# Feynman path-integral approach to simulating q. circuits and interferometers


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# Simulation of general quantum circuits

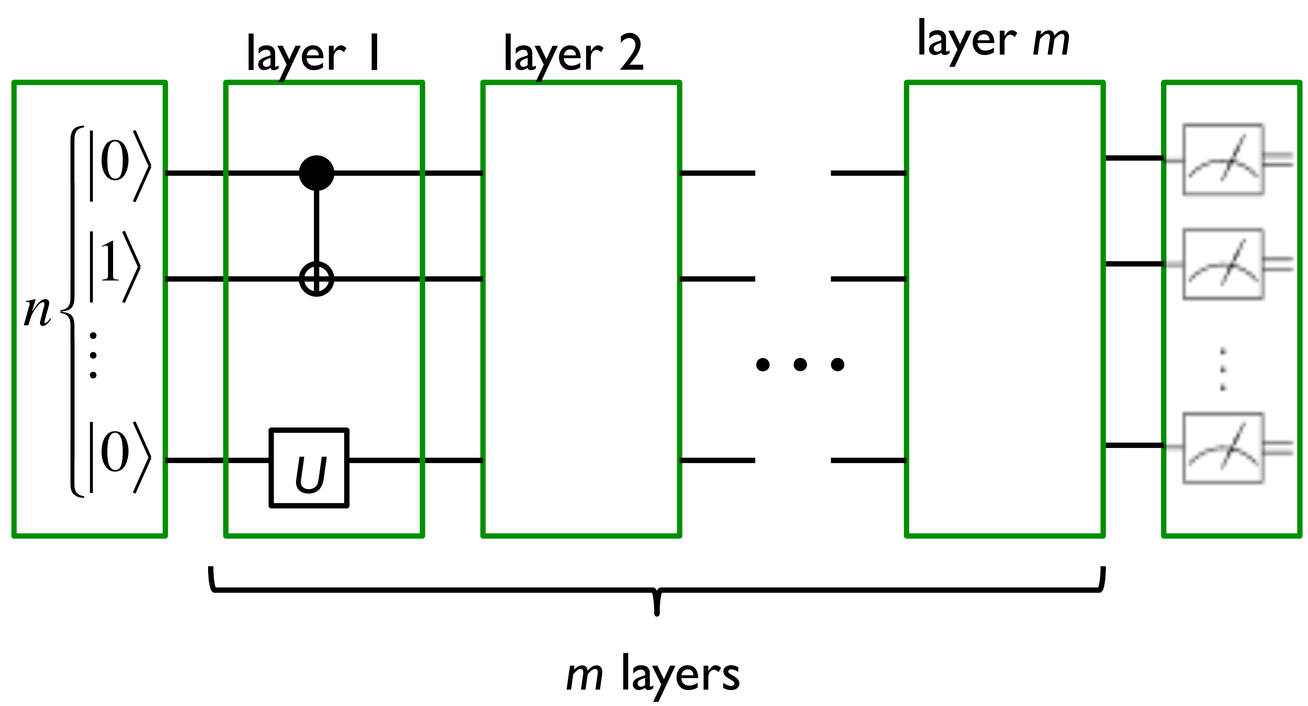
2 general approaches to simulation of general quantum circuits:

- Brute-force calculation - **Schrodinger** approach
- Calculation with polynomial-sized memory – **Feynman** approach

# Schrodinger simulation: exp(n) time, exp(n) space

This is the approach most students of QM would take. Setting:

- *n* qubits
- depth *m* (= number of temporal layers of gates)




- Simulation:

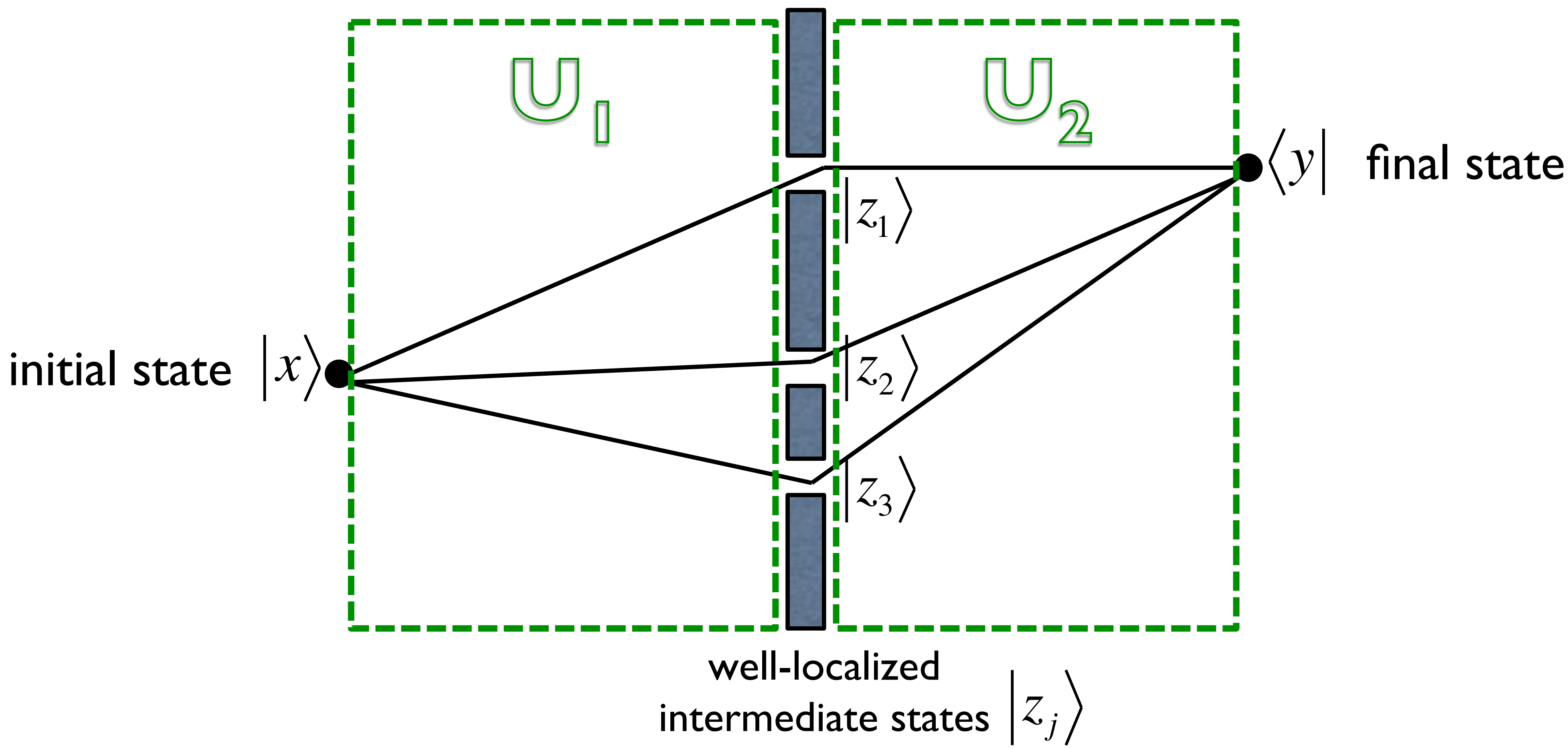
1. Initialize input state;
2. Calculate state after first layer of one- and two-qubit gates;
3. Repeat step 2 above until we get the final state;
4. Directly obtain the amplitude corresponding to the final states of interest.

- Complexity:
  - *m2^n* time
  - *2^n* space (to store wavefunction amplitudes)

# Another approach: Feynman's path integral

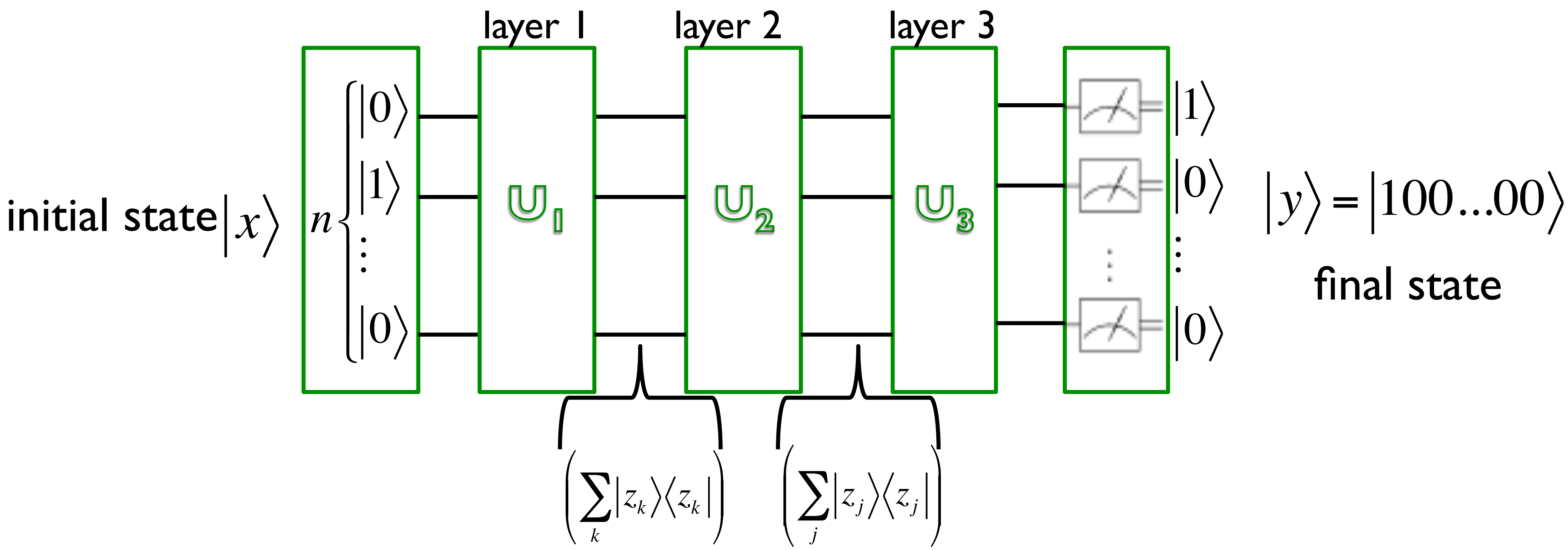
- Let us look at how we can compute amplitudes using Feynman's path integrals:

- **Goal**: calculate $\langle y|U_2 U_1|x\rangle$




$$\langle y|U_2 U_1|x\rangle = \langle y|U_2 \underbrace{\left(\sum_{j=1}^{3} |z_j\rangle\langle z_j|\right)}_{=1} U_1|x\rangle = \underbrace{\sum_{j=1}^{3} \langle y|U_2|z_j\rangle\langle z_j|U_1|x\rangle}_{\text{sum over path amplitudes}}$$

# Simulation using Feynman's sum over path amplitudes




$$\langle y|U_3U_2U_1|x\rangle = \langle y|U_3\left(\sum_j |z_j\rangle\langle z_j|\right)U_2\left(\sum_k |z_k\rangle\langle z_k|\right)U_1|x\rangle = \sum_{j,k}\langle y|U_3|z_j\rangle\langle z_j|U_2|z_k\rangle\langle z_k|U_1|x\rangle$$

If we want the amplitude that the top qubit's measurement be 1:

$$\langle 1anything|U_3U_2U_1|x\rangle = \sum_w \sum_{j,k}\langle 1w|U_3|z_j\rangle\langle z_j|U_2|z_k\rangle\langle z_k|U_1|x\rangle$$

Complexity for m unitary layers:

- *exp(n)* time
- *poly(n,m)* space ( not **exponential** like the Schrodinger scheme)

# Refinements and applications

- Simulating circuits with:
  - *n* qubits
  - *m* gates
  - depth *d*

- Aaronson and Chen (2016) algorithms: [arXiv:1612.05903]

  1. poly(n,m) space, m^O(n) time
  2. poly(n,m) space, d^O(n) time
  3. "Smooth tradeoff" with Schrodinger's scheme:
     - Halve memory use in S. scheme => multiply time use by *d*

- Application (together with other tricks) [Pednault et al., arXiv:1710.05867]

  - 7x7=49 2D grid, random circuit, depth 27
  - 2 days of IBM Vulcan IBM Blue Gene/Q supercomputer (Lawrence Livermore Labs)
  - 4.5 TB memory use, computation of 2^38 amplitudes
  - related paper simulates 7x8=56 qubit circuit of depth 27 [Boixo et al., arXiv:1712.05903]

- Feynman approach used in **Google's quantum advantage** paper (2019)

- Other schemes:
  - contracting tensor networks (Markov, Shi 2008).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