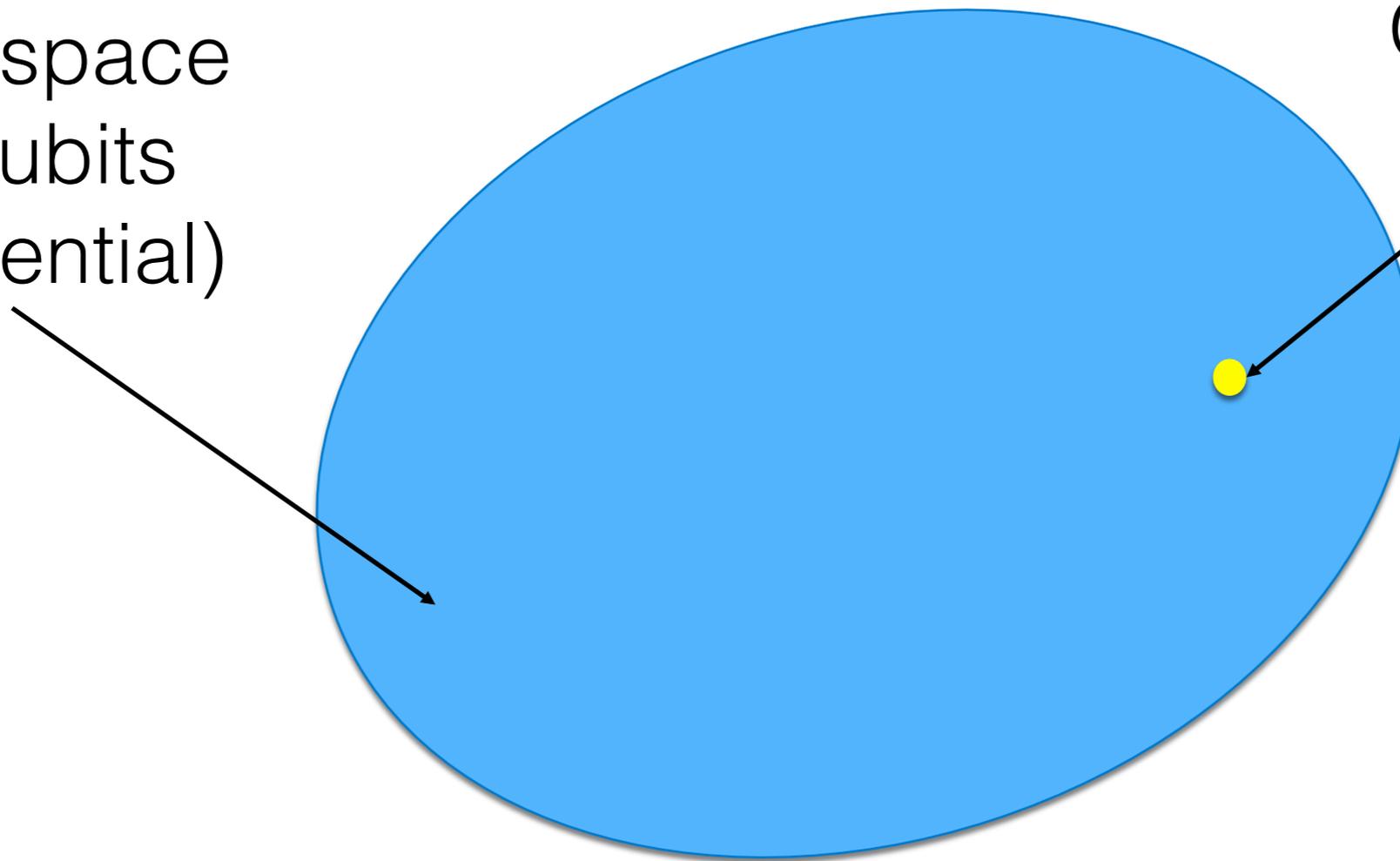


Variational Principle

$$\langle \mathcal{H} \rangle(\vec{\theta}) = \langle \psi(\vec{\theta}) | \mathcal{H} | \psi(\vec{\theta}) \rangle \geq E_{\text{gnd}}.$$

Hilbert space
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Ground state

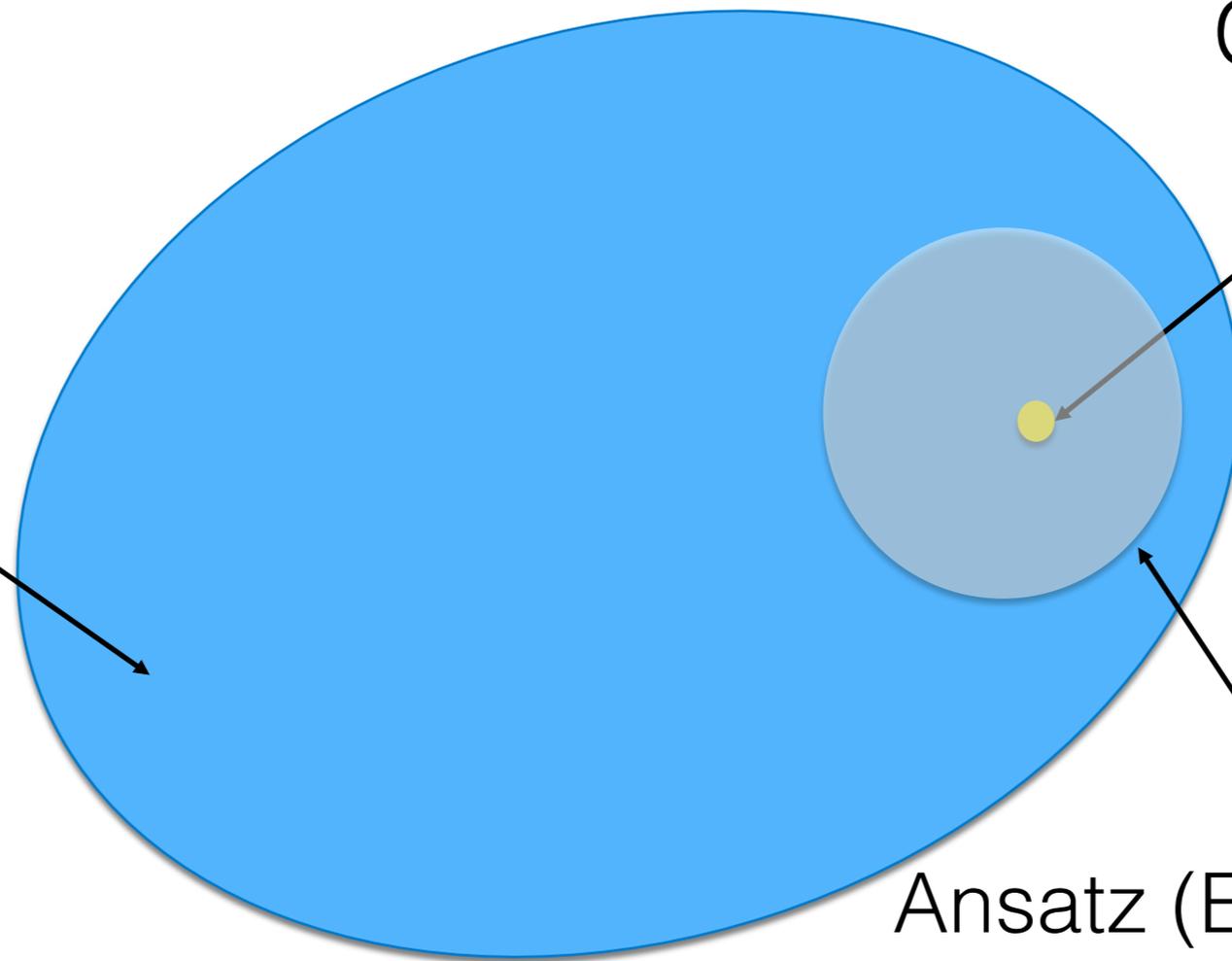


Variational Principle

$$\langle \mathcal{H} \rangle(\vec{\theta}) = \langle \psi(\vec{\theta}) | \mathcal{H} | \psi(\vec{\theta}) \rangle \geq E_{\text{gnd}}.$$

Hilbert space
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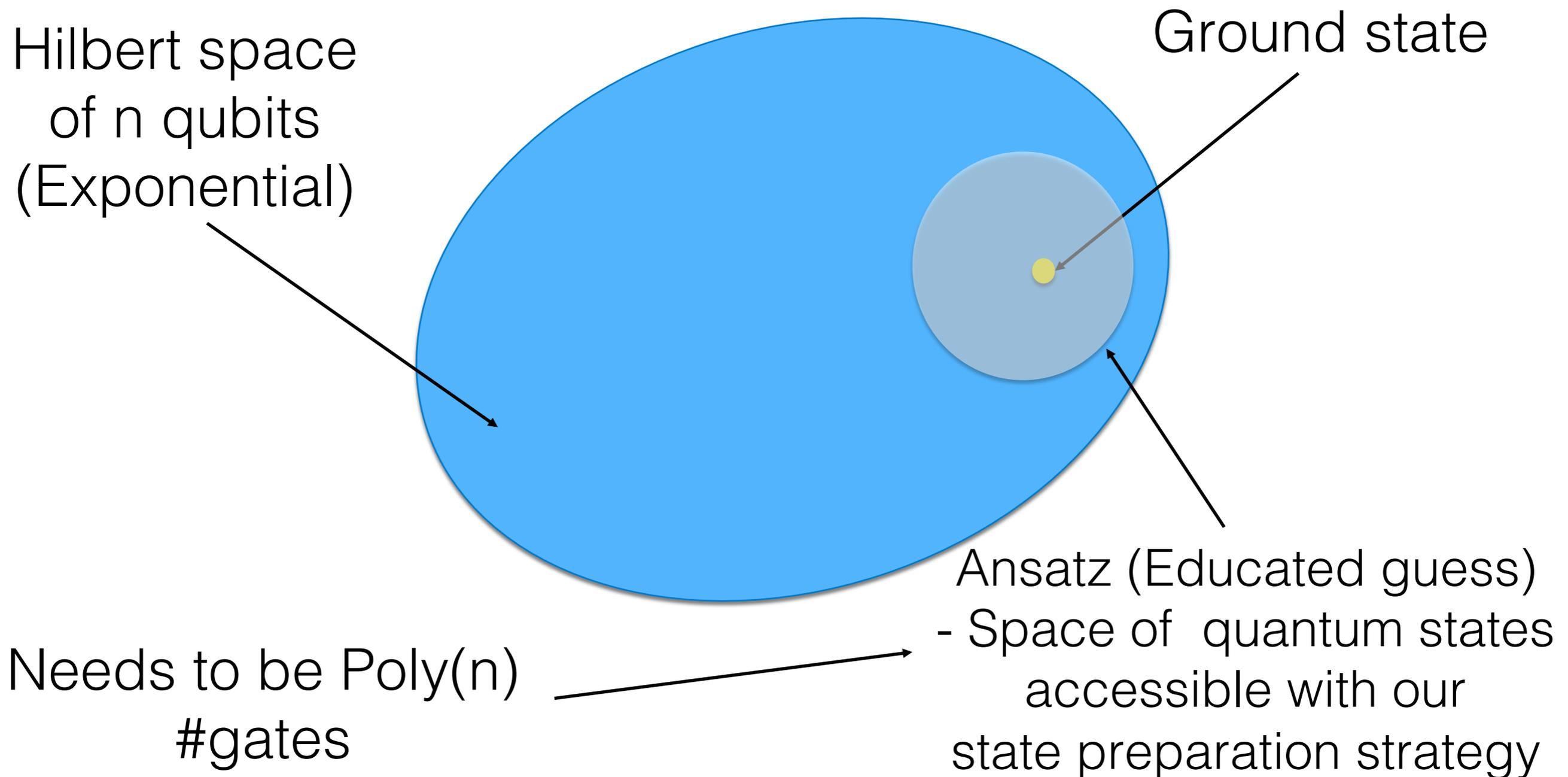
Ground state



Ansatz (Educated guess)
- Space of quantum states
accessible with our
state preparation strategy

Variational Principle

$$\langle \mathcal{H} \rangle(\vec{\theta}) = \langle \psi(\vec{\theta}) | \mathcal{H} | \psi(\vec{\theta}) \rangle \geq E_{\text{gnd}}.$$



Ansatz

Unitary coupled cluster

$$|\Psi\rangle = e^{T - T^\dagger} |\Phi\rangle_{\text{ref}}$$

$(T - T^\dagger)$ Anti-hermitian

$$T = T_1 + T_2 + \dots$$

$$T_1 = \sum_{pr} t_p^r \hat{a}_p^\dagger \hat{a}_r$$

Jordan-Wigner

$$\hat{a}_j \rightarrow I^{\otimes j-1} \otimes \sigma_+ \otimes \sigma_z^{\otimes N-j}$$

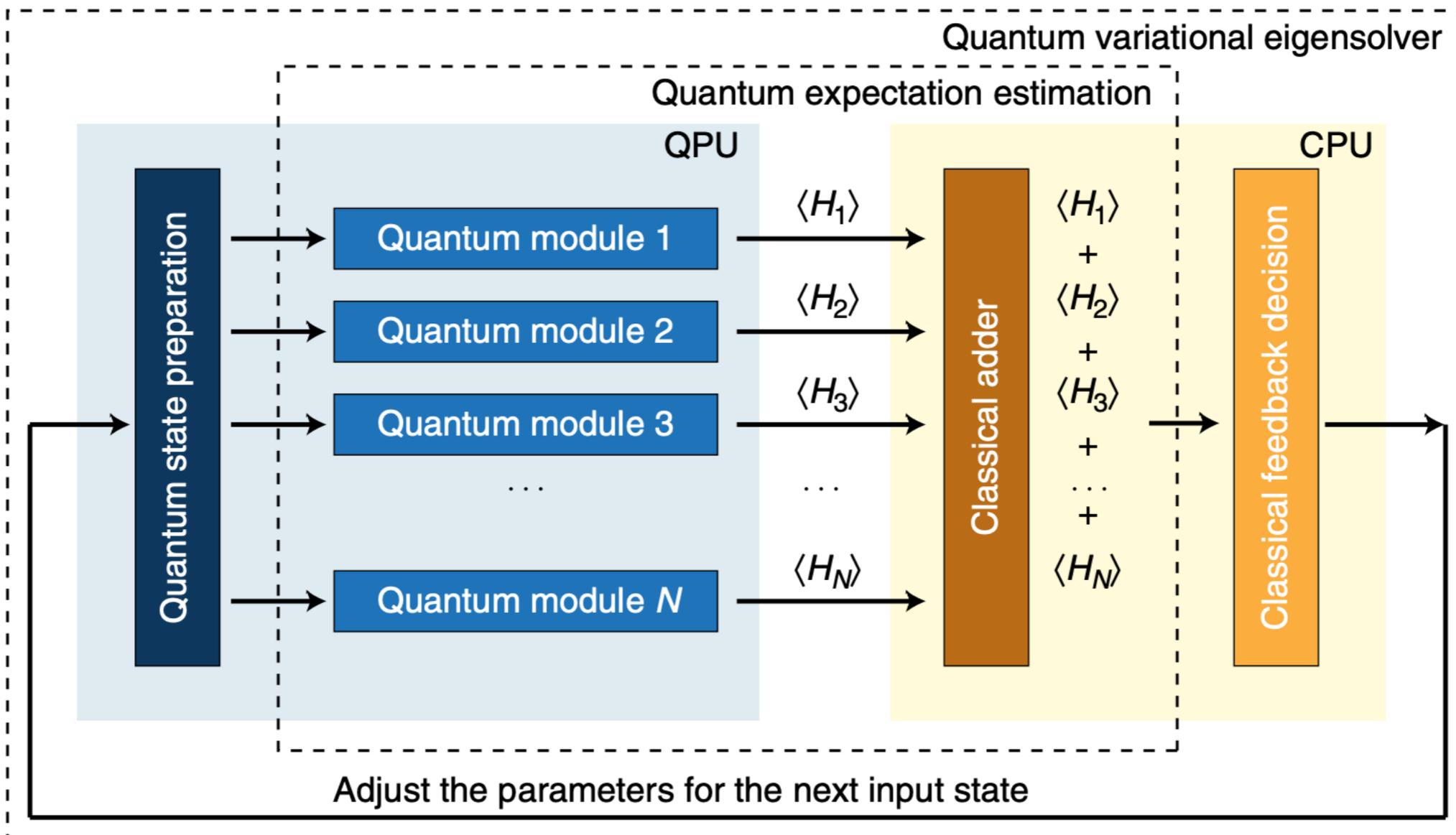
$$\hat{a}_j^\dagger \rightarrow I^{\otimes j-1} \otimes \sigma_- \otimes \sigma_z^{\otimes N-j}$$

$$T_2 = \sum_{pqrs} t_{pq}^{rs} \hat{a}_p^\dagger \hat{a}_q^\dagger \hat{a}_r \hat{a}_s$$

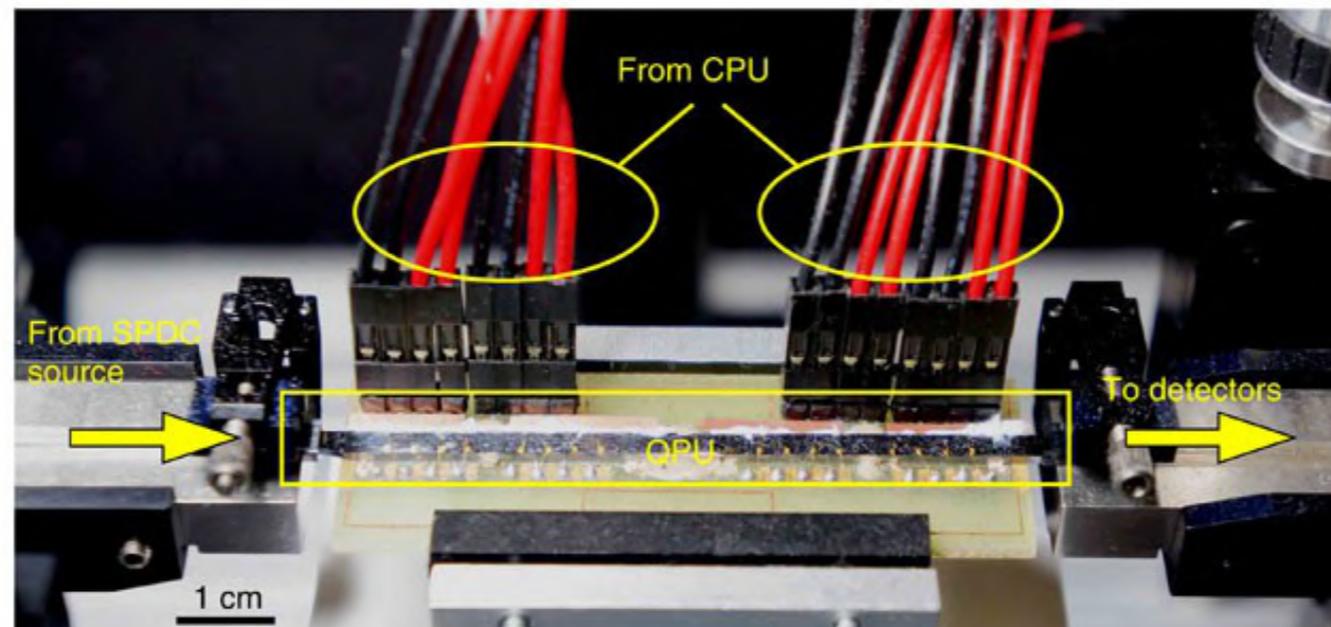
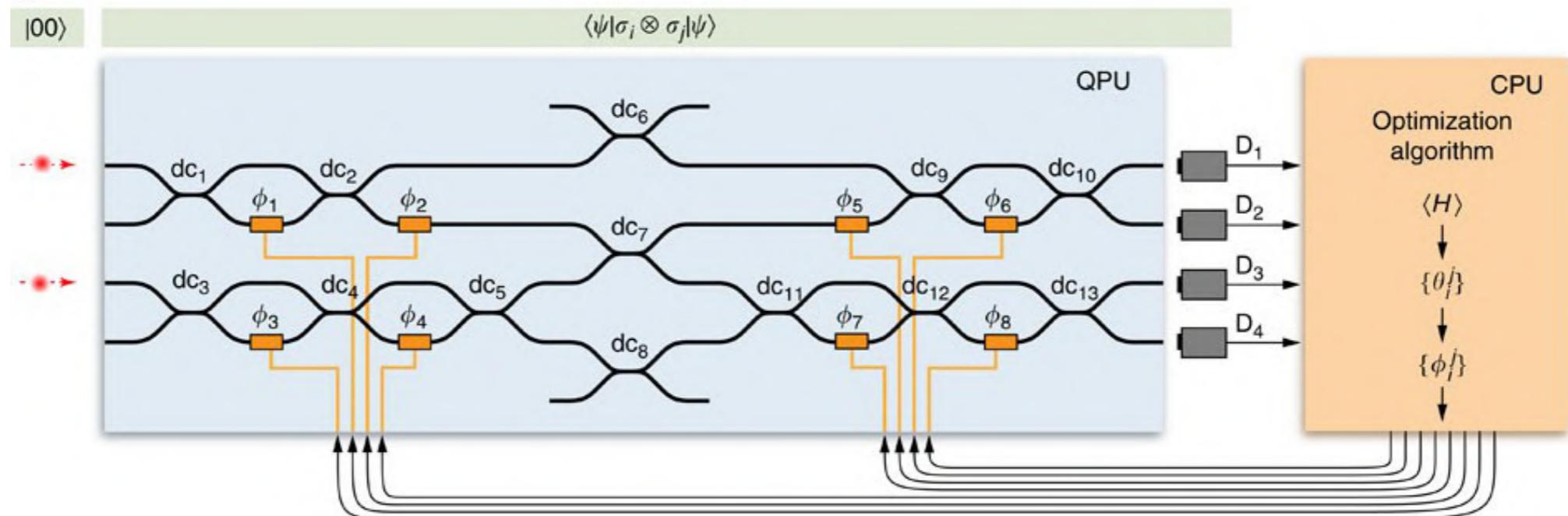
VQE

$$\langle \mathcal{H} \rangle(\vec{\theta}) = \langle \psi(\vec{\theta}) | \mathcal{H} | \psi(\vec{\theta}) \rangle \geq E_{\text{gnd}}.$$

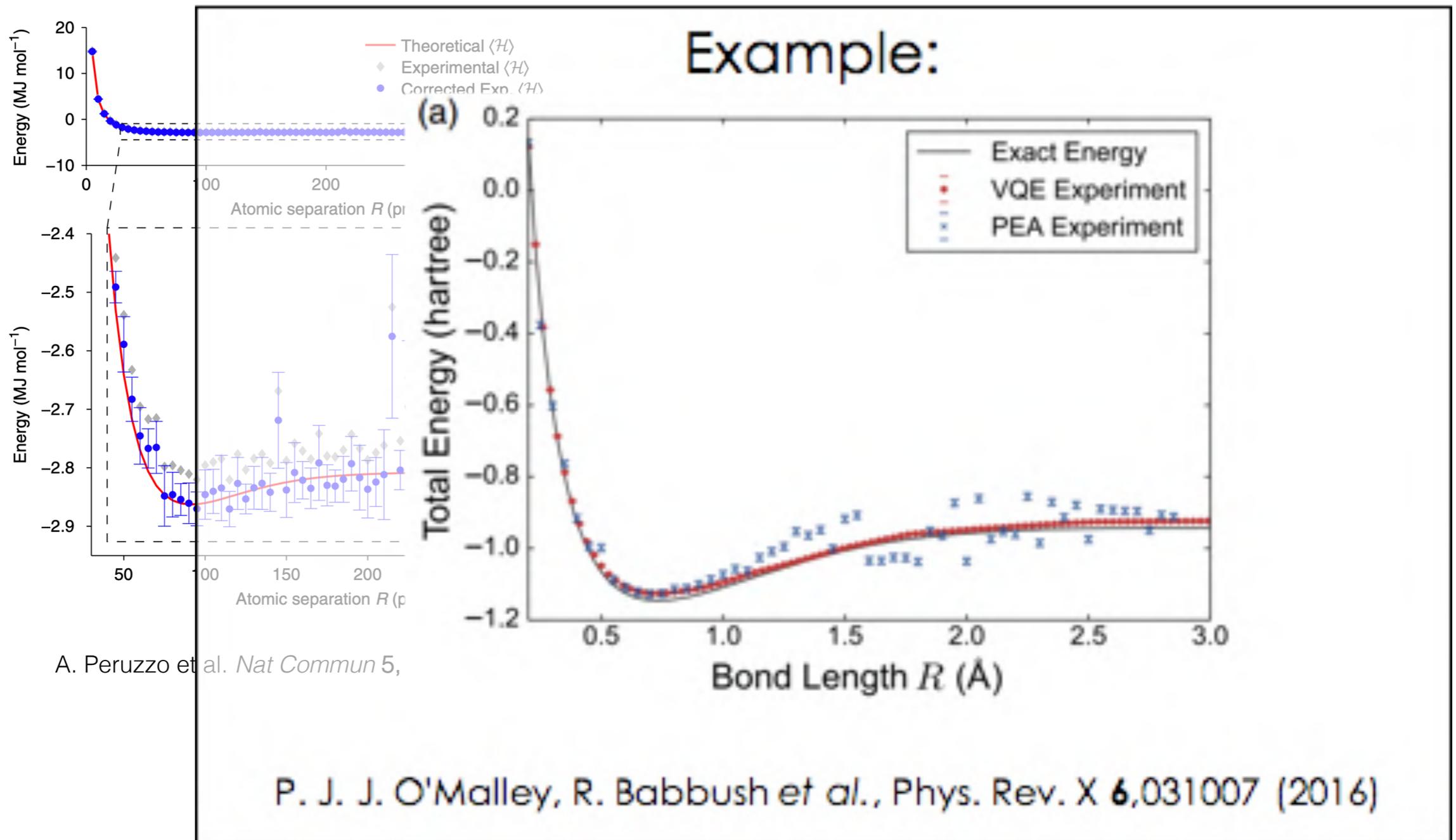
$$\mathcal{H} = \sum_{i\alpha} h_{\alpha}^i \sigma_{\alpha}^i + \sum_{ij\alpha\beta} h_{\alpha\beta}^{ij} \sigma_{\alpha}^i \sigma_{\beta}^j + \dots$$



First experimental dem.



VQE experiment



VQE summary

- VQE is great for noise resilience
- It is expected to be applied to pre-threshold devices
- As it is, it can target only the ground state but excited states are equally fundamental
- So far main solution was to use an expensive folding spectra method

WAVES - Witness Assisted Variational Eigenstate Solver

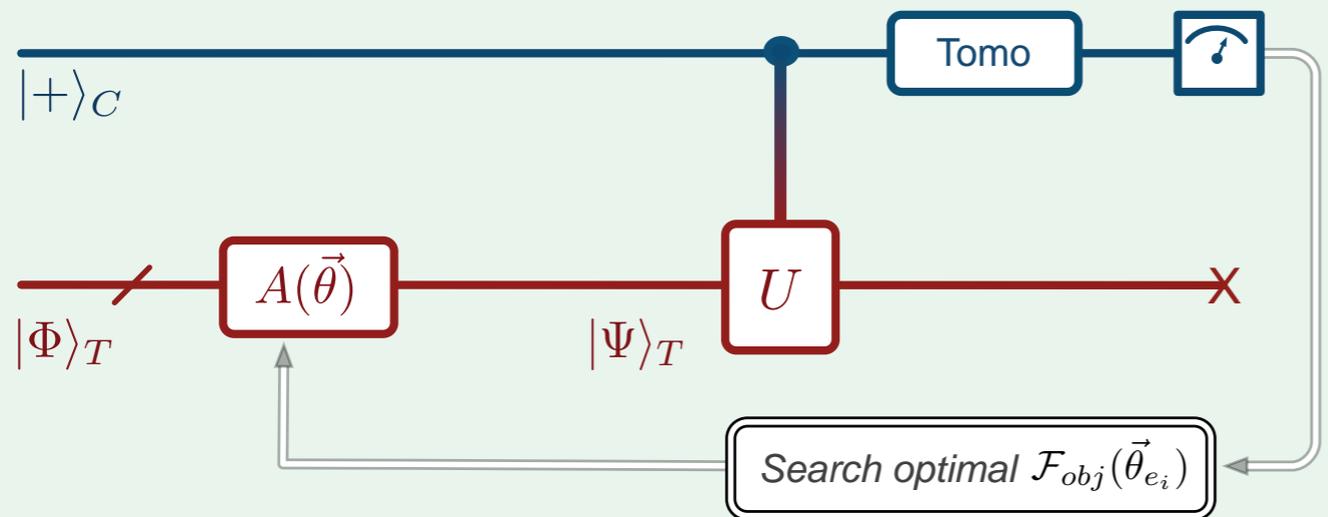
Ground-state search
using \mathcal{F}_{obj} and $\hat{E}_{p_0} = \hat{I}$

Excited-states search
using \mathcal{S} and guessed \hat{E}_{p_i}

Eigenvalues estimation
using *IPEA*

$$U = e^{-iHt}$$

Ground-state search:



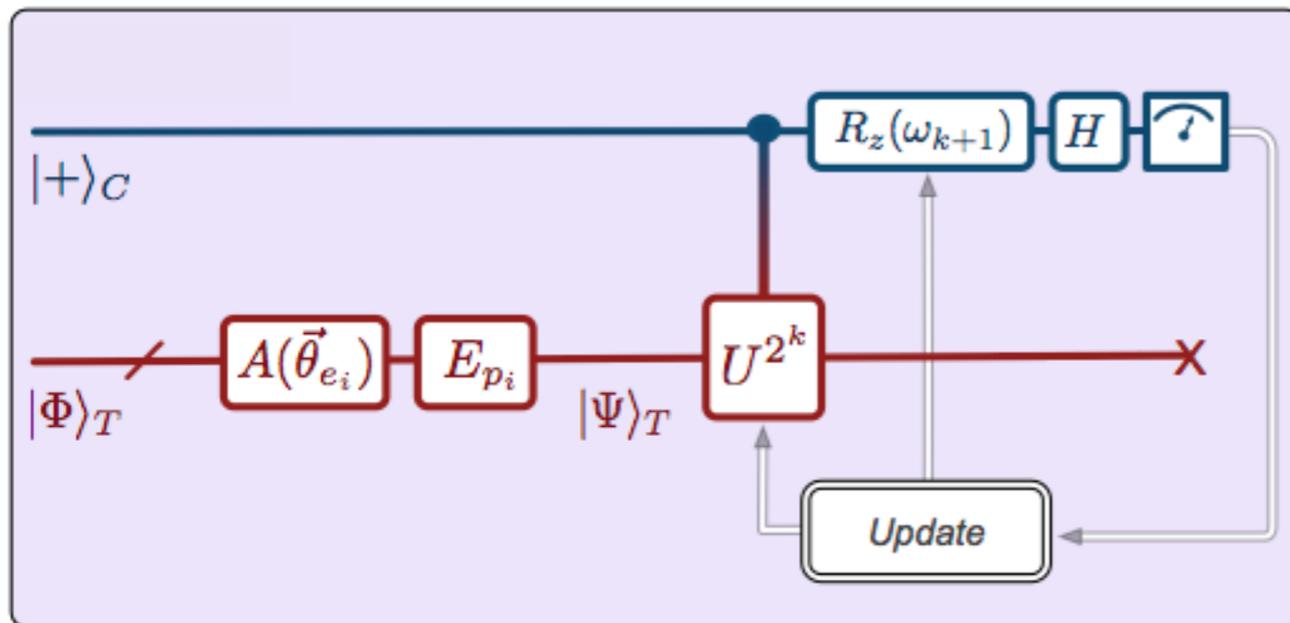
$$\hat{U} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix}$$

$$|+\rangle_c \rightarrow |0\rangle + e^{i\phi}|1\rangle$$

From the phase we can estimate the eigenvalue (i.e. the energy)

WAVES - Witness Assisted Variational Eigenstate Solver

If P is 1 $|\Psi_T\rangle$ is eigenstate
 If P is <1 $|\Psi_T\rangle$ is NOT eigenstate



The purity can be used as eigenstate witness.

Helmholtz free energy

$$\mathcal{F}'_{obj}(\mathcal{S}, \mathcal{E}) = \mathcal{E} + T\mathcal{S}(\rho)$$

$$\mathcal{F}_{obj}(\mathcal{P}, \mathcal{E}) = \mathcal{E} - T\mathcal{P}$$

$$\mathcal{S} = -\text{Tr}(\rho \ln \rho) \quad \mathcal{P} = 1 - \text{Tr}(\rho^2)$$

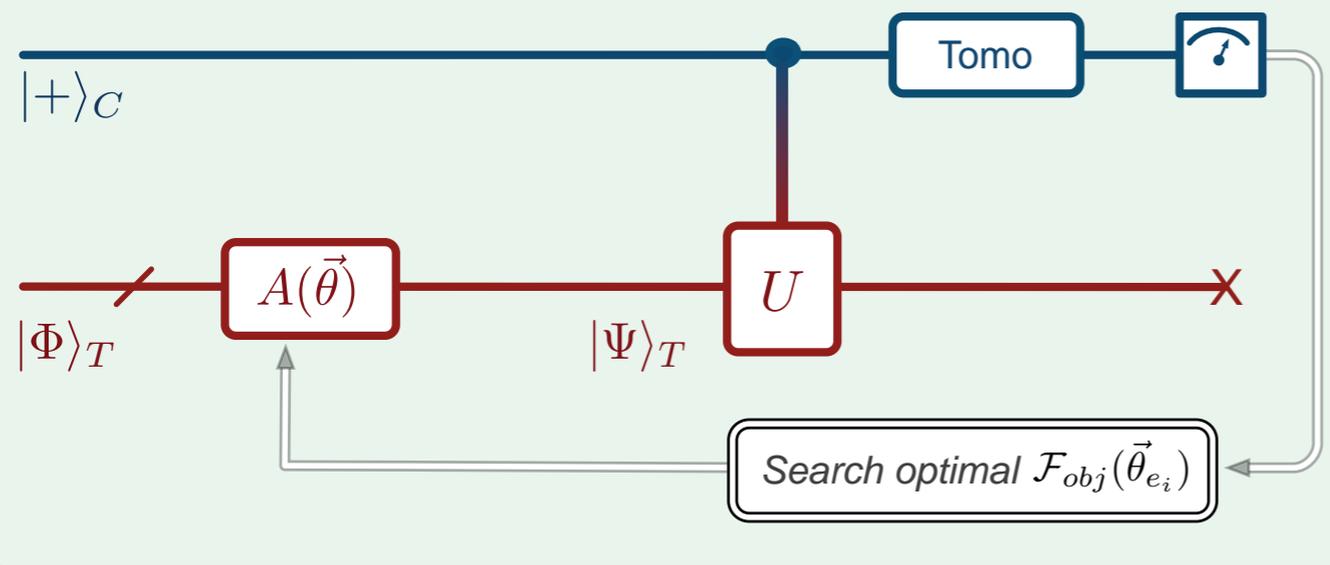
WAVES - Witness Assisted Variational Eigenstate Solver

Ground-state search
using \mathcal{F}_{obj} and $\hat{E}_{p_0} = \hat{I}$

Excited-states search
using \mathcal{S} and guessed \hat{E}_{p_i}

Eigenvalues estimation
using *IPEA*

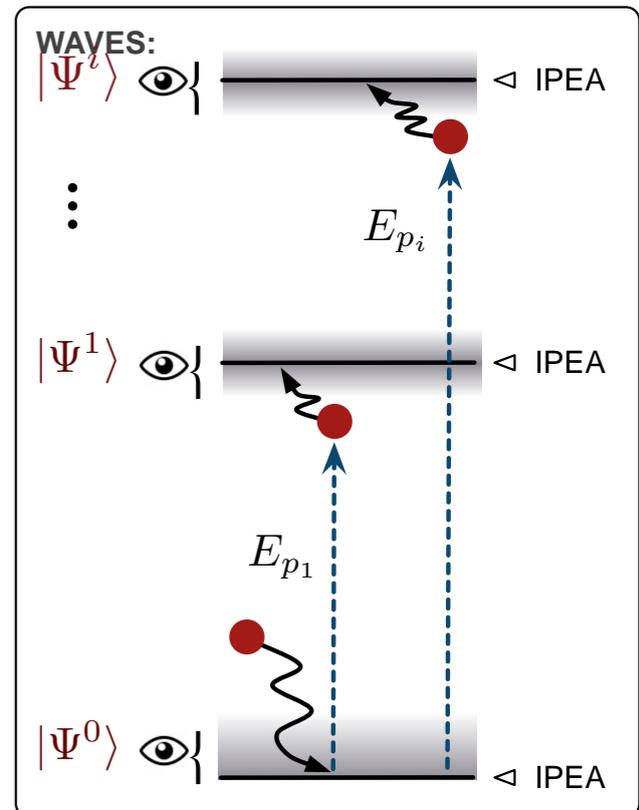
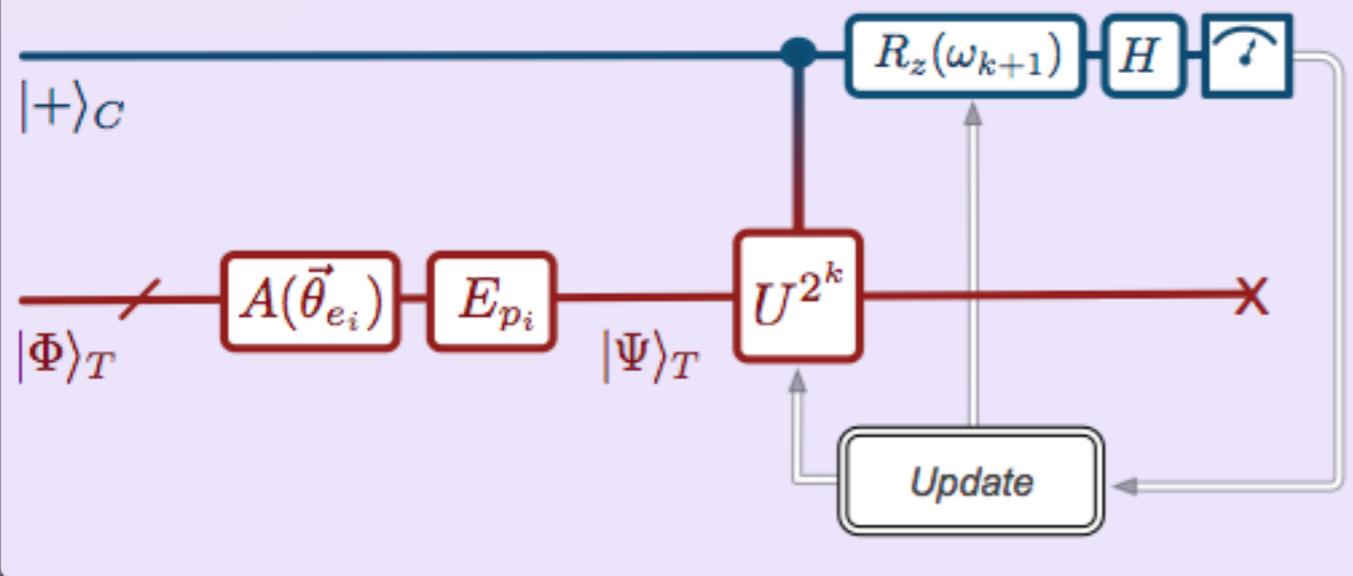
Ground-state search:

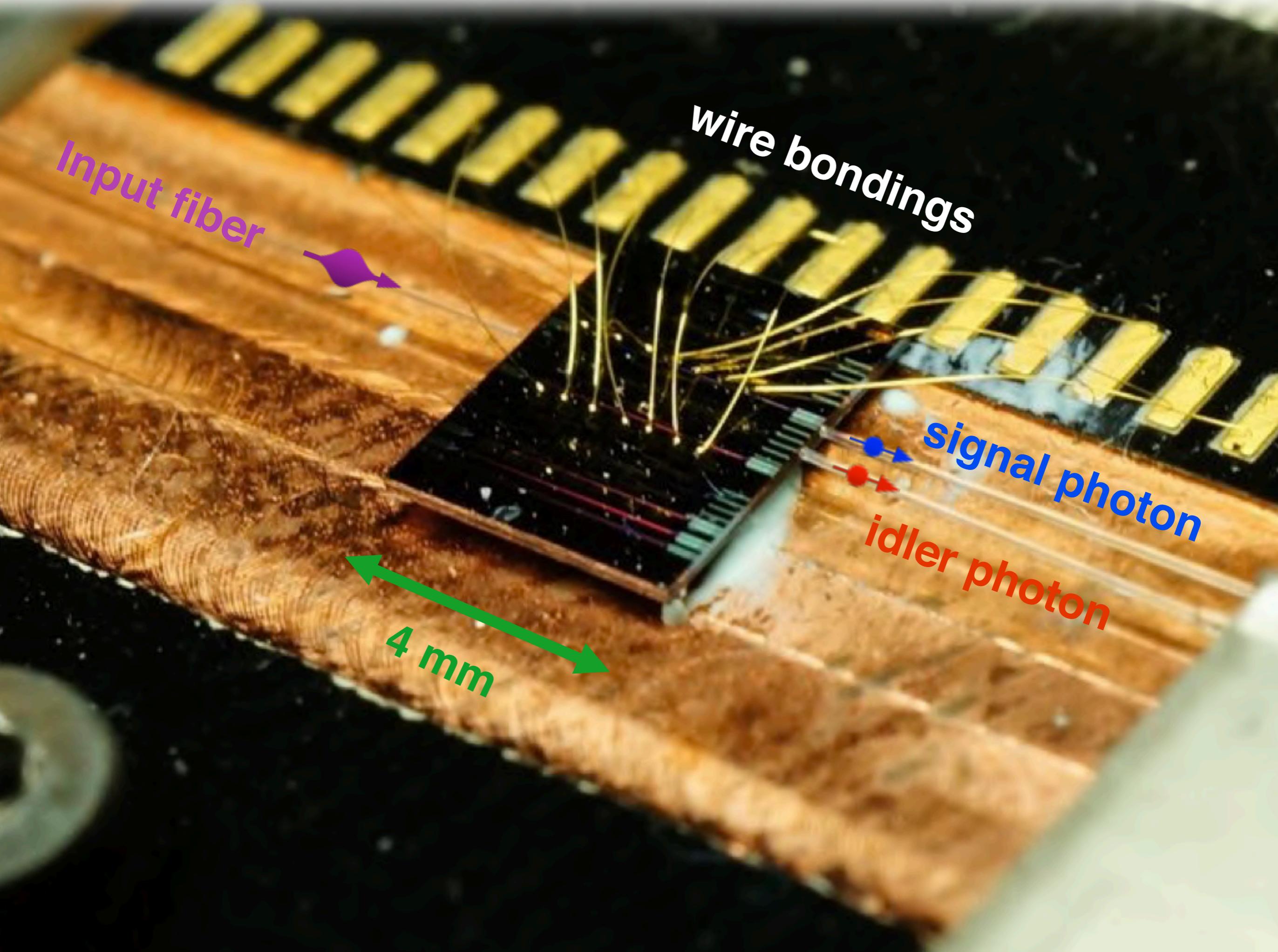


$$\mathcal{F}_{obj} = \mathcal{E} + T\mathcal{S}$$

$$T \rightarrow \infty$$

Targeting excited states





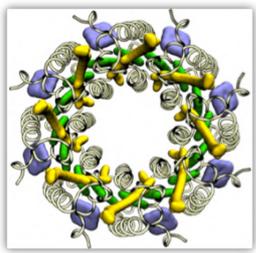
Input fiber

wire bondings

signal photon

idler photon

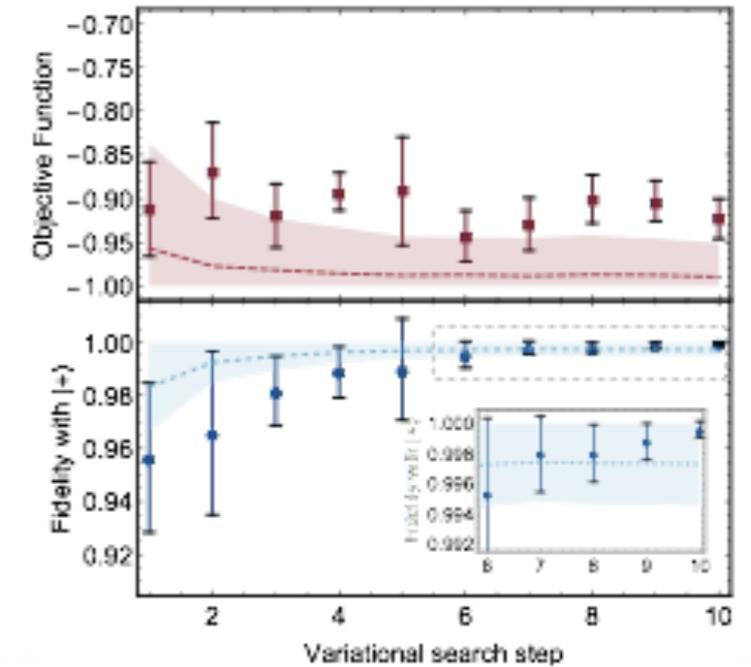
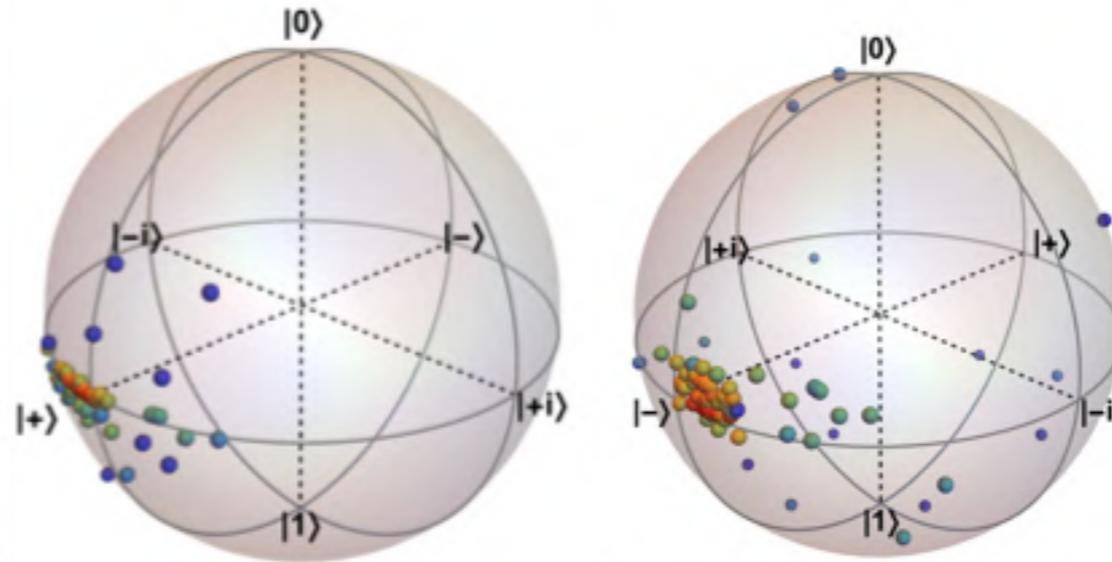
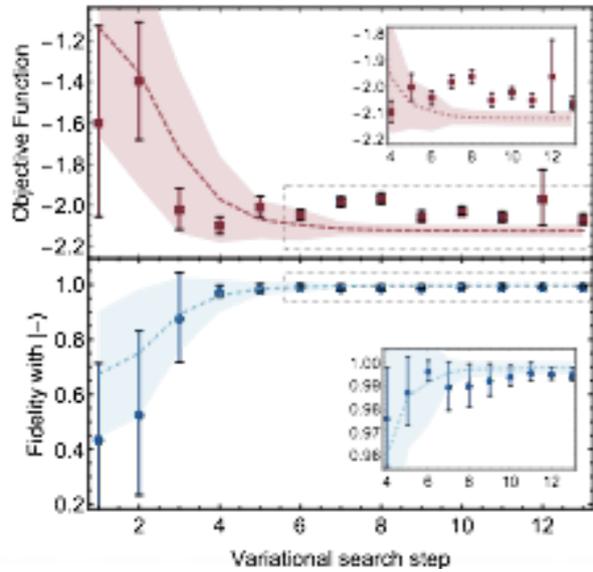
4 mm



Experiment

Ground-state search
using \mathcal{F}_{obj} and $\hat{E}_{p_0} = \hat{I}$

Excited-states search
using \mathcal{S} and guessed \hat{E}_{p_i}



F=99.5%

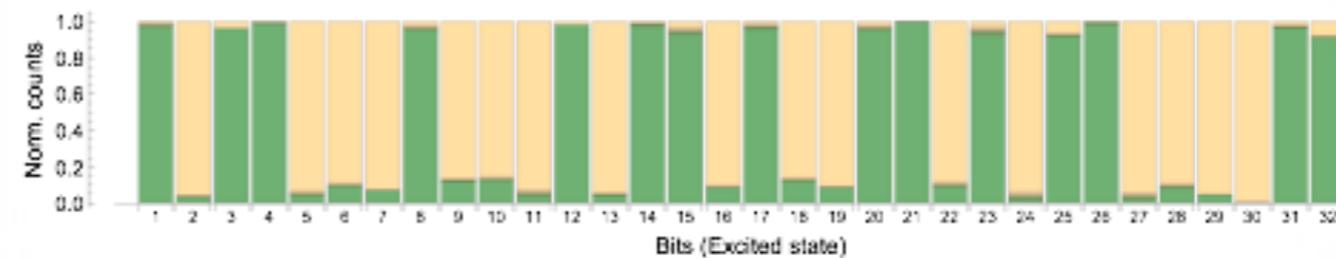
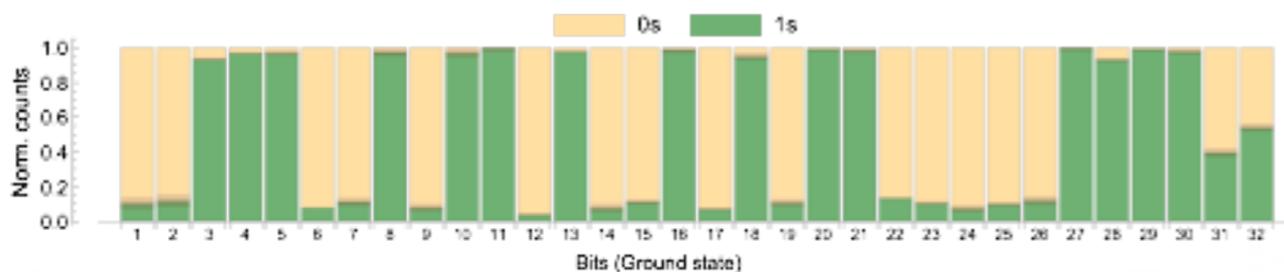
$$\mathcal{F}'_{obj}(\mathcal{S}, \mathcal{E}) = \mathcal{E} + T\mathcal{S}(\rho)$$

Santagati et. al (2016) in preparation

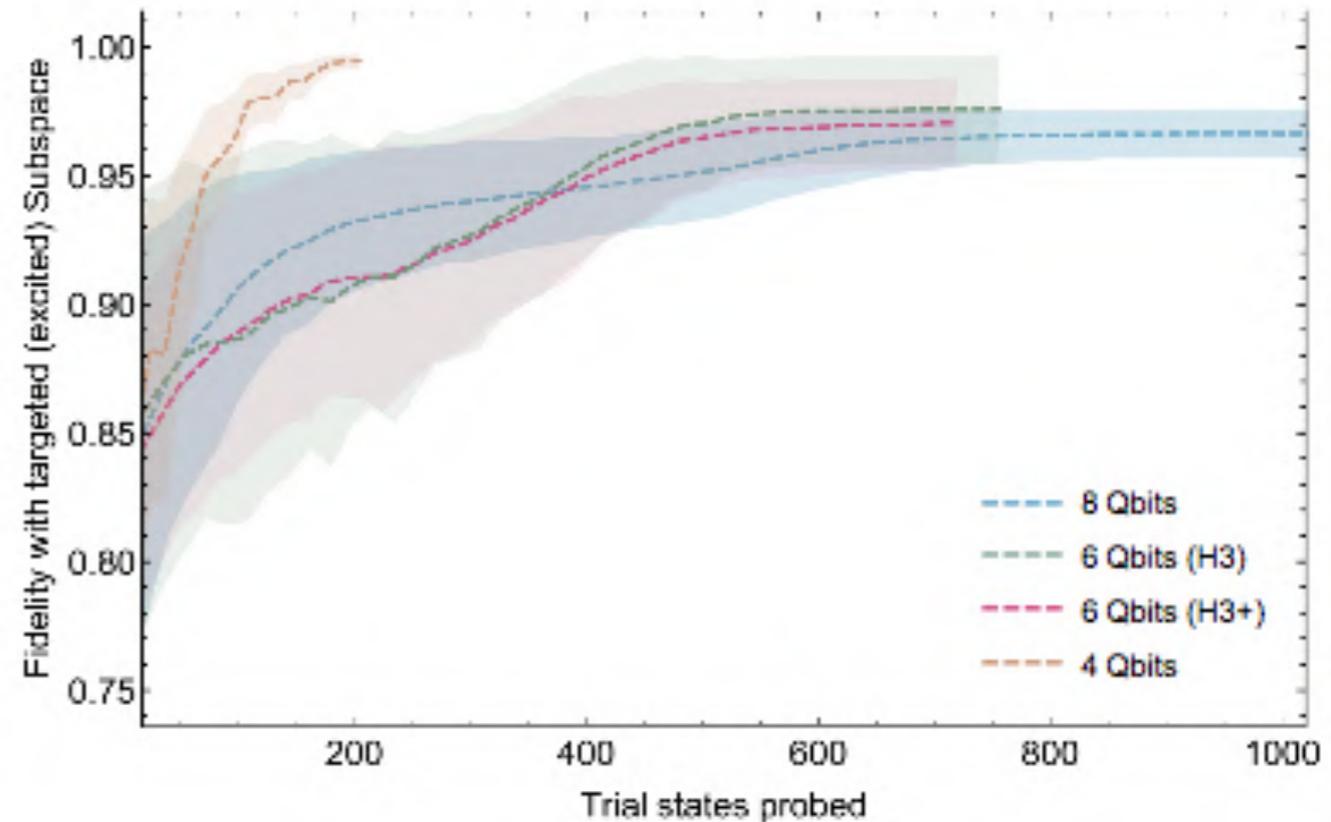
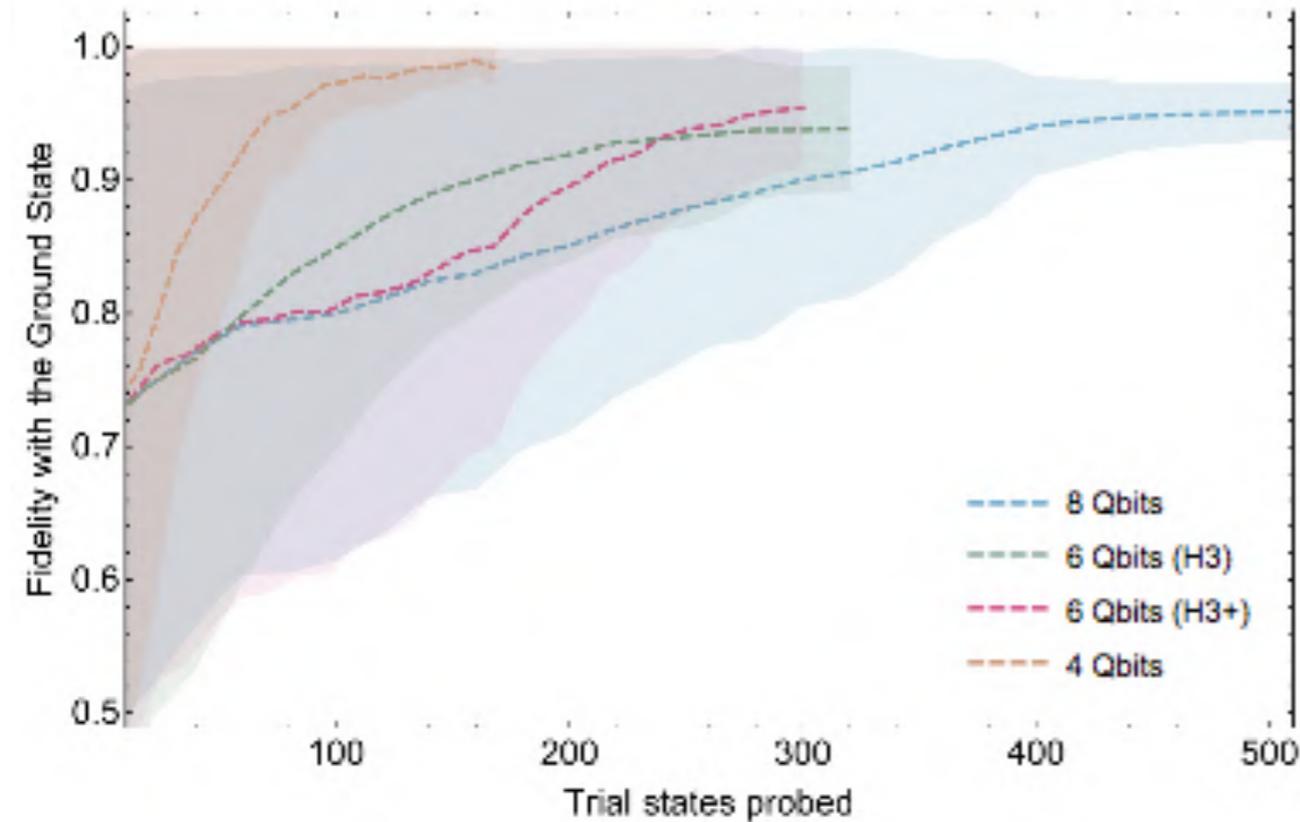
F=99.9%

IPEA

32 bits



WAVES: Simulations



Simulating H2, H3, H3+ and H4
with
4,6 and 8 qubits

Restrictions on the ansatz state
preparation seems to limit the initial
fidelity

Ansatz: UUCC

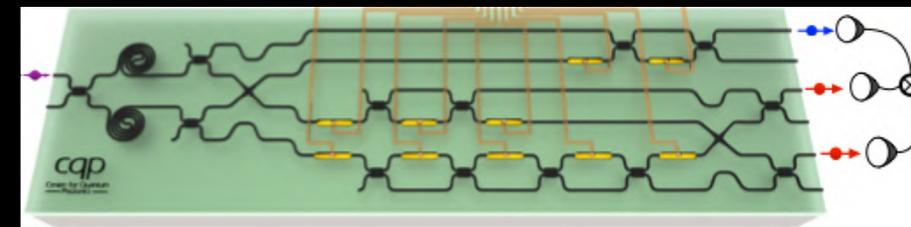
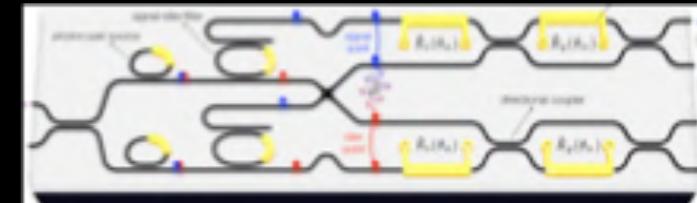
$$U(\vec{t}) = \exp \left[\sum_{ij} t_{ij} (a_i^\dagger a_j - a_j^\dagger a_i) + \sum_{ijkl} t_{ijkl} (a_i^\dagger a_j a_k^\dagger a_l - a_l^\dagger a_k a_j^\dagger a_i) \right]$$

Excitation operators

$$E_{ij} = \exp \left[\frac{\pi}{2} (a_i^\dagger a_j - a_j^\dagger a_i) \right]$$

CONCLUSION

- VQE
- Witness Assisted Variational Eigensolver
- Implementation in silicon quantum photonics



CAP



Barreto

O'Brien

Wilkes

Sinclair

Piekarek

Tyler

Kennard

Sibson

Villareal

Bonneau

Santagati

Silverstone

Thompson

Wang

Gentile

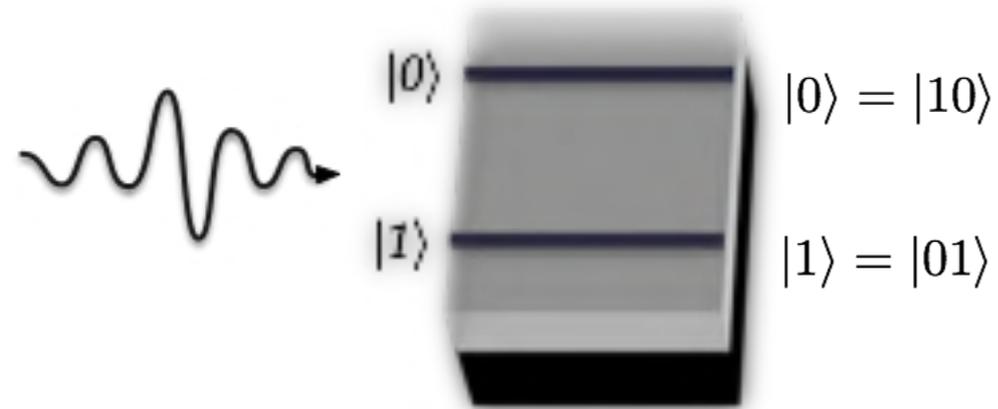
Erven

Adcock

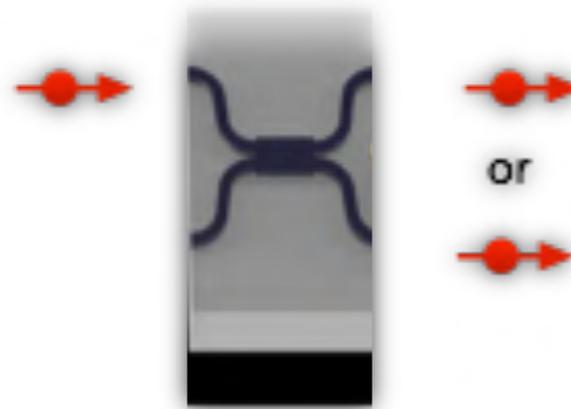
Paesani

Q.I. with integrated photonics

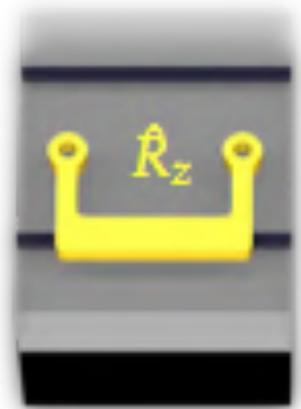
path-encoded qubit



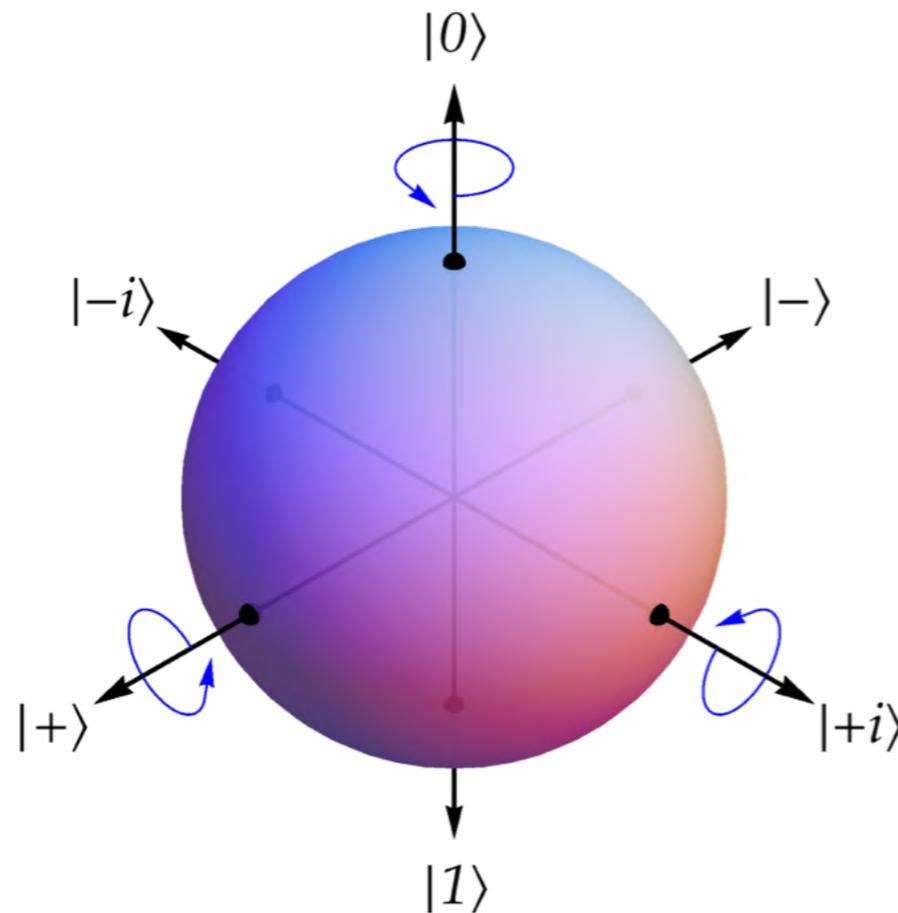
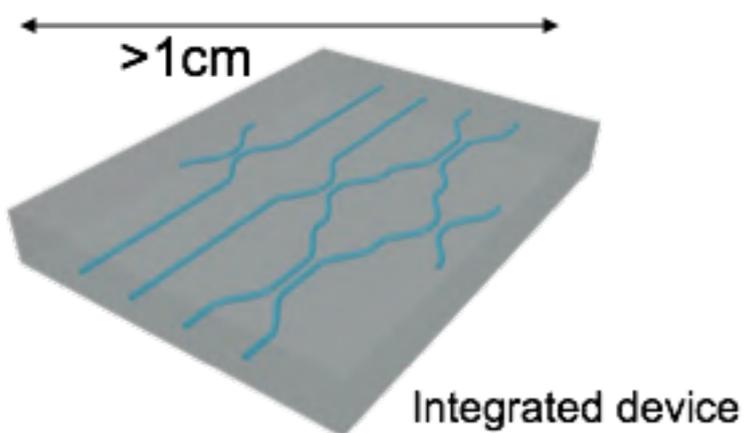
Directional Coupler



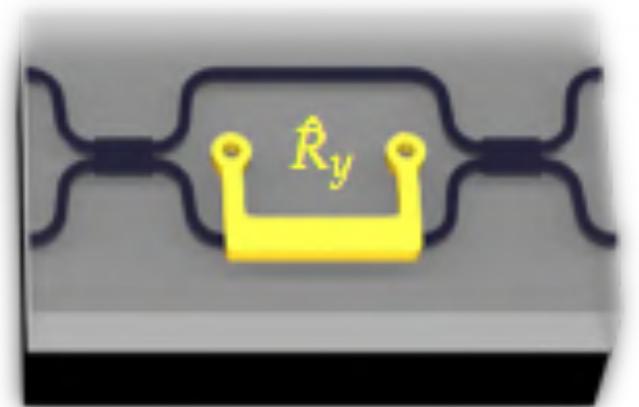
Thermal phase shifter



- Path encode qubits
- very long coherence time
- single and 2 qubits gate

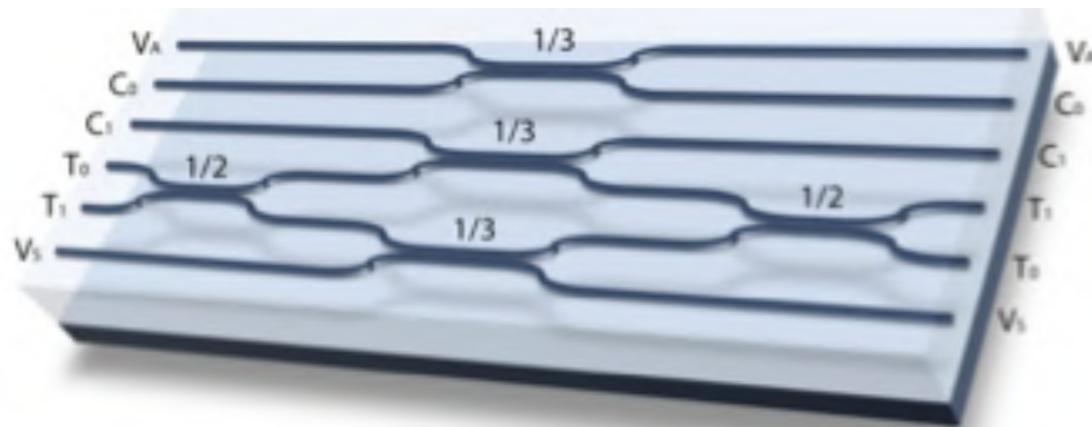


Interferometers



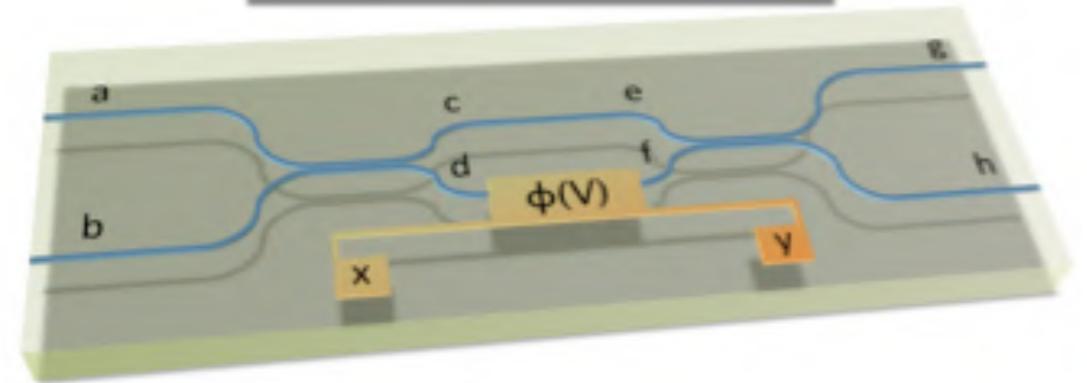
Silica on silicon integrated quantum photonic circuits

CNOT Gate



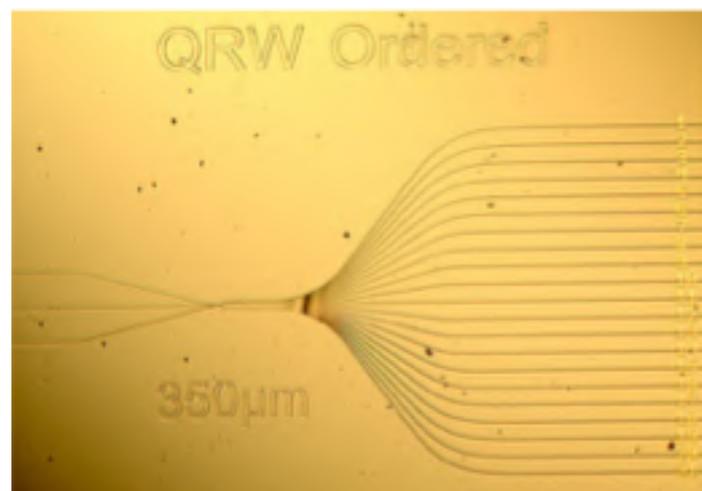
Politi, Cryan, Rarity, Yu, and O'Brien
Science 320, 5876 (2008)

Single qubit Gate



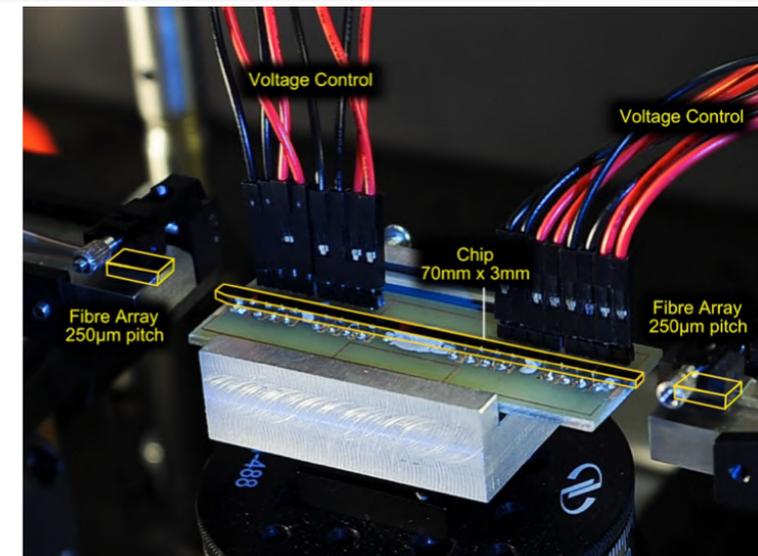
Politi, Cryan, Rarity, Yu, and O'Brien
Science 320, 5876 (2008)

Quantum Walks



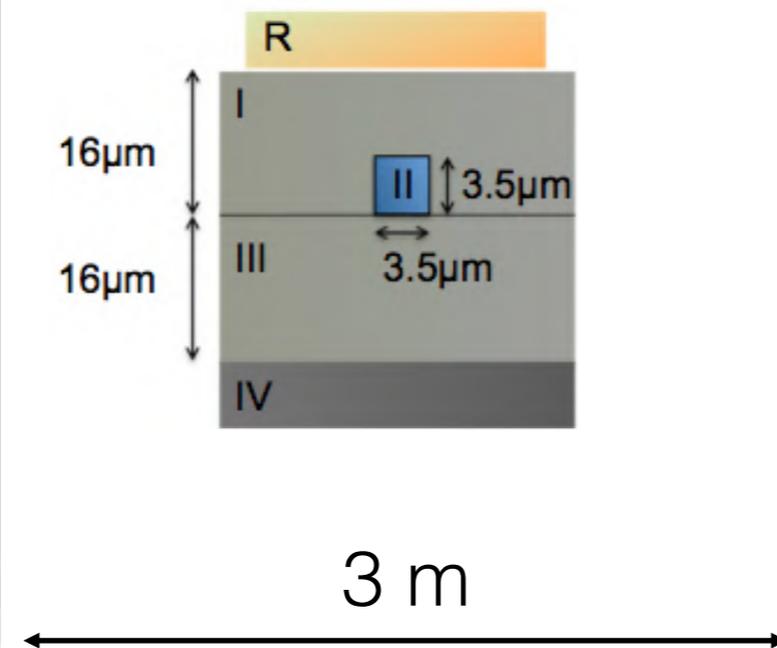
Peruzzo, Lobino, Matthews, Matsuda, *et al.*
Science 329 1500 (2010)

Generate arbitrary states

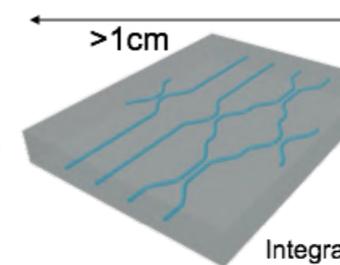
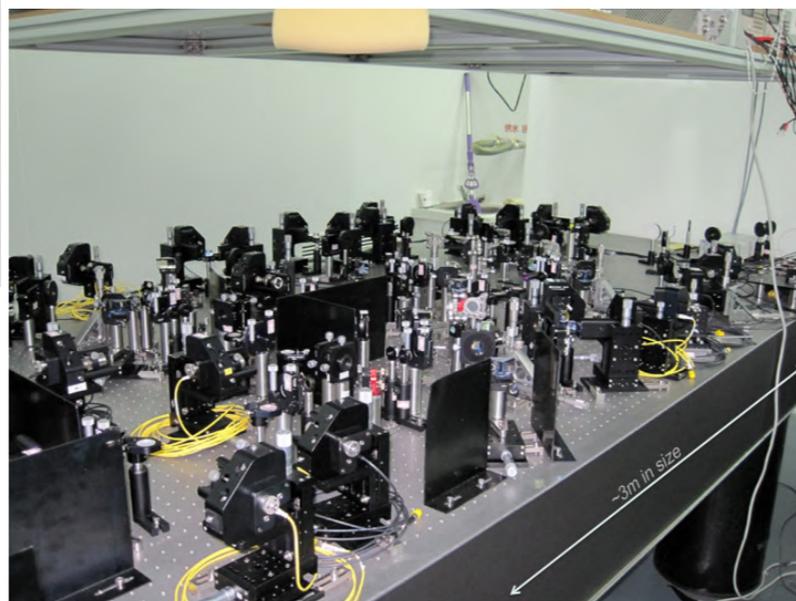
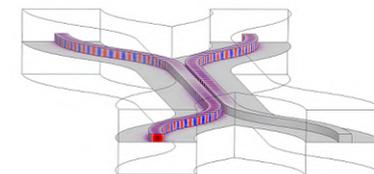
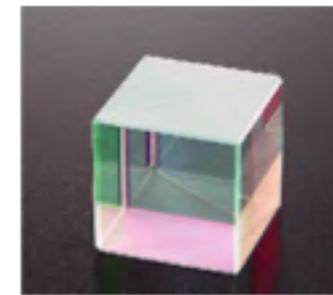


Shadbolt *et al.*
Nature Photonics (2011)

Silica on silicon integrated quantum photonic circuits



Beam Splitter



Integrated device



10^4 components...

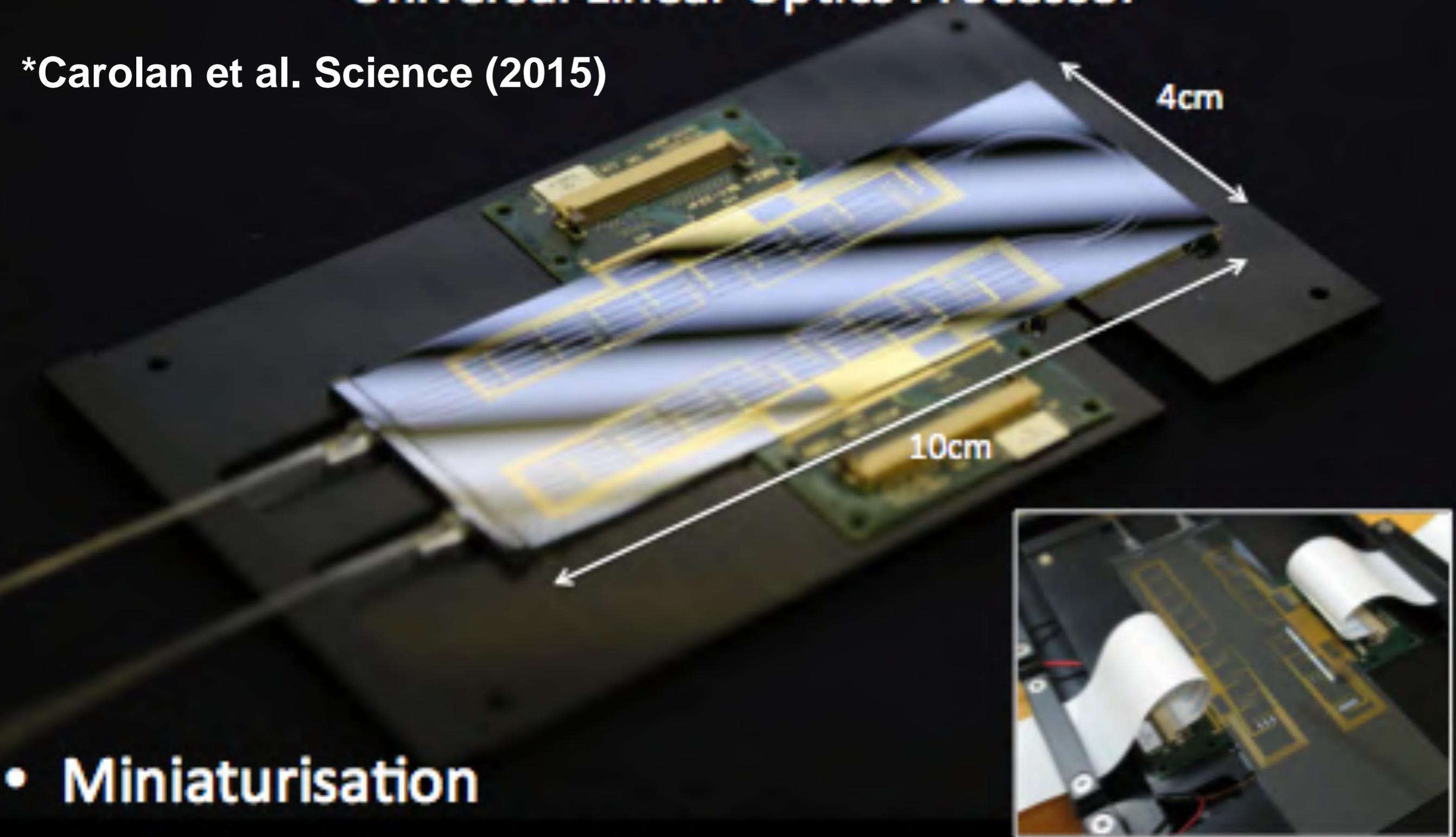


- Silica waveguide on a silicon wafer
- Photo-lithographically processed with sub μm accuracy
- Low loss coupling to fibre-optics

- Size / Compactness
 - Stability
 - Complexity
- Route to scalability?

Universal Linear Optics Processor

*Carolan et al. Science (2015)

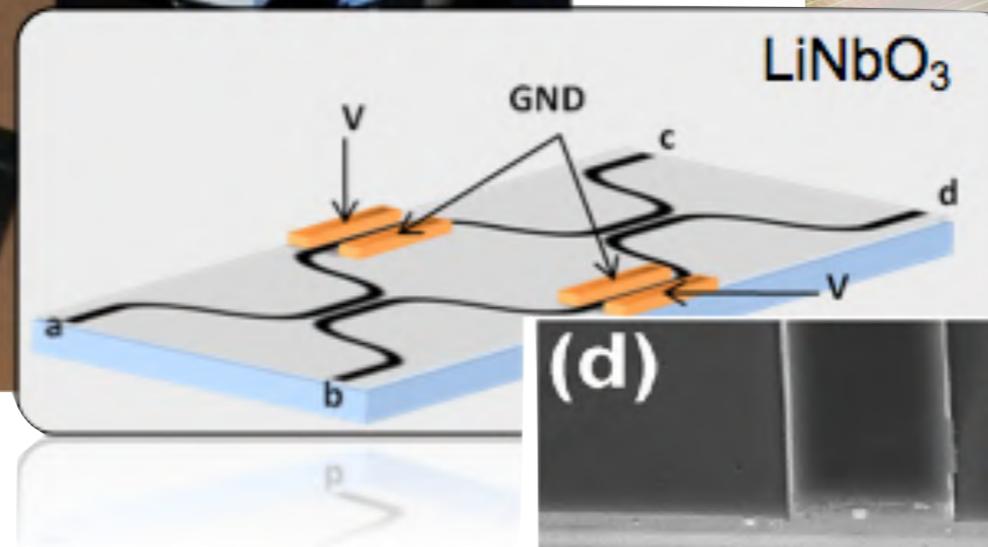
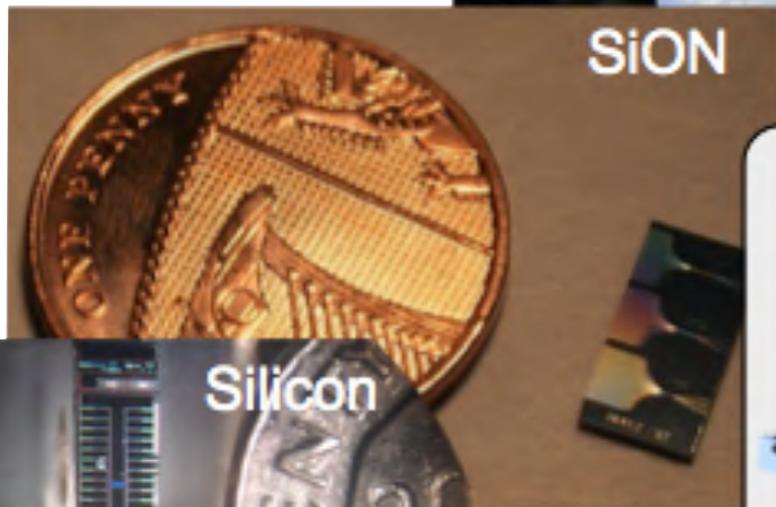
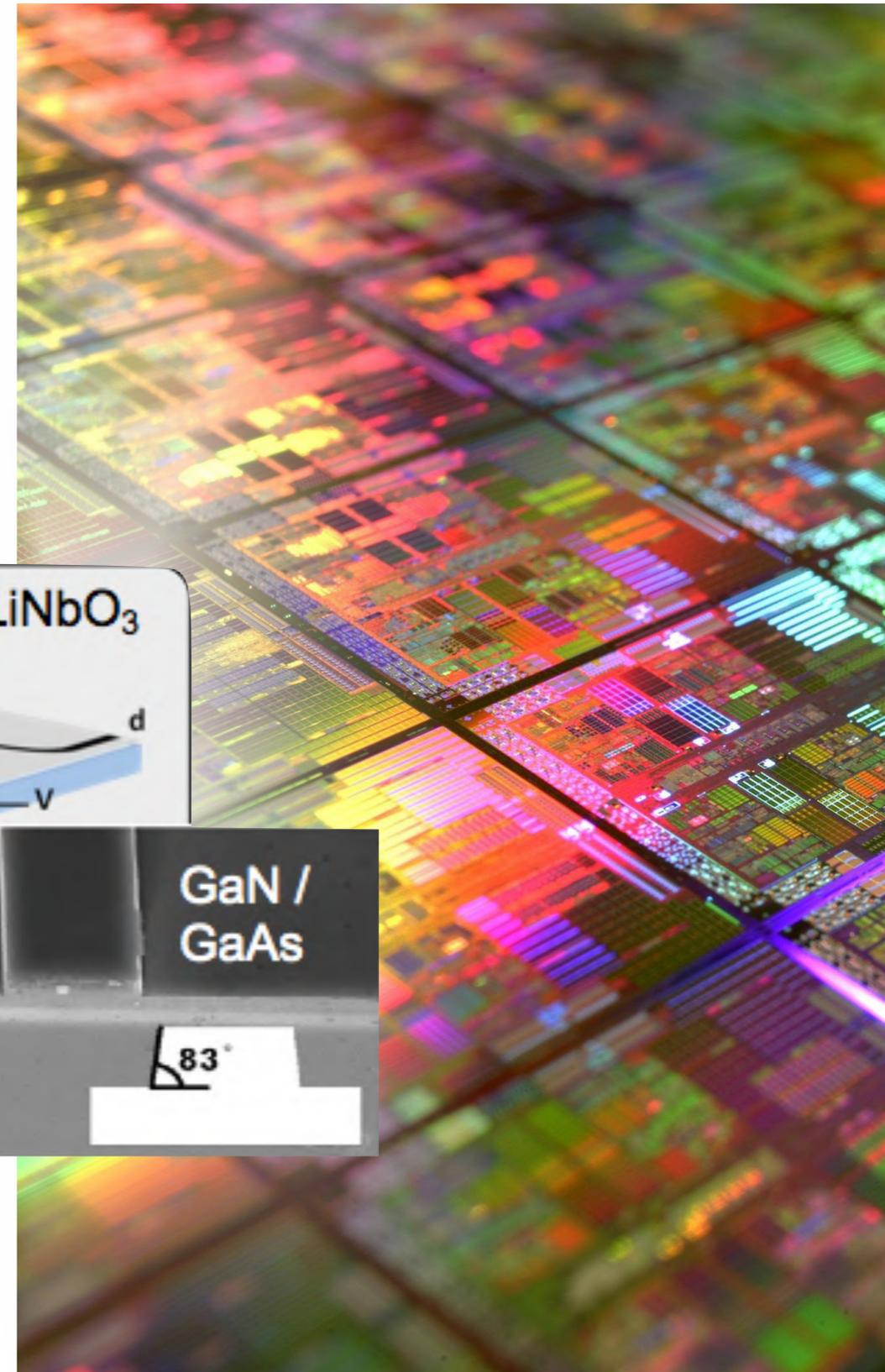
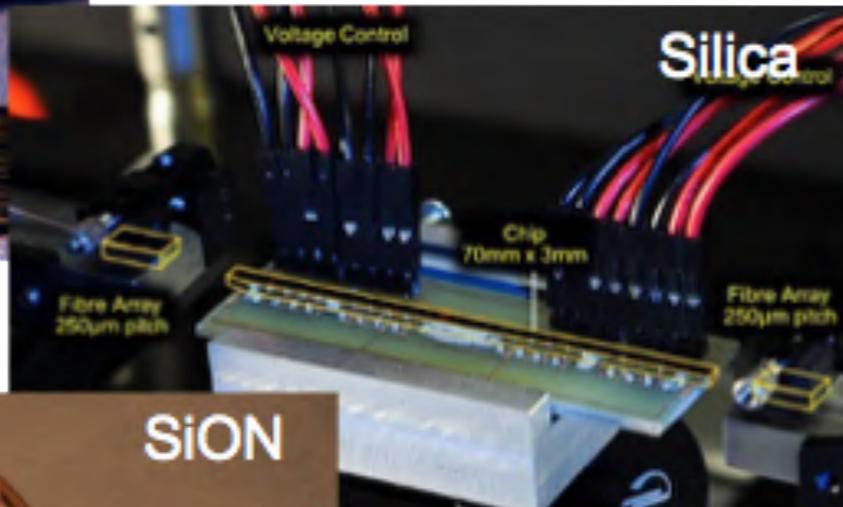


- **Miniaturisation**
- **Additional functionality** (sources, detectors)
- **Full integration**

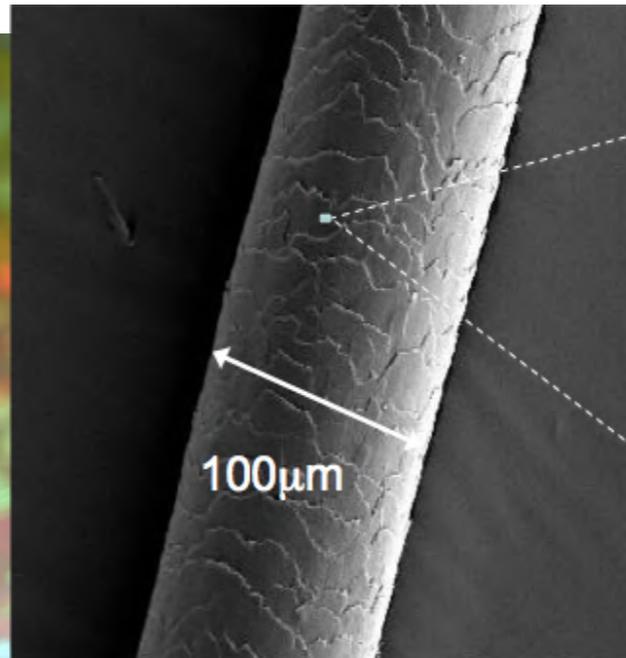
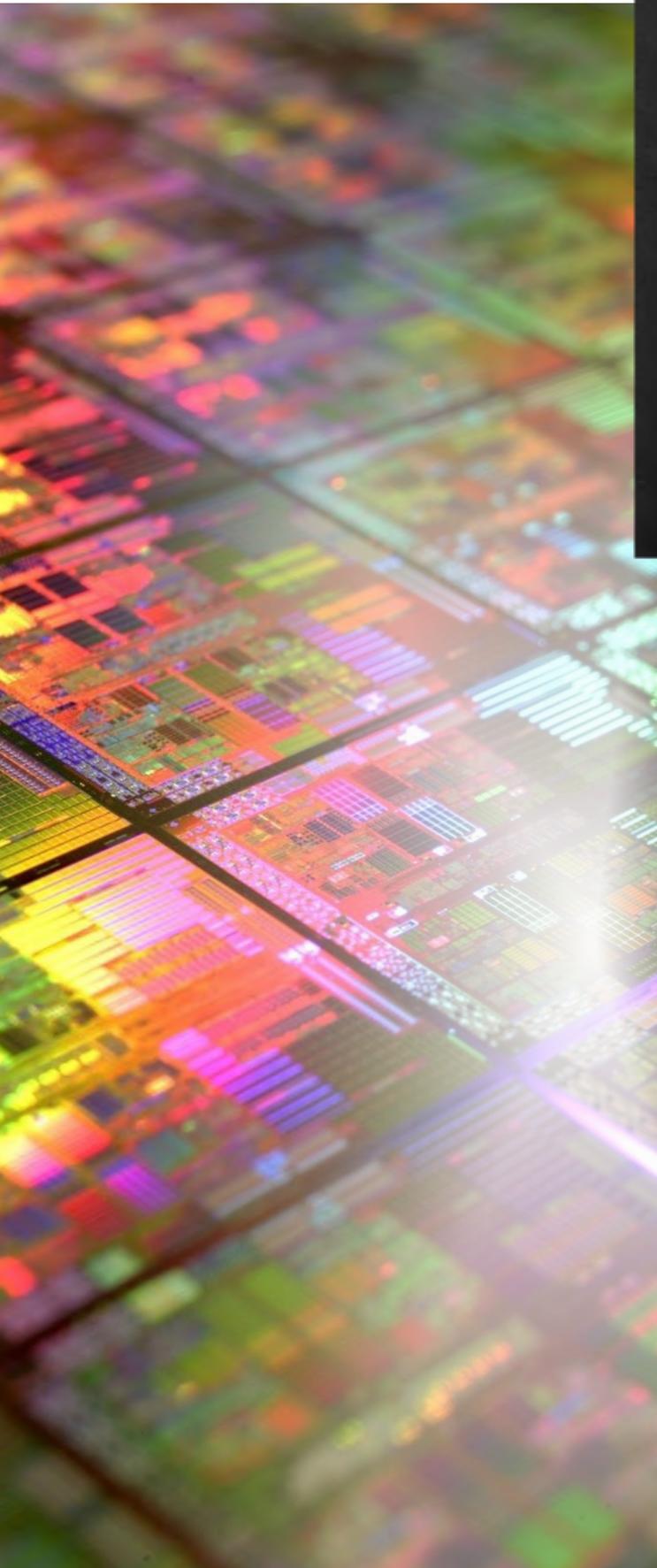


University of
BRISTOL

Materials choices

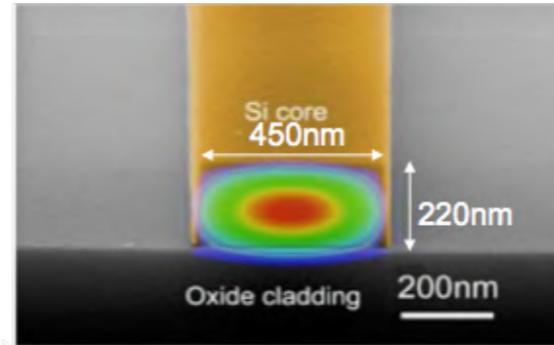


Silicon Quantum photonics



Si waveguide is 200 times smaller

Silicon waveguide

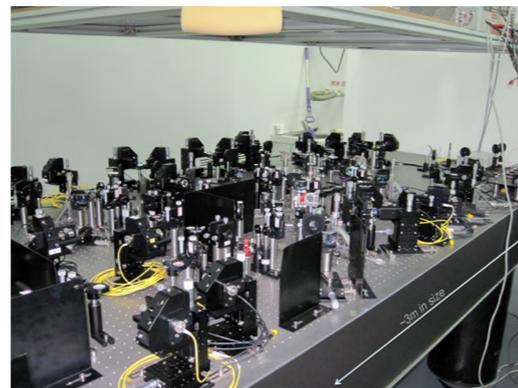


Ultra-high confinement of light

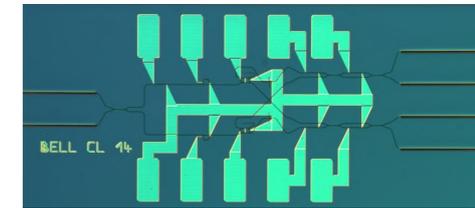
silicon

- Ultra-compact waveguides
 - Small bend radius ($<1\mu\text{m}$)
 - High component density
- Mature semiconductor fabrication processing
- High non-linearity efficient sources
- High confinement efficient detectors
- Filters have been demonstrated
- Integration with electronics

3 m



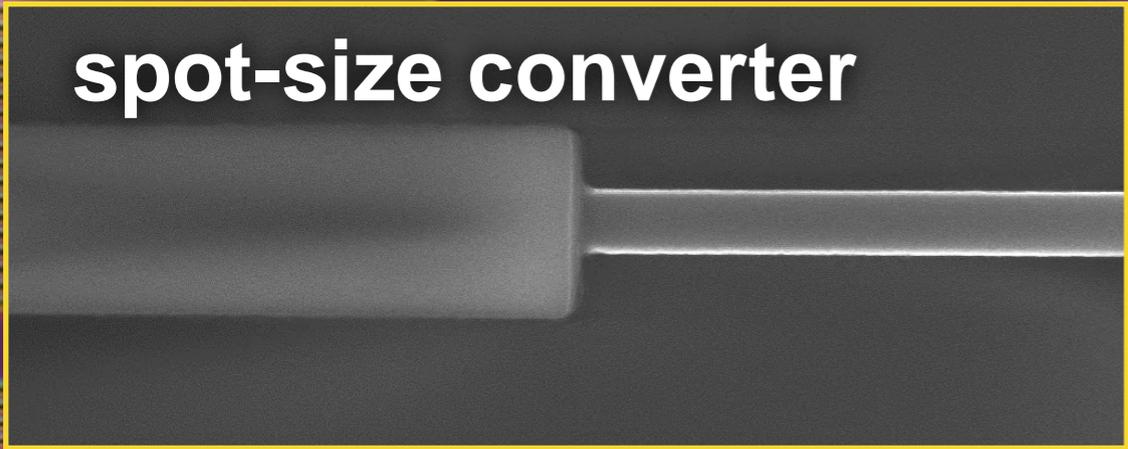
> 1 cm



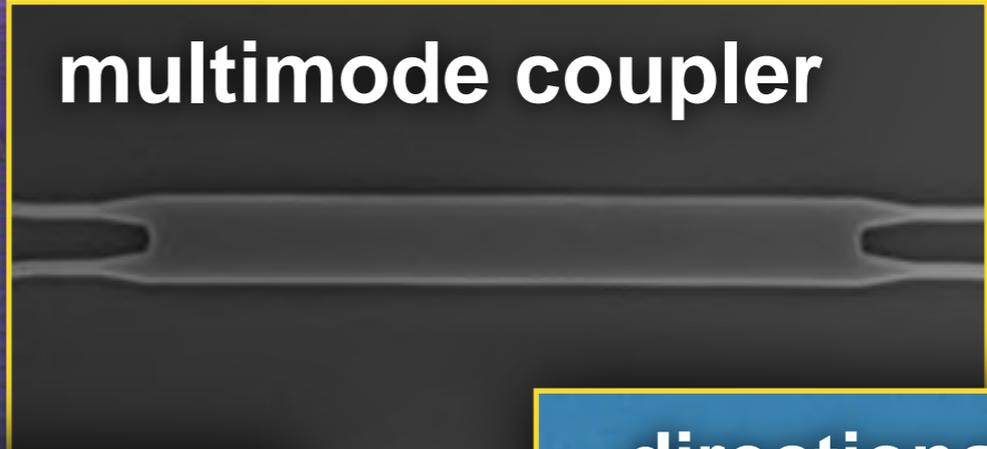
1.4 mm

Silicon photonics components

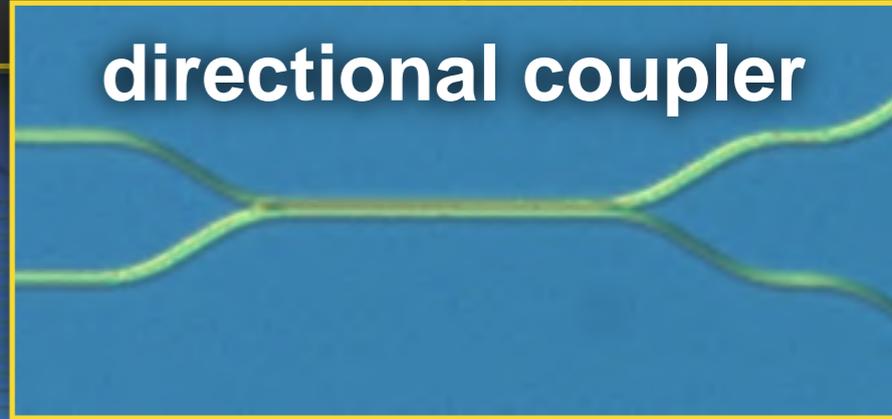
spot-size converter



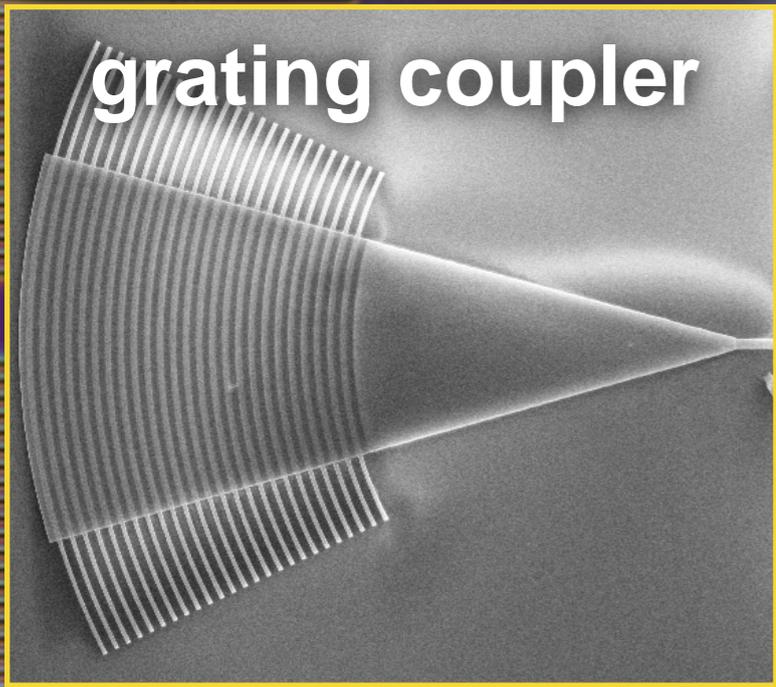
multimode coupler



directional coupler



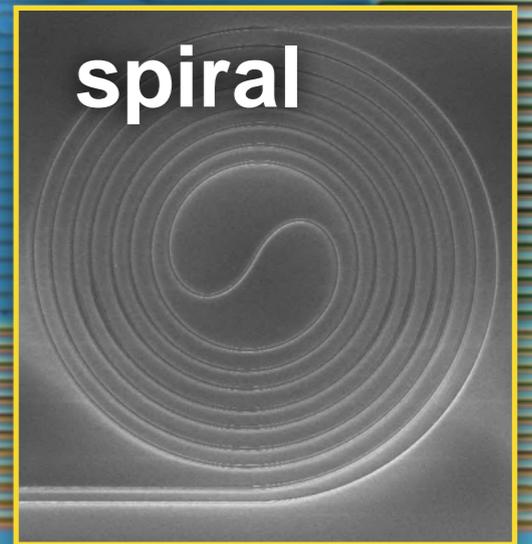
grating coupler



phase shifter



spiral



resonator

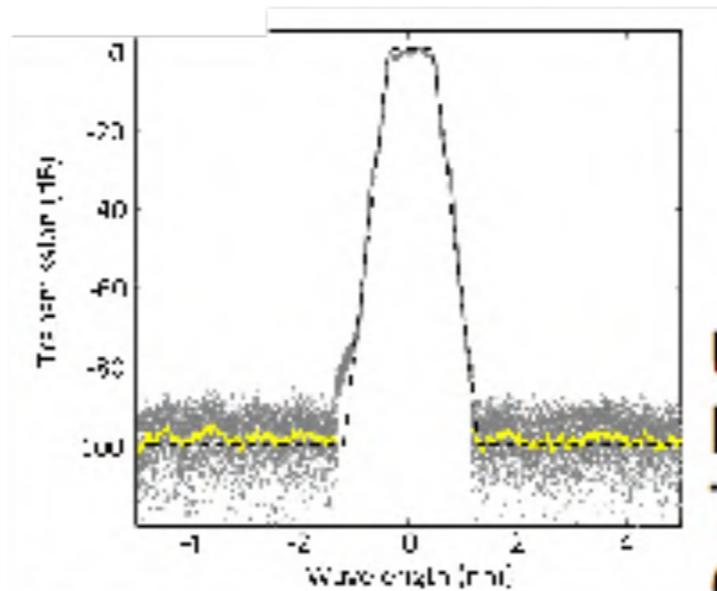
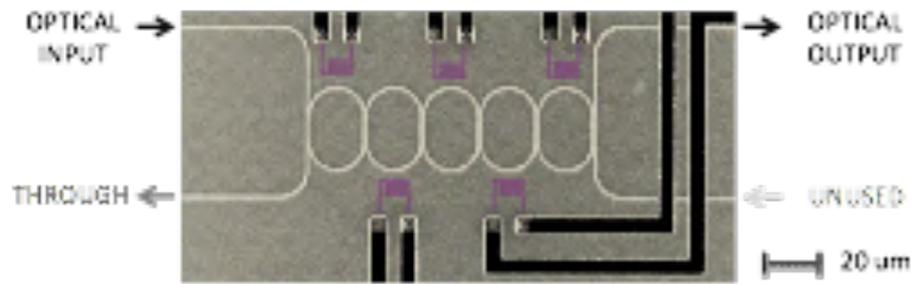


filter



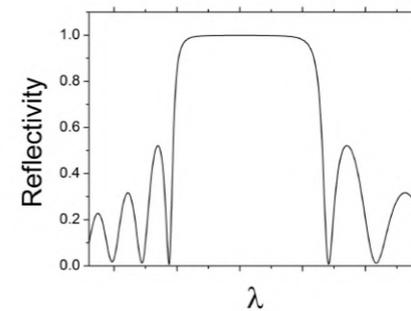
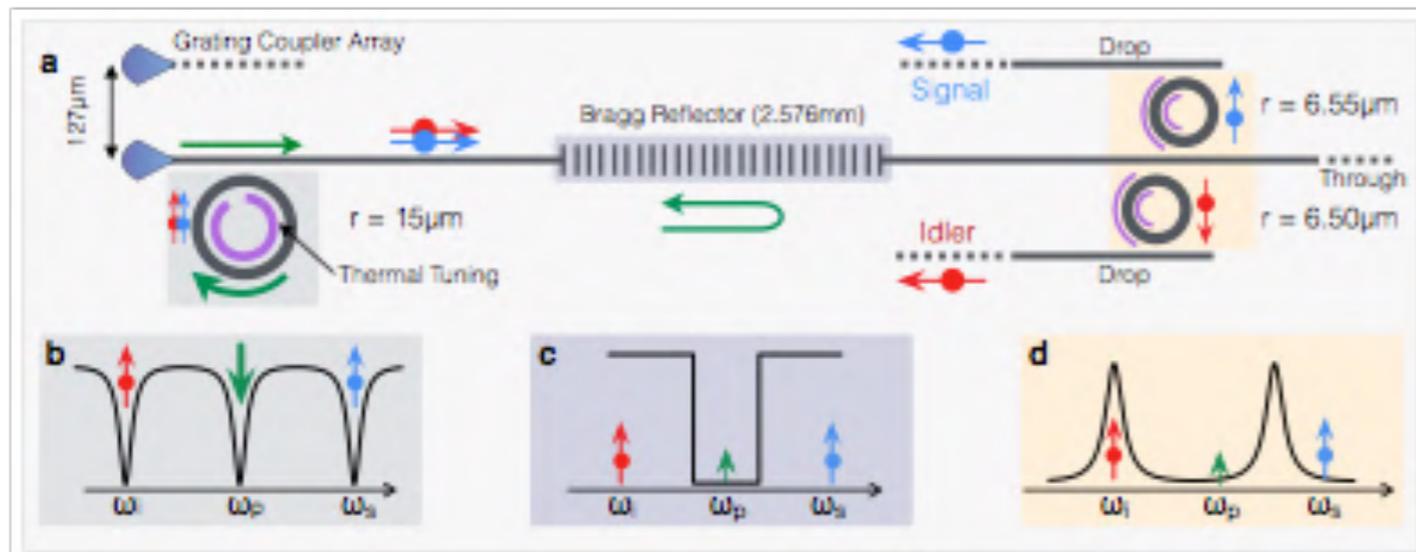
Integrated filters in SOI

>100 dB extinction ratio



UCSD: Ong, Kumar, Mookherjea, IEEE Photon. Technol. Lett. **25**, 1543 (2013).

Integration with photon source

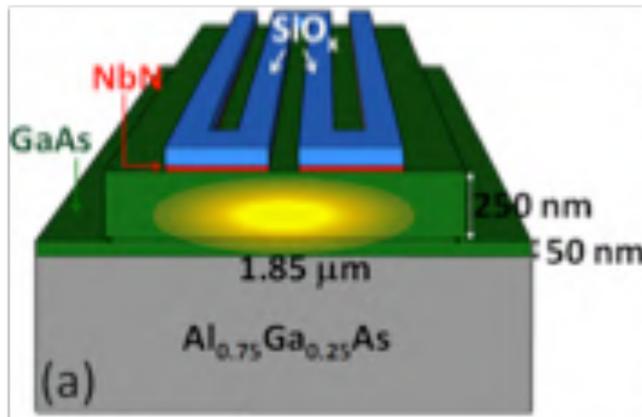


95 dB
CAR 50Hz

MIT: Harris, D. Grassani, Simbula, Pant, Galli, Baehr-Jones, Hochberg, Englund, Bajoni, Galland, (2014).

Detectors

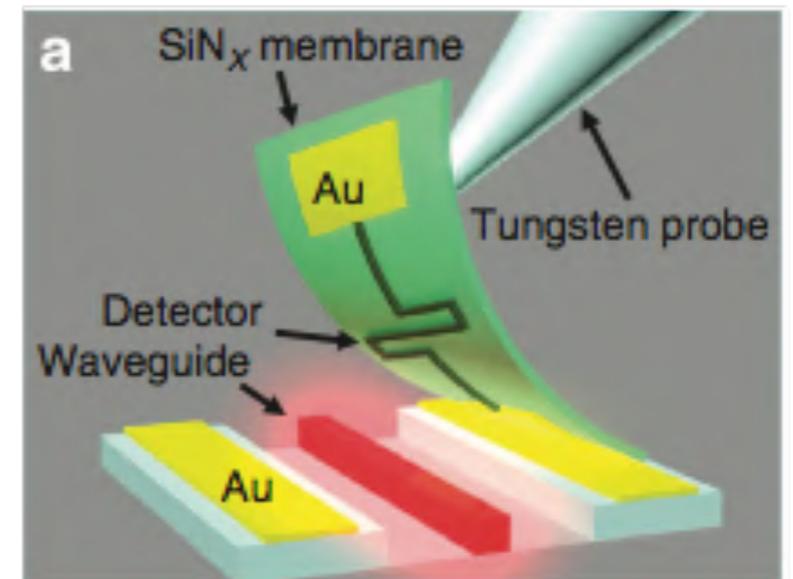
GaAs waveguide superconducting detector



$\eta=20\%$

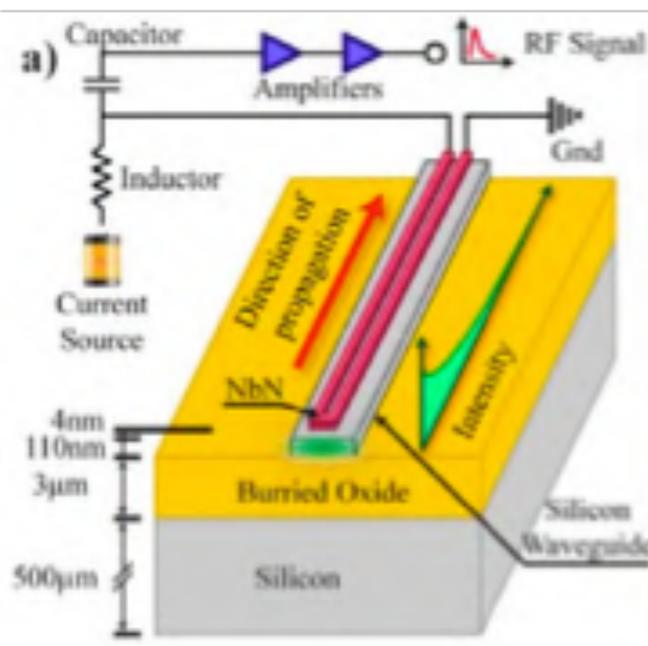
Sprengers, Gaggero, Sahin, Jahanmirinejad, Frucci, Mattioli, Leoni, Beetz, Lermer, Kamp, Höfling, Sanjines, Fiore, *Appl. Phys. Lett.* **99**, 181110 (2011)

Flip-chip integration



Najafi, Mower, Harris, Bellei, Dane, Lee, Hu, Kharel, Marsili, Assefa, Berggren, Englund, *Nat Comms* **6**, 5873 (2015).

Silicon waveguide superconducting detector



$\eta=91\%$

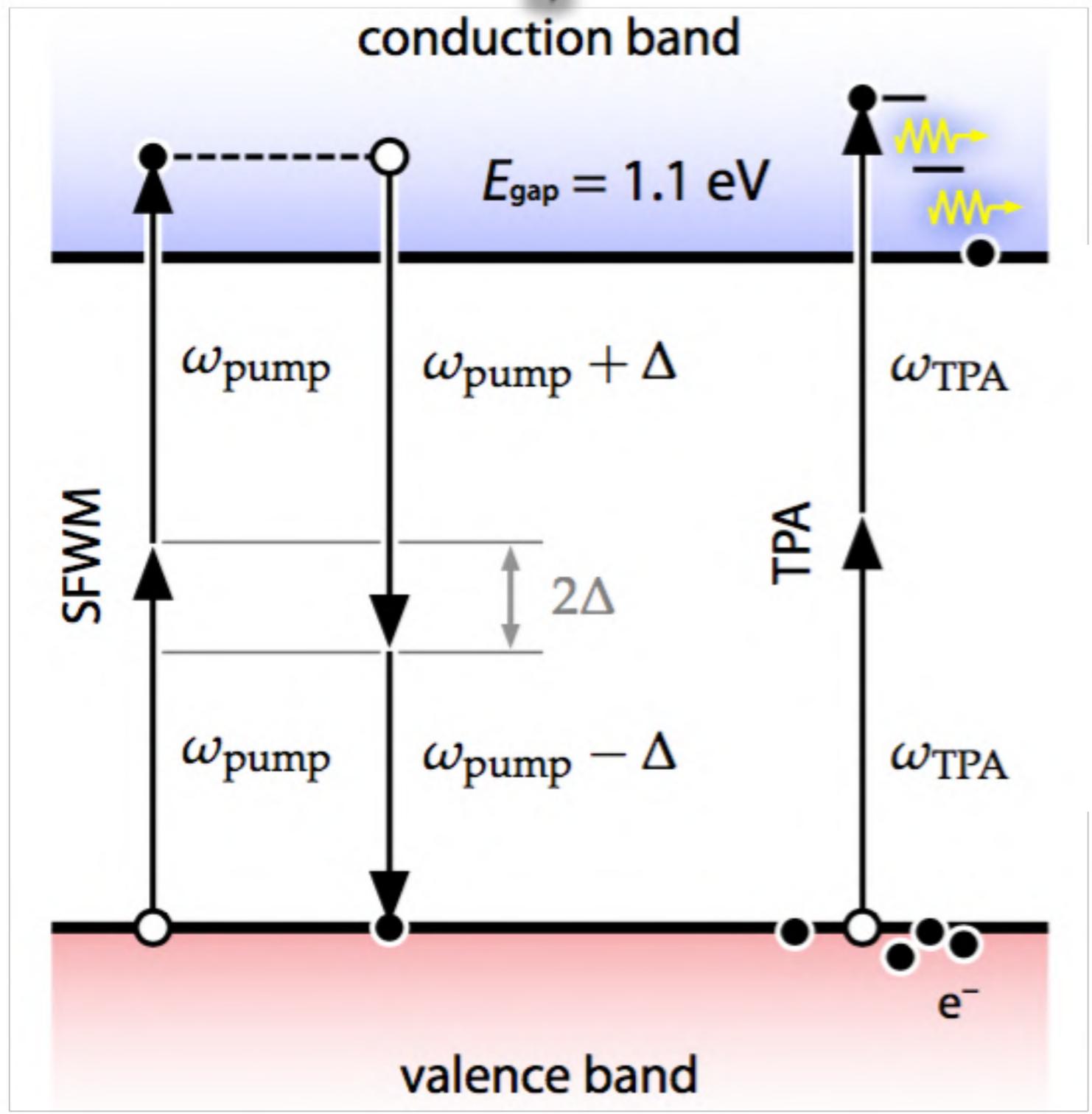
Pernice, Schuck, Minaeva, Gol'tsman, Sergienko, Tang, *Nat Comms* **3**, 1325 (2012)

S

Photon energy
e

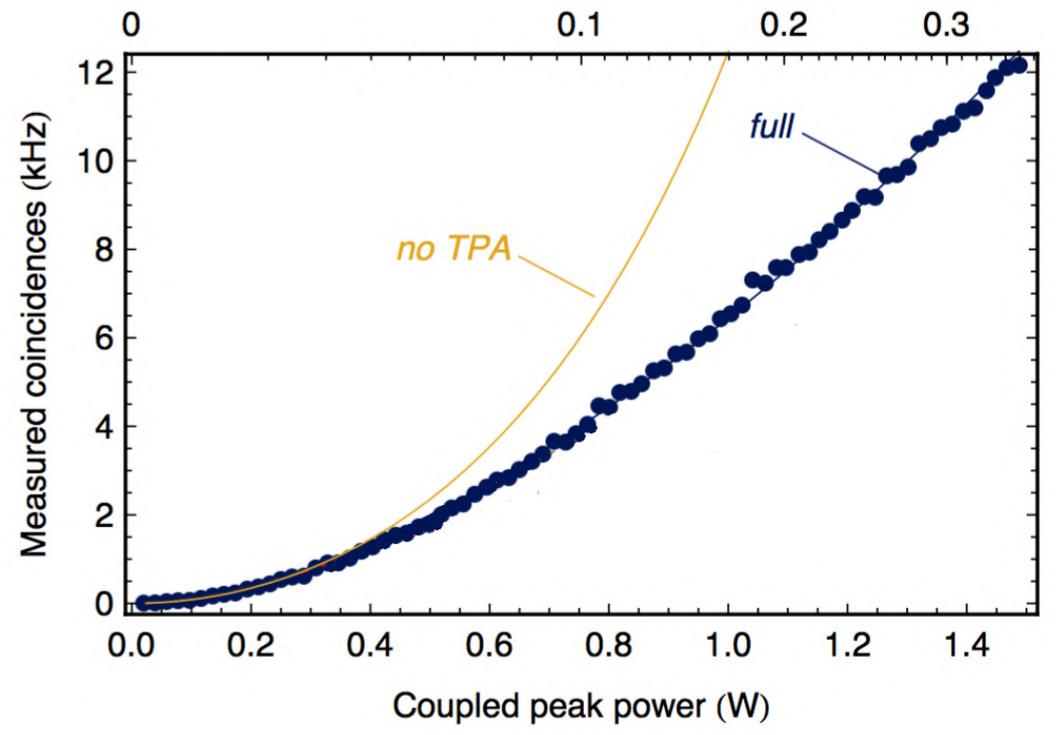
2Δ

Δ



FWM

WM
nearity



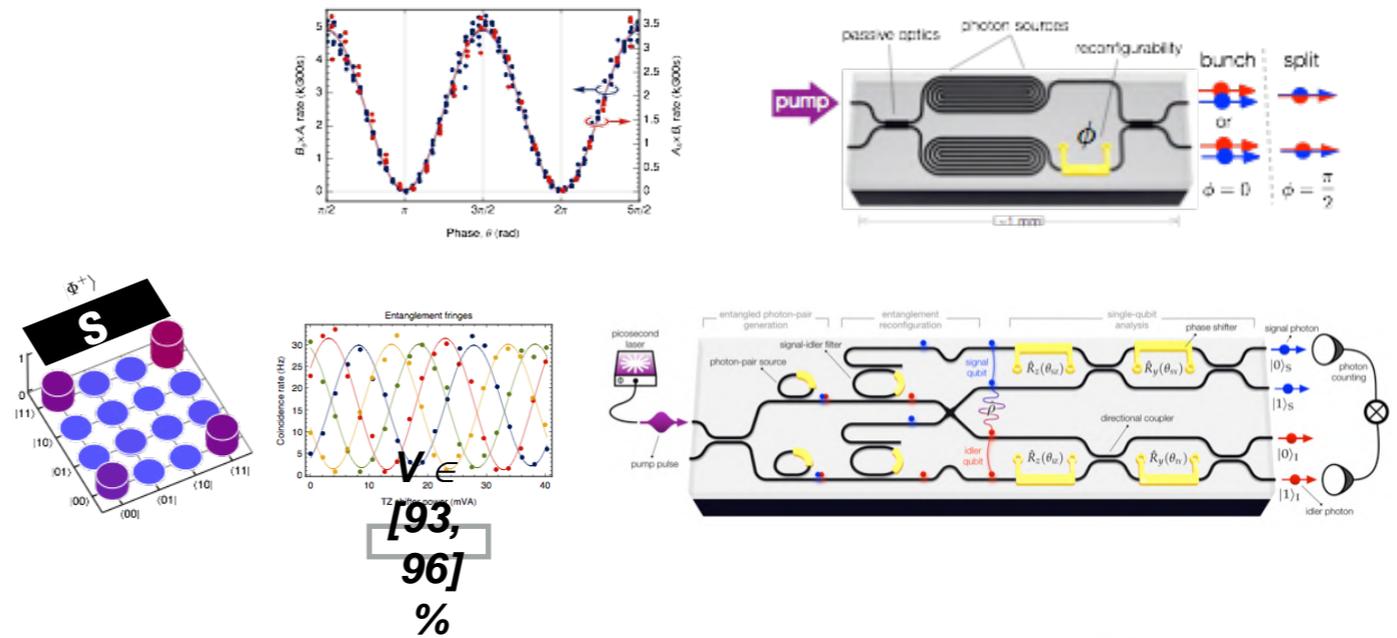
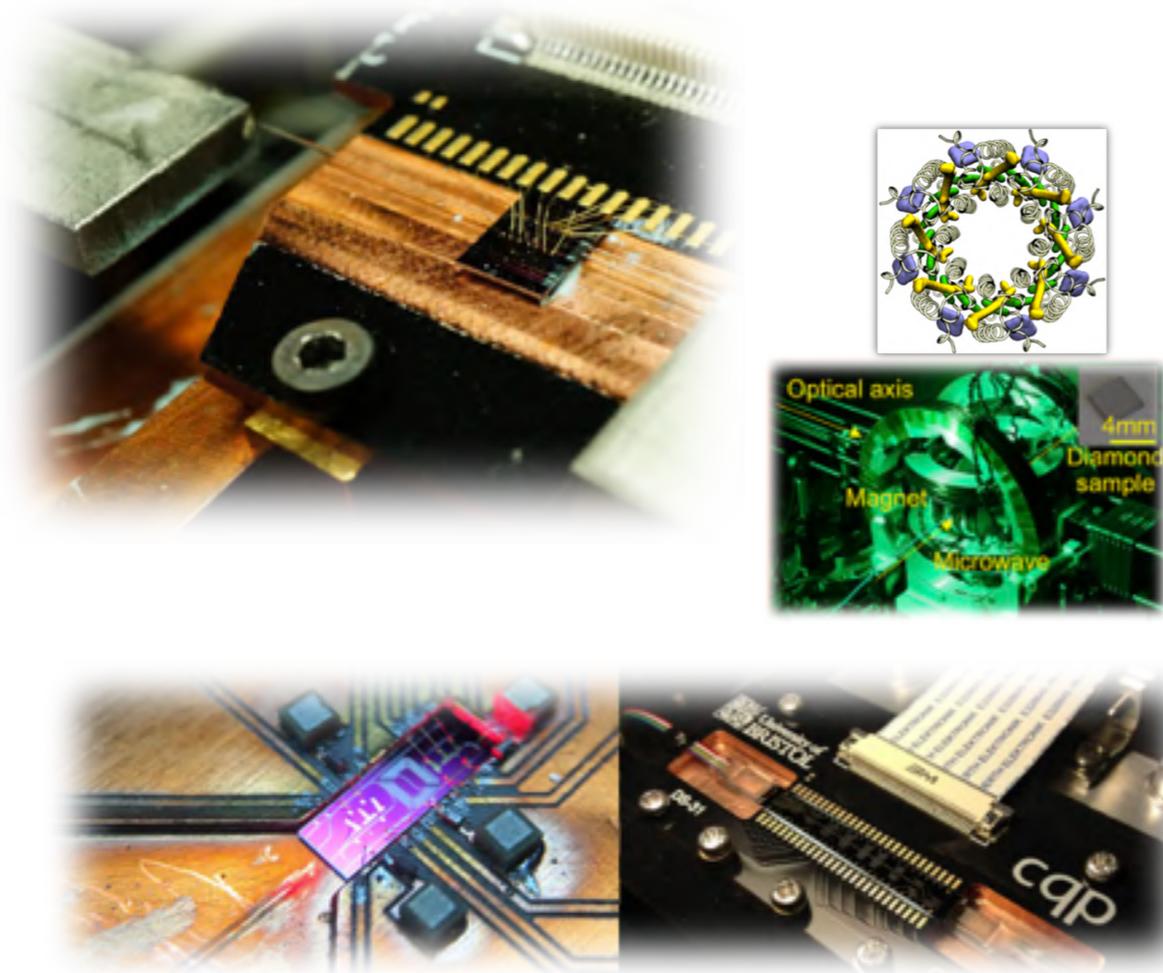
width



Priem, Dumon, Bogaerts et al.
2005

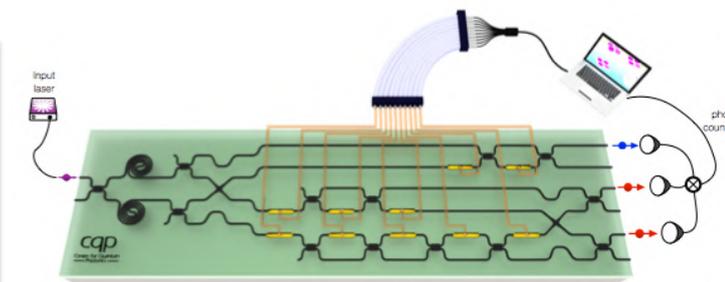
Silverstone et al. 2016

Quantum information processing and communication in Si QP

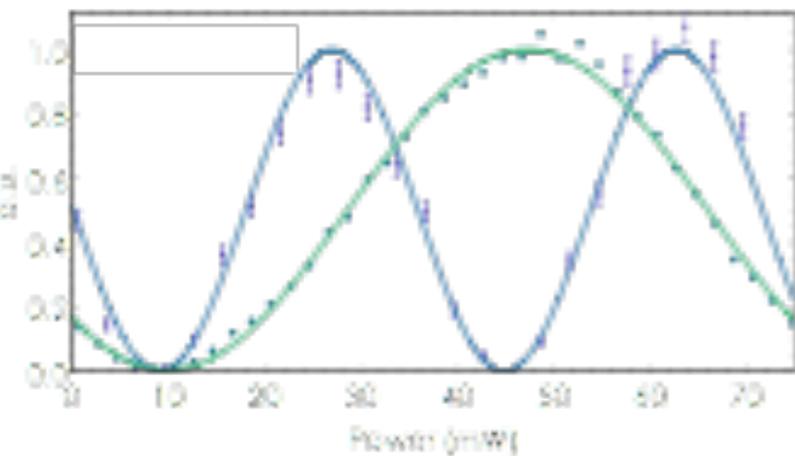
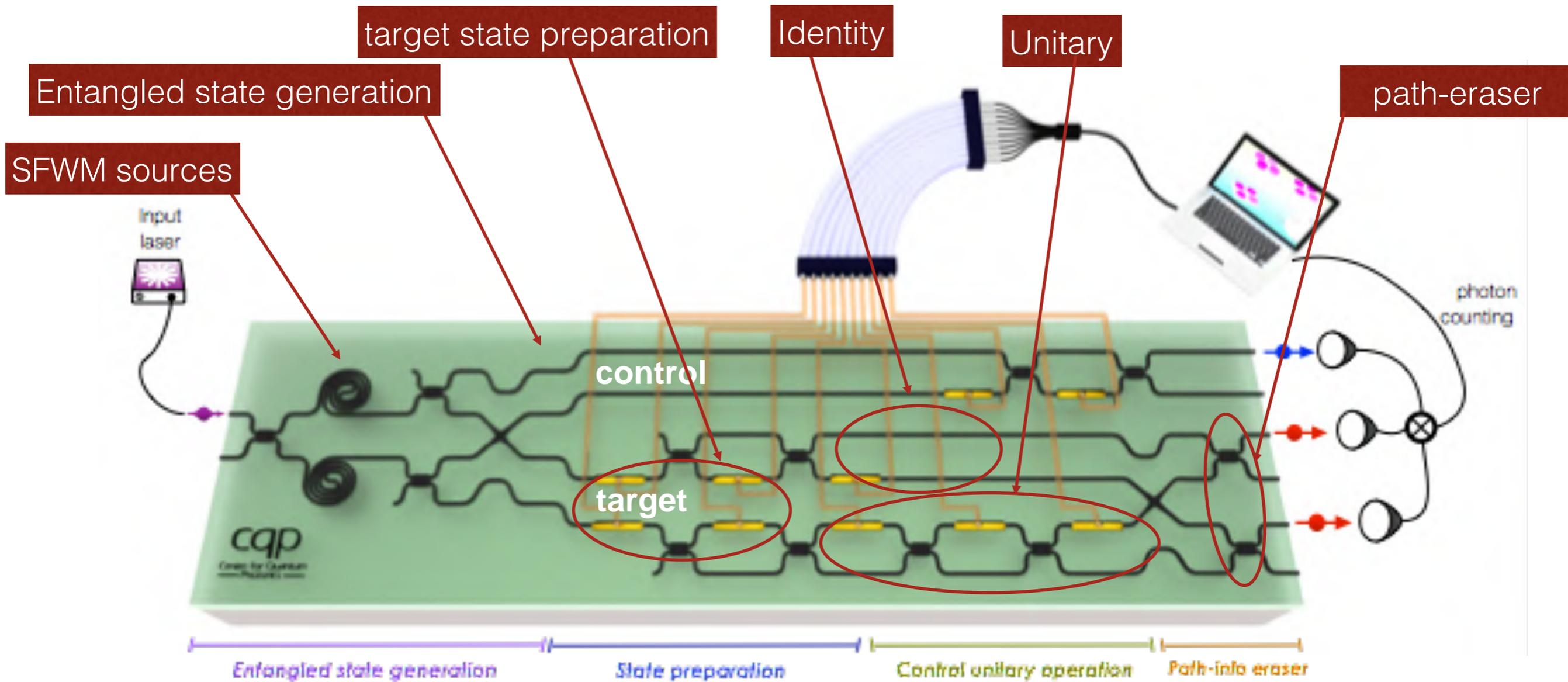


- Quantum simulation
- Bayesian Phase Estimation
- Hamiltonian Learning

Santagati et al. (2016)
 Paesani et al. (2016)
 Wang et al. (2016)



CU in Integrated Optics



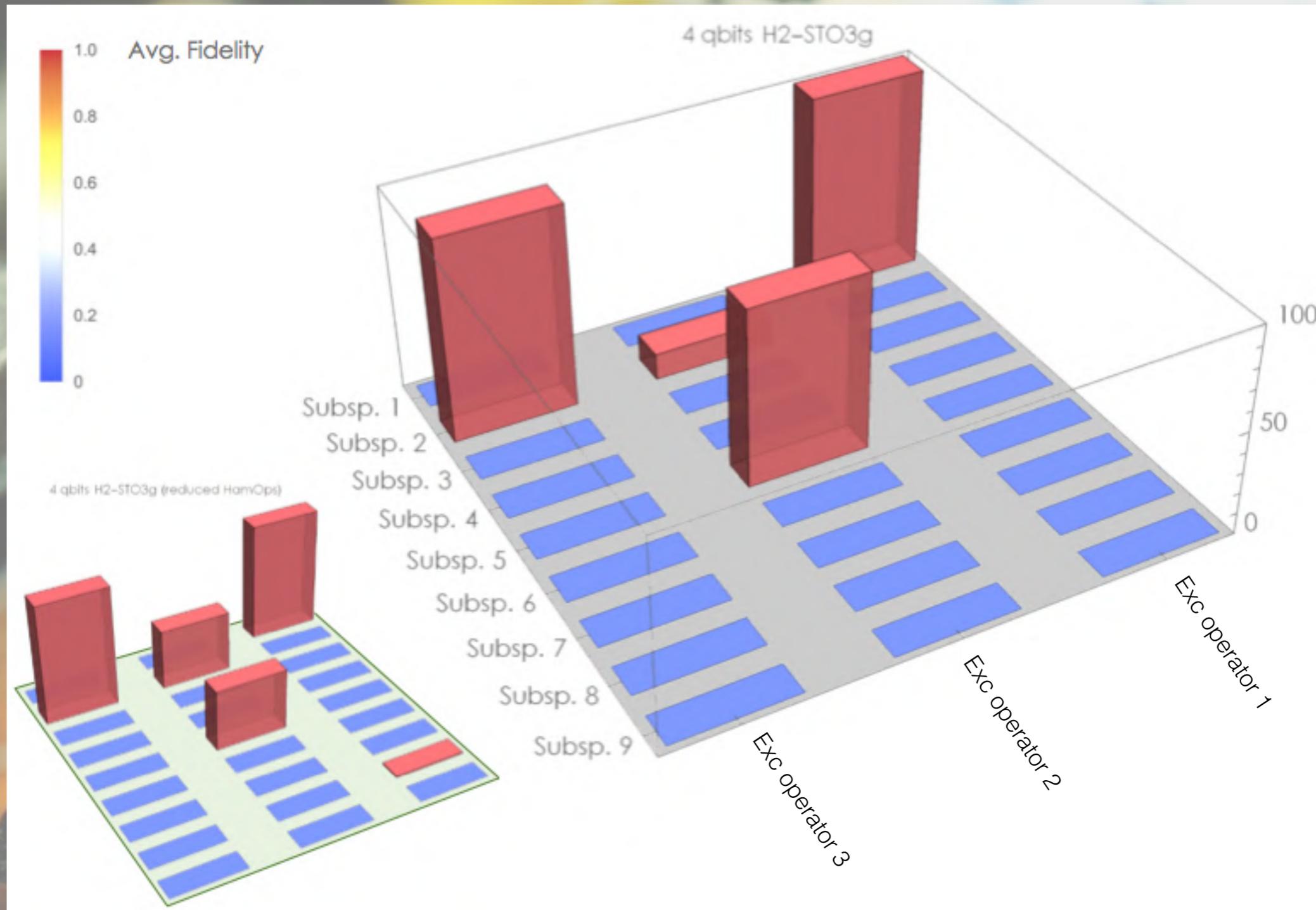
$\mathcal{V}_{C0} = 0.994 \pm 0.001$

$\mathcal{V}_{C1} = 1.00 \pm 0.02$

$$|0\rangle_c \otimes I|\psi\rangle_T + |1\rangle_c \otimes U|\psi\rangle_T$$

Zou et al. (2013)

Simulations



Typical Silicon quantum photonics experiment

