Efficient Bell-Measurements

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QLOC seminar – 22 September 2021

Fundação para a Ciência e a Tecnologia
MINISTÉRIO DA EDUCAÇÃO E CIÊNCIA

INTERNATIONAL IBERIAN NANOTECHNOLOGY LABORATORY

PRESENTATION OUTLINE

1. INTRODUCTION

2. ¾ EFFICIENT BELL-MEASUREMENT

   2.1 Arbitrarily complete Bell-state measurement using only linear optical elements, *Grice*

   2.2 ¾ efficient Bell measurement with passive linear optics and unentangled ancillae, *Ewert and van Loock*

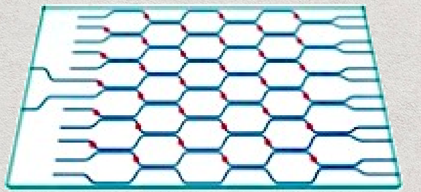
3. BOOSTED TYPE-II FUSION

   From three-photon GHZ states to ballistic universal quantum computation, *Gimeno-Segovia, Shadbolt, Browne and Rudolph*

Introduction:

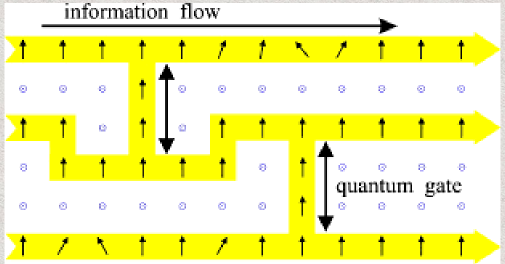
QUANTUM COMPUTATION WITH PHOTONS

MODELS

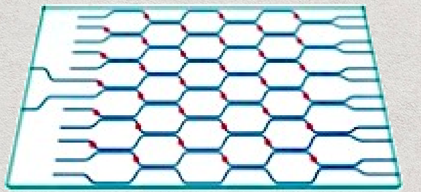


BOSON SAMPLING

MODELS



MEASUREMENT-BASED QUANTUM COMPUTATION



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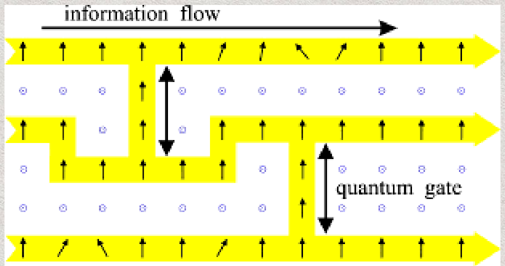
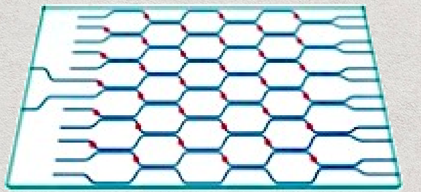


MEASUREMENT-BASED
QUANTUM COMPUTATION



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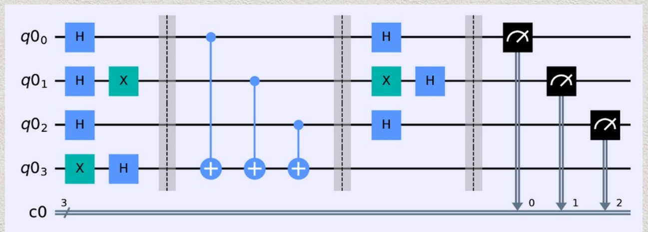
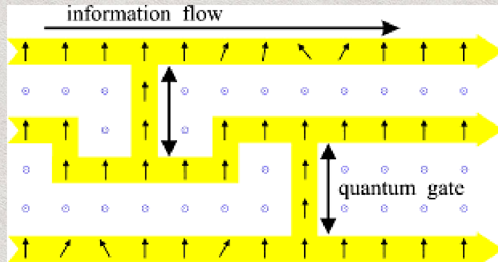
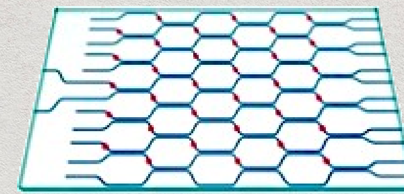


CIRCUIT MODEL

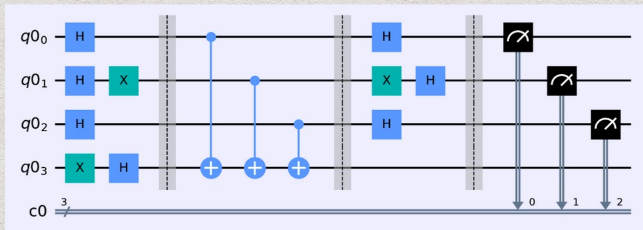
MODELS



MEASUREMENT-BASED QUANTUM COMPUTATION



BOSON SAMPLING



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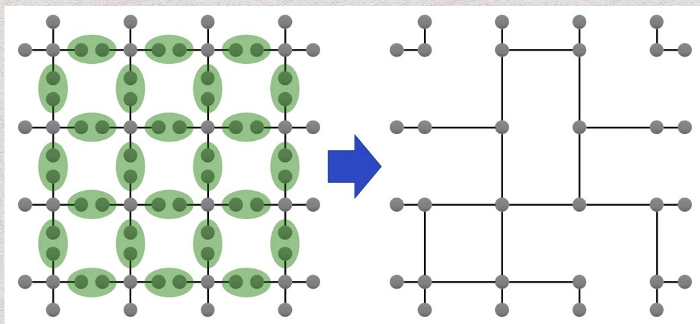

FUSION-BASED QUANTUM COMPUTATION

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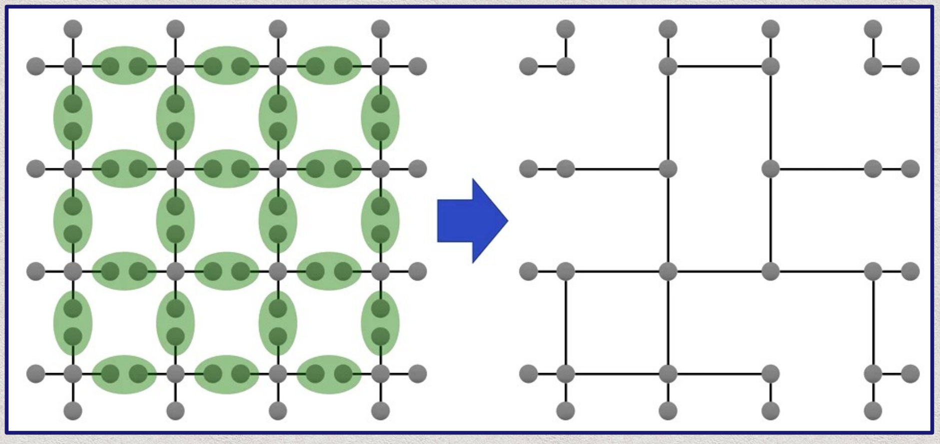
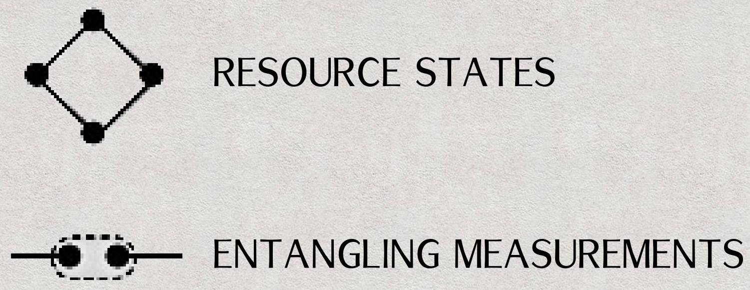
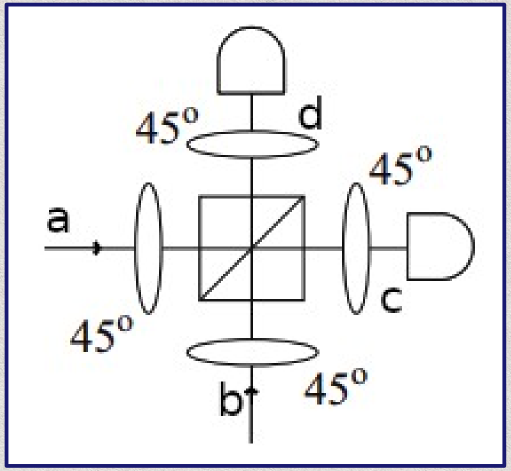


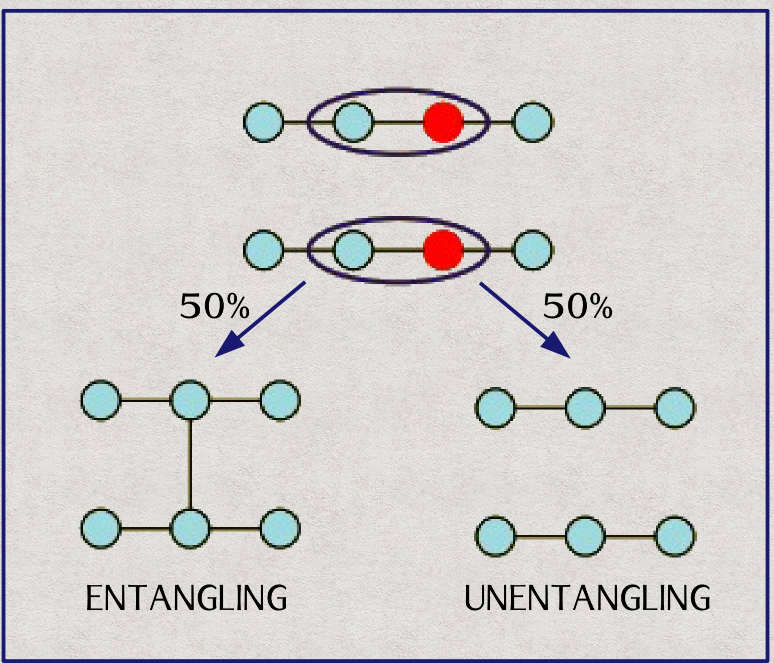


Introduced by:

S. Bartolucci, P. Birchall, H. Bombin, H. Cable, C. Dawson, M. Gimeno-Segovia, E. Johnston,K. Kieling, N. Nickerson, M. Pant, et al. *Fusion-based quantum computation*. arXiv preprintarXiv:2101.09310, 2021

FUSION: ENTANGLING MEASUREMENTS





Original proposal of fusion gates:

E. Browne and T. Rudolph. *Resource-efficient linear optical quantum computation.* Physical Review Letters, 95(1):010501, 2005

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FUSION: ENTANGLING MEASUREMENTS

- STANDARD BELL-MEASUREMENT IN THE ROTATED BASIS
- 50% EFFICIENT

Original proposal of fusion gates:

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Introduction:
STANDARD
BELL-MEASUREMENT

EXAMPLE 1: Bell states in polarization modes encoding:

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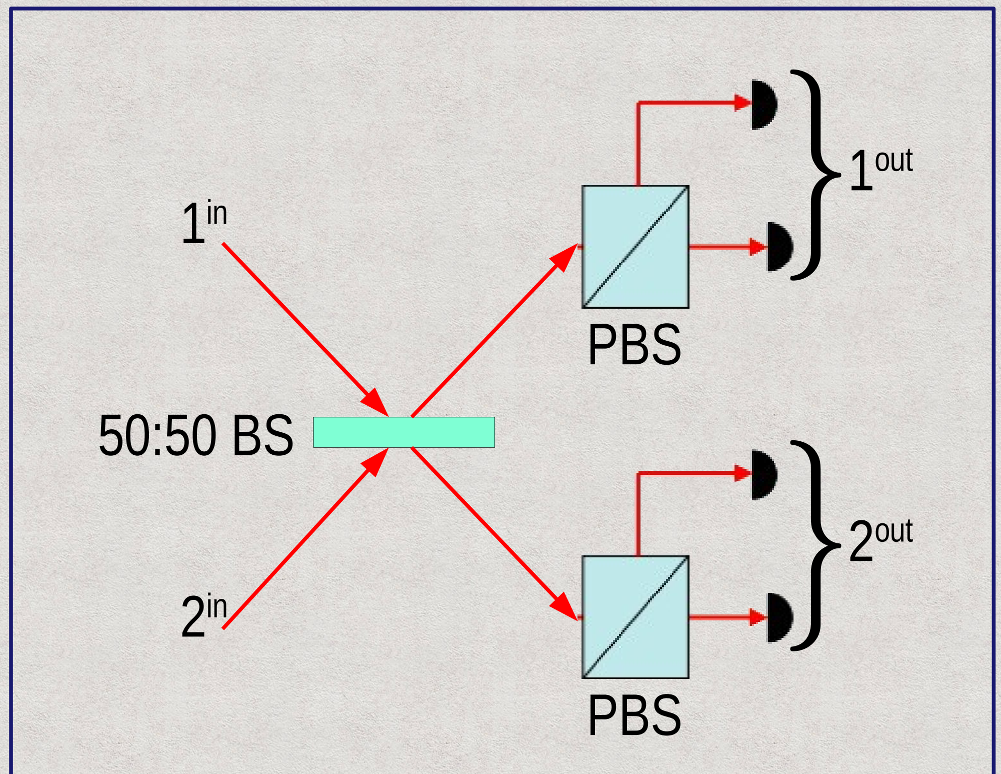
EXAMPLE 1: Bell states in polarization modes encoding:

$$\begin{cases} |\,\mathbf{0}\rangle \Longrightarrow |\,H\rangle \\ |\,\mathbf{1}\rangle \Longrightarrow |\,V\rangle \end{cases} \quad \text{s.t.} \quad \begin{cases} |\,\Phi_+\rangle = \frac{1}{\sqrt{2}}\Big(\,|\,HH\rangle + |\,VV\rangle\Big) \\ |\,\Phi_-\rangle = \frac{1}{\sqrt{2}}\Big(\,|\,HH\rangle - |\,VV\rangle\Big) \\ |\,\Psi_+\rangle = \frac{1}{\sqrt{2}}\Big(\,|\,HV\rangle + |\,VH\rangle\Big) \\ |\,\Psi_-\rangle = \frac{1}{\sqrt{2}}\Big(\,|\,HV\rangle - |\,VH\rangle\Big) \end{cases}$$

How can we measure them?

EXAMPLE 1: Bell states in polarization modes encoding:

Experimental setup



Photon-number and -polarization resolving detectors

EXAMPLE 1: Bell states in polarization modes encoding:

Outputs:

$$\boxed{\begin{aligned} |\Psi_+\rangle &\longrightarrow \frac{i}{\sqrt{2}}\Big(|H_1\rangle|V_1\rangle + |H_2\rangle|V_2\rangle\Big) \\ |\Psi_-\rangle &\longrightarrow \frac{1}{\sqrt{2}}\Big(|H_1\rangle|V_2\rangle - |V_1\rangle|H_2\rangle\Big) \end{aligned}}$$

$$\boxed{|\Phi_\pm\rangle \longrightarrow \frac{i}{2}\Big(|2_{H_1}\rangle + |2_{H_2}\rangle \pm |2_{V_1}\rangle \pm |2_{V_2}\rangle\Big)}$$

50% efficient

EXAMPLE 2: Bell states in dual-rail encoding:

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Dual-rail encoding:

$$\begin{cases} |\,\mathbf{0}\rangle \Longrightarrow |\,10\rangle = a^{\dagger}\,|\,00\rangle \\ |\,\mathbf{1}\rangle \Longrightarrow |\,01\rangle = b^{\dagger}\,|\,00\rangle \end{cases}$$

Photon in the first waveguide ⇒ Logical Zero

Photon in the second waveguide ⇒ Logical One

where: $|\,0\rangle_a \otimes |\,0\rangle_b \equiv |\,00\rangle$

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EXAMPLE 2: Bell states in dual-rail encoding:

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s.t.

$$\begin{cases} |\,\Phi_+\rangle = \frac{1}{\sqrt{2}}\Big(\,|\,1010\rangle + |\,0101\rangle\Big) \\ |\,\Phi_-\rangle = \frac{1}{\sqrt{2}}\Big(\,|\,1010\rangle - |\,0101\rangle\Big) \\ |\,\Psi_+\rangle = \frac{1}{\sqrt{2}}\Big(\,|\,1001\rangle + |\,0110\rangle\Big) \\ |\,\Psi_-\rangle = \frac{1}{\sqrt{2}}\Big(\,|\,1001\rangle - |\,0110\rangle\Big) \end{cases}$$

How can we measure them?

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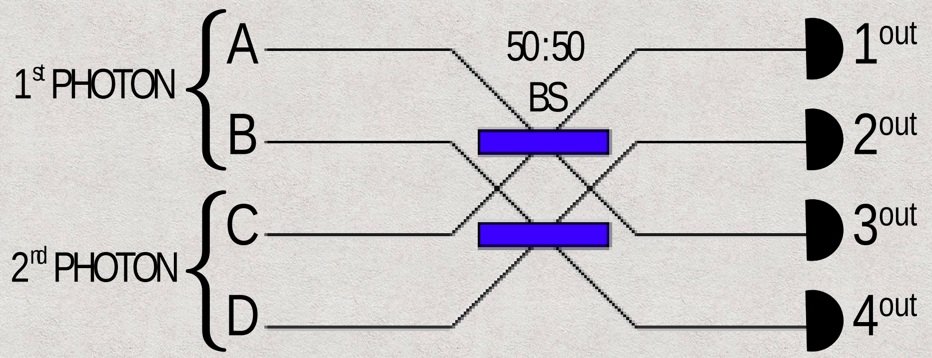
Experimental setup:



$$\begin{pmatrix} \hat{o}_i^\dagger \\ \hat{o}_j^\dagger \end{pmatrix} \xrightarrow{BS} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \begin{pmatrix} \hat{o}_i^\dagger \\ \hat{o}_j^\dagger \end{pmatrix}$$

Photon-number resolving detectors

EXAMPLE 2: Bell states in dual-rail encoding:

Outputs:

$$|\,\Psi_+\rangle \longrightarrow \frac{i}{\sqrt{2}}\Big(\,|\,1100\rangle + |\,0011\rangle\Big)$$

$$|\,\Psi_-\rangle \longrightarrow \frac{1}{\sqrt{2}}\Big(\,|\,1001\rangle - |\,0110\rangle\Big)$$

$$|\,\Phi_\pm\rangle \longrightarrow \frac{i}{2}\Big(\,|\,2000\rangle + |\,0020\rangle \pm |\,0200\rangle \pm |\,0002\rangle\Big)$$

50% efficient

¾ EFFICIENT BELL-MEASUREMENT (1)

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Reference:

PHYSICAL REVIEW A **84**, 042331 (2011)

Arbitrarily complete Bell-state measurement using only linear optical elements

W. P. Grice*

Computational Sciences and Engineering Division, Oak Ridge National Laboratory, Tennessee, USA

(Received 24 May 2011; published 19 October 2011)

A complete Bell-state measurement is not possible using only linear-optic elements, and most schemes achieve a success rate of no more than 50%, distinguishing, for example, two of the four Bell states but returning degenerate results for the other two. It is shown here that the introduction of a pair of ancillary entangled photons improves the success rate to 75%. More generally, the addition of $2^N - 2$ ancillary photons yields a linear-optic Bell-state measurement with a success rate of $1 - 1/2^N$.

⟹ Measure of Bell-states in polarization encoding

BOOSTING:

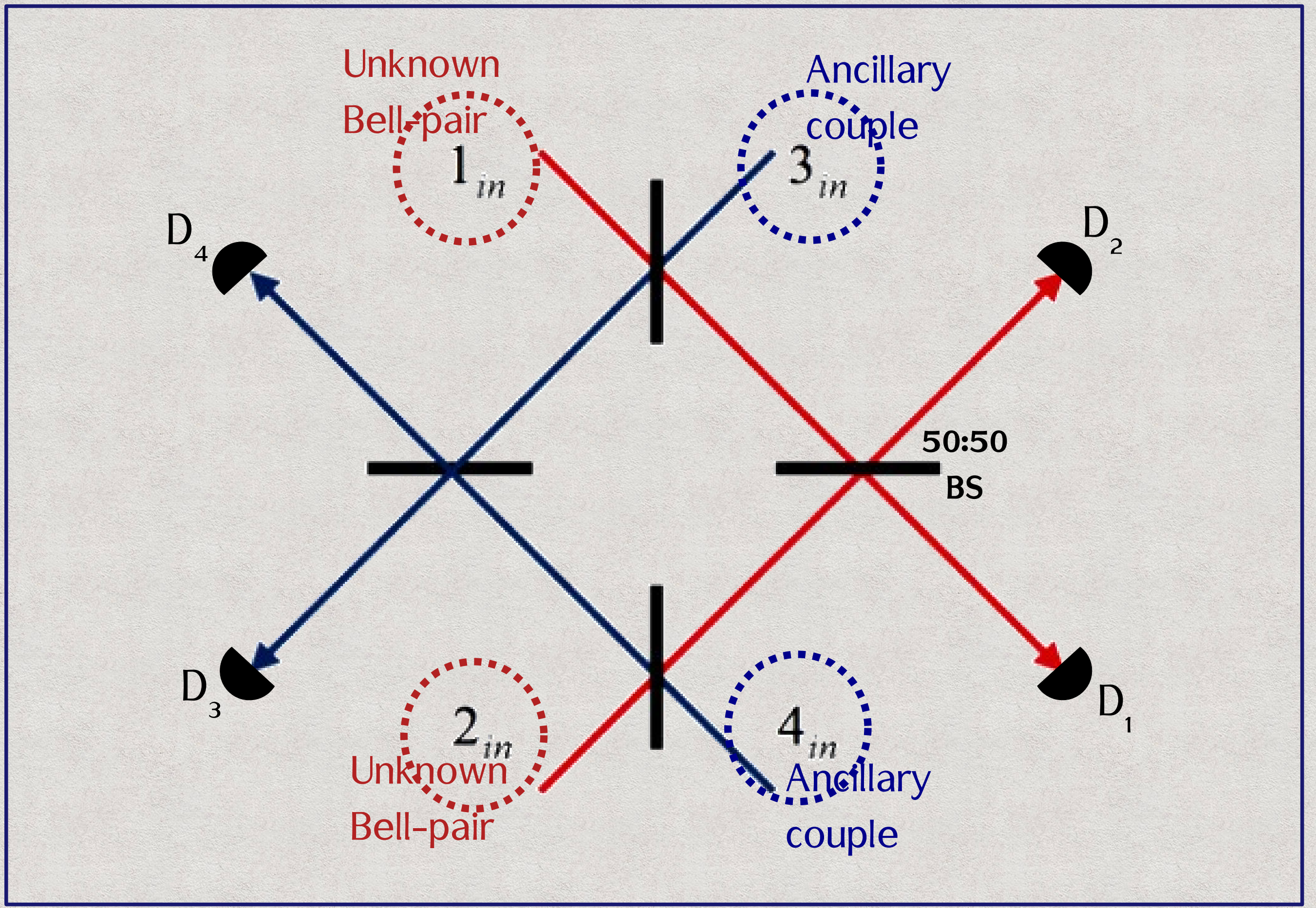
⟹ Ancillary photons in the state: $|\,\Upsilon_1\rangle = \frac{1}{\sqrt{2}}\left[\hat{h}_3^\dagger \hat{h}_4^\dagger + \hat{v}_3^\dagger \hat{v}_4^\dagger\right] |\,\varnothing\rangle$

unentangled with the initial Bell-state

⇓

degeneracy in simple BSM can be broken by interfering $|\,\Phi^{\pm}\rangle$ with something having a very similar form

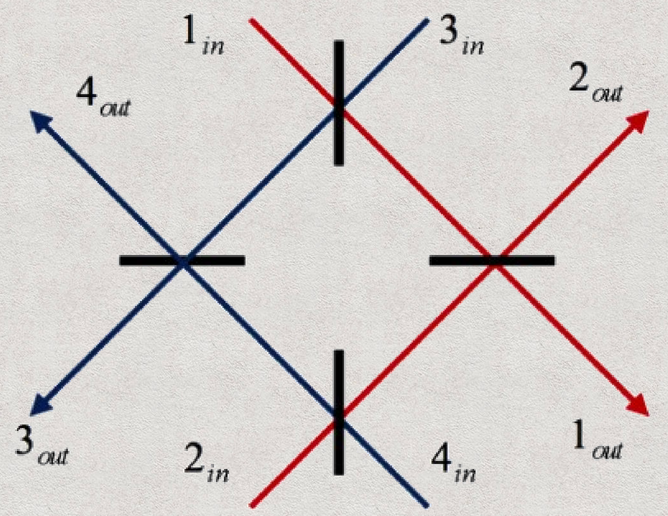
Experimental setup:



D_i: photon-number and -polarization resolving detector

The apparatus is symmetrical

$$\begin{pmatrix} \hat{a}_1^\dagger \\ \hat{a}_2^\dagger \\ \hat{a}_3^\dagger \\ \hat{a}_4^\dagger \end{pmatrix}_{\text{in}} \rightarrow \frac{1}{2}\begin{pmatrix} 1 & i & i & -1 \\ i & 1 & -1 & i \\ i & -1 & 1 & i \\ -1 & i & i & 1 \end{pmatrix}\begin{pmatrix} \hat{a}_1^\dagger \\ \hat{a}_2^\dagger \\ \hat{a}_3^\dagger \\ \hat{a}_4^\dagger \end{pmatrix}_{\text{out}}.$$



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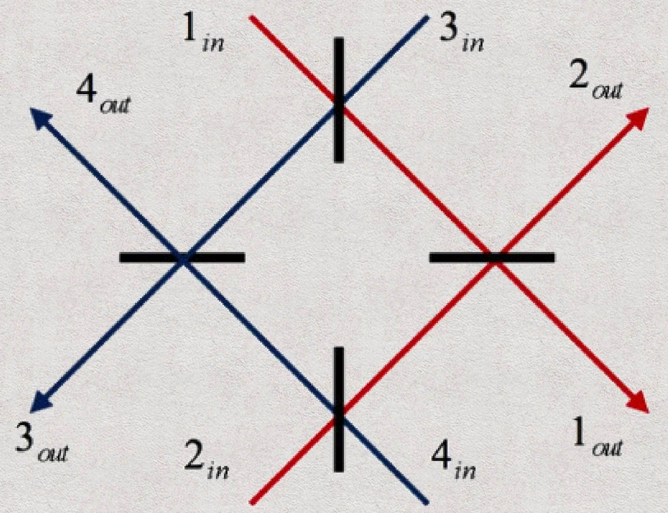

LABELS:

n_H, n_V = number of horizontally and vertically polarized photons at the outputs

$n_{[i]}$ = total number of photons at detector i

$$n_H, n_V$$

EVEN $\longrightarrow$ $| \Psi^{\pm} \rangle | \Upsilon_1 \rangle$

ODD $\longrightarrow$ $| \Phi^{\pm} \rangle | \Upsilon_1 \rangle$

n_H, n_V

EVEN → $|\Psi^{\pm}\rangle|\Upsilon_1\rangle$

ODD → $|\Phi^{\pm}\rangle|\Upsilon_1\rangle$

$|\Psi^{\pm}\rangle$ = clearly detectable!

$$n_{[1,3]} = n_{[1]} + n_{[3]}$$

$$\begin{cases} \text{is even for } |\Psi^{+}\rangle|\Upsilon_1\rangle \\ \text{is odd for } |\Psi^{-}\rangle|\Upsilon_1\rangle \end{cases}$$

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n_H, n_V

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✔

50% cases: same polarization

$\Rightarrow$ indistinguishability ✘

50% cases: half and half

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$$\Longrightarrow \mathcal{P}_{succ} = \frac{1}{4}\Big(1 + 1 + 0.5 + 0.5\Big) = \frac{3}{4}$$

Near-unit efficient Bell Measurement

MORE ANCILLARY PHOTONS

↓

MORE IMPROVEMENT

Near-unit efficient Bell Measurement

MORE ANCILLARY PHOTONS

↓

MORE IMPROVEMENT

STRATEGY:

- each input is mixed at a 50:50 BS with one photon of an entangled state;
- the BS outputs are sent to identical arrangements of BSs;

Near-unit efficient Bell Measurement

Input state: $$|\,\zeta\rangle\,\underbrace{|\,\Upsilon_1\rangle \ldots |\,\Upsilon_{N-1}\rangle}$$

UNKNOWN
BELL-STATE

N-1 ANCILLARY STATES, GHZ-TYPE:

$$\boxed{|\,\Upsilon_j\rangle \equiv \frac{1}{\sqrt{2}}\left[\hat{h}^{\dagger}_{2^j+1}\ldots\hat{h}^{\dagger}_{2^{j+1}} + \hat{v}^{\dagger}_{2^j+1}\ldots\hat{v}^{\dagger}_{2^{j+1}}\right]|\,0\rangle}$$

Near-unit efficient Bell Measurement

Input state: $\mid\zeta\rangle\underbrace{\mid\Upsilon_1\rangle\ldots\mid\Upsilon_{N-1}\rangle}$

UNKNOWN BELL-STATE

N-1 ANCILLARY STATES, GHZ-TYPE:

$$\boxed{\mid\Upsilon_j\rangle\equiv\frac{1}{\sqrt{2}}\left[\hat{h}^\dagger_{2^j+1}\ldots\hat{h}^\dagger_{2^{j+1}}+\hat{v}^\dagger_{2^j+1}\ldots\hat{v}^\dagger_{2^{j+1}}\right]\mid 0\rangle}$$

Example:

$$\begin{cases}\mid\Upsilon_1\rangle\equiv\frac{1}{\sqrt{2}}\left[\hat{h}^\dagger_3\hat{h}^\dagger_4+\hat{v}^\dagger_3\hat{v}^\dagger_4\right]\mid 0\rangle\quad\text{for}\quad j=1\\ \mid\Upsilon_2\rangle\equiv\frac{1}{\sqrt{2}}\left[\hat{h}^\dagger_5\hat{h}^\dagger_6\hat{h}^\dagger_7\hat{h}^\dagger_8+\hat{v}^\dagger_5\hat{v}^\dagger_6\hat{v}^\dagger_7\hat{v}^\dagger_8\right]\mid 0\rangle\quad\text{for}\quad j=2\end{cases}$$

Near-unit efficient Bell Measurement

$$n_H, n_V$$

EVEN

$$|\Psi^{\pm}\rangle\, |\Upsilon_1\rangle \ldots |\Upsilon_{N-1}\rangle$$

ODD

$$|\Phi^{\pm}\rangle\, |\Upsilon_1\rangle \ldots |\Upsilon_{N-1}\rangle$$

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Near-unit efficient Bell Measurement

$$n_H, n_V$$

EVEN: $|\Psi^\pm\rangle\,|\Upsilon_1\rangle \dots |\Upsilon_{N-1}\rangle$

ODD: $|\Phi^\pm\rangle\,|\Upsilon_1\rangle \dots |\Upsilon_{N-1}\rangle$

$|\Psi^\pm\rangle$ = clearly detectable!

$\mathbf{n_{odd}}$ = total number of photons on odd outputs

$$\begin{cases} \text{is even for } |\Psi^+\rangle\,|\Upsilon_1\rangle \dots |\Upsilon_{N-1}\rangle \\ \text{is odd for } |\Psi^-\rangle\,|\Upsilon_1\rangle \dots |\Upsilon_{N-1}\rangle \end{cases}$$

Near-unit efficient Bell Measurement

$$\begin{tikzcd} & \boxed{n_H, n_V} \arrow[dl, "\text{EVEN}"'] \arrow[dr, "\text{ODD}"] & \\ \boxed{|\Psi^{\pm}\rangle \, |\Upsilon_1\rangle \ldots |\Upsilon_{N-1}\rangle} & & \boxed{|\Phi^{\pm}\rangle \, |\Upsilon_1\rangle \ldots |\Upsilon_{N-1}\rangle} \end{tikzcd}$$

The degeneracy rate is reduced by half for each additional ancillary state.

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Near-unit efficient Bell Measurement

$$|\Phi^{\pm}\rangle\,|\Upsilon_1\rangle\ldots|\Upsilon_{N-1}\rangle$$

can be decomposed

$$|\Xi_N^{\pm}\rangle + |\Xi_{N-1}^{\pm}\rangle\,|\Upsilon_{N-1}\rangle + |\Xi_{N-2}^{\pm}\rangle\,|\Upsilon_{N-2}\rangle\,|\Upsilon_{N-1}\rangle + \ldots$$
$$\ldots + |\Xi_2^{\pm}\rangle\,|\Upsilon_2\rangle\,|\Upsilon_3\rangle\ldots|\Upsilon_{N-1}\rangle + |\Gamma_N^{\pm}\rangle$$

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Near-unit efficient Bell Measurement

$$\boxed{|\,\Phi^{\pm}\rangle\,|\,\Upsilon_1\rangle\ldots|\,\Upsilon_{N-1}\rangle}$$

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$$|\,\Xi_N^{\pm}\rangle + |\,\Xi_{N-1}^{\pm}\rangle\,|\,\Upsilon_{N-1}\rangle + |\,\Xi_{N-2}^{\pm}\rangle\,|\,\Upsilon_{N-2}\rangle\,|\,\Upsilon_{N-1}\rangle + \ldots$$
$$\ldots + |\,\Xi_2^{\pm}\rangle\,|\,\Upsilon_2\rangle\,|\,\Upsilon_3\rangle\ldots|\,\Upsilon_{N-1}\rangle + |\,\Gamma_N^{\pm}\rangle$$

THIS REMAINS THE ONLY TERM WITH DEGENERACY
AND IT HAPPENS WITH A PROBABILITY OF $1/2^N$

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Near-unit efficient Bell Measurement

$\Rightarrow$ BSM success rate approaches unity as $2^N \rightarrow \infty$

$$|\,\Xi_N^{\pm}\rangle + |\,\Xi_{N-1}^{\pm}\rangle\,|\,\Upsilon_{N-1}\rangle + |\,\Xi_{N-2}^{\pm}\rangle\,|\,\Upsilon_{N-2}\rangle\,|\,\Upsilon_{N-1}\rangle + \ldots$$
$$\ldots + |\,\Xi_2^{\pm}\rangle\,|\,\Upsilon_2\rangle\,|\,\Upsilon_3\rangle \ldots |\,\Upsilon_{N-1}\rangle + |\,\Gamma_N^{\pm}\rangle$$

THIS REMAINS THE ONLY TERM WITH DEGENERACY AND IT HAPPENS WITH A PROBABILITY OF $1/2^N$

Near-unit efficient Bell Measurement

But…

Near-unit efficient Bell Measurement

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GENERATING LARGE ENTANGLED STATES IS EXPENSIVE

The number of entangled photons required makes it impractical to reach even 95% (~30 photons)

Near-unit efficient Bell Measurement

But...

GENERATING LARGE ENTANGLED STATES IS EXPENSIVE

The number of entangled photons required makes it impractical to reach even 95% (~30 photons)

Each stage:

→ total number of photons doubles;

→ total number of outputs doubles.

REQUIREMENT OF BIG STABLE EXPERIMENTAL APPARATUS

¾ EFFICIENT BELL-MEASUREMENT (2)

Reference:

3/4-efficient Bell measurement with passive linear optics and unentangled ancillae

Fabian Ewert* and Peter van Loock
Institute of Physics, Johannes-Gutenberg Universität Mainz, Staudingerweg 7, 55128 Mainz, Germany

(Dated: March 20, 2014)

It is well known that an unambiguous discrimination of the four optically encoded Bell states is possible with a probability of 50% at best, when using static, passive linear optics and arbitrarily many vacuum mode ancillae. By adding unentangled single-photon ancillae, we are able to surpass this limit and reach a success probability of at least 75%. We discuss the error robustness of the proposed scheme and a generalization to reach a success probability arbitrarily close to 100%.

Measure of Bell-states in dual-rail encoding

BOOSTING:

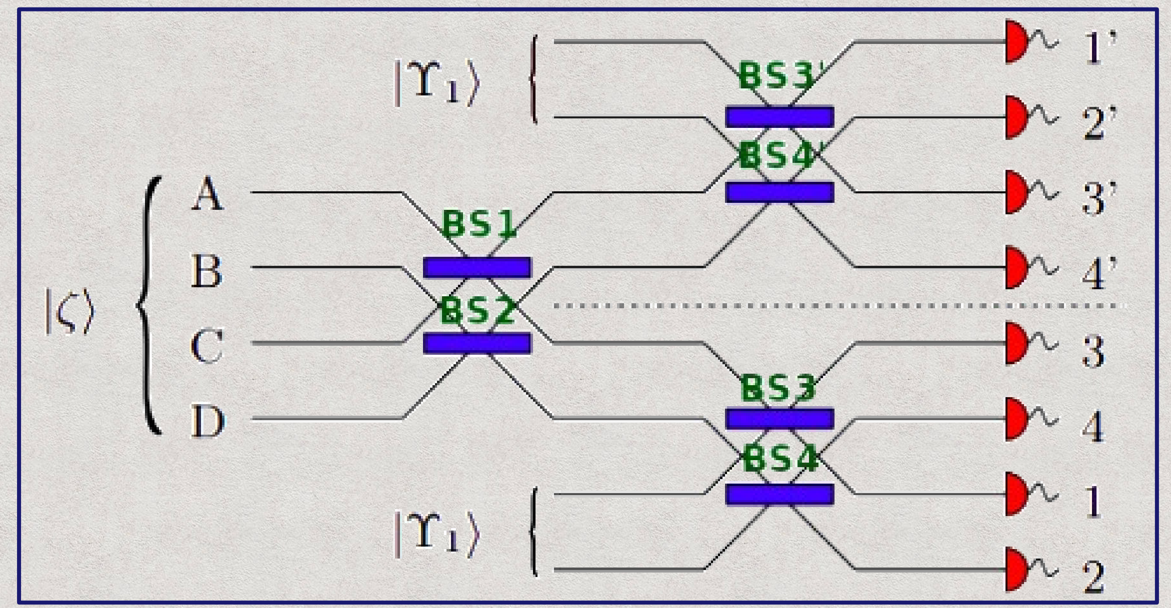
Ancillary photons in the state: $|\Upsilon_1\rangle = \frac{1}{\sqrt{2}}\Big(|20\rangle + |02\rangle\Big)$

Easily obtainable by sending two single photons through a BS (HOM effect).

Main advantage of this scheme over Grice's:

Deterministic single-photon sources are employed as opposed to a probabilistically generated Bell pair.

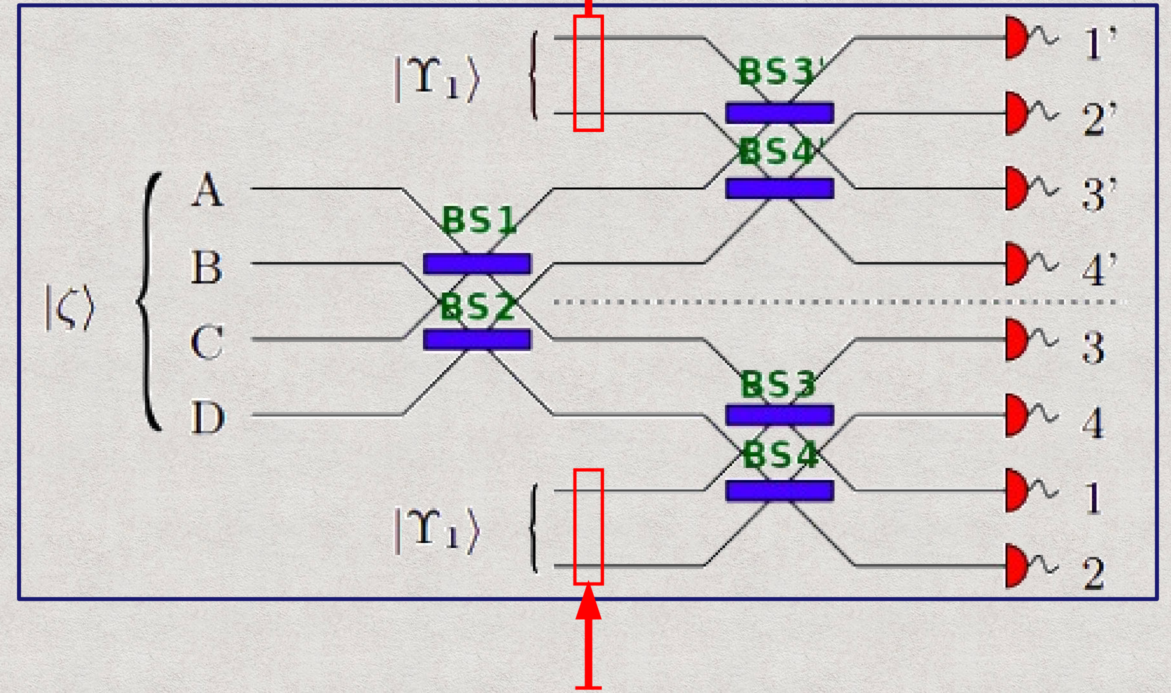
Experimental setup:



Where: $|\zeta\rangle \in \{\Psi^{\pm}, \Phi^{\pm}\}$

Experimental setup:

$$|\Upsilon_1\rangle = \frac{1}{\sqrt{2}}\Big(|20\rangle + |02\rangle\Big)$$



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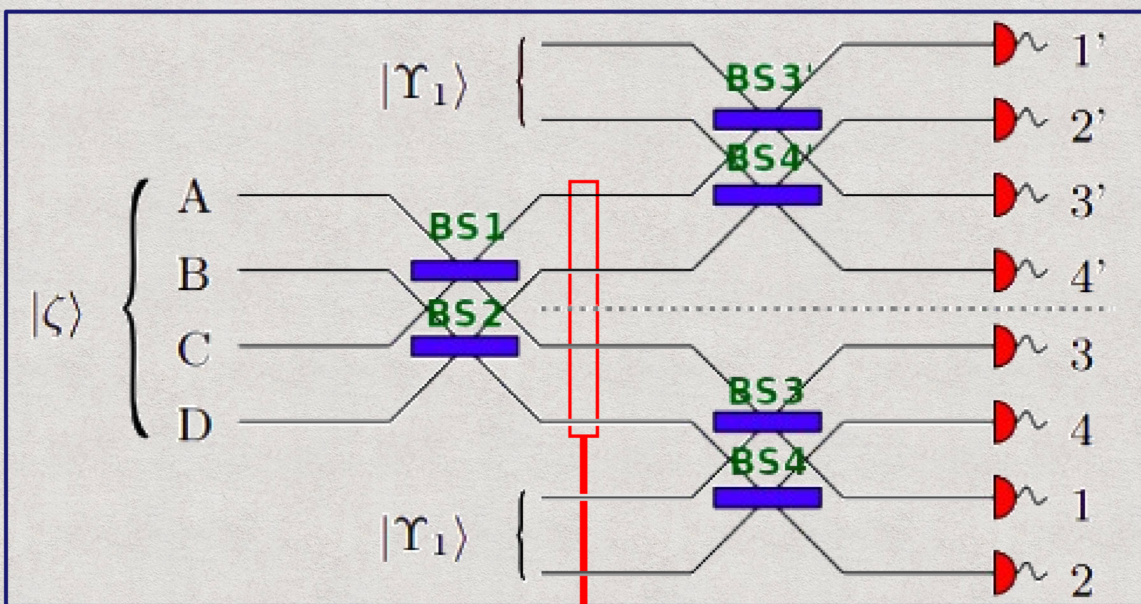
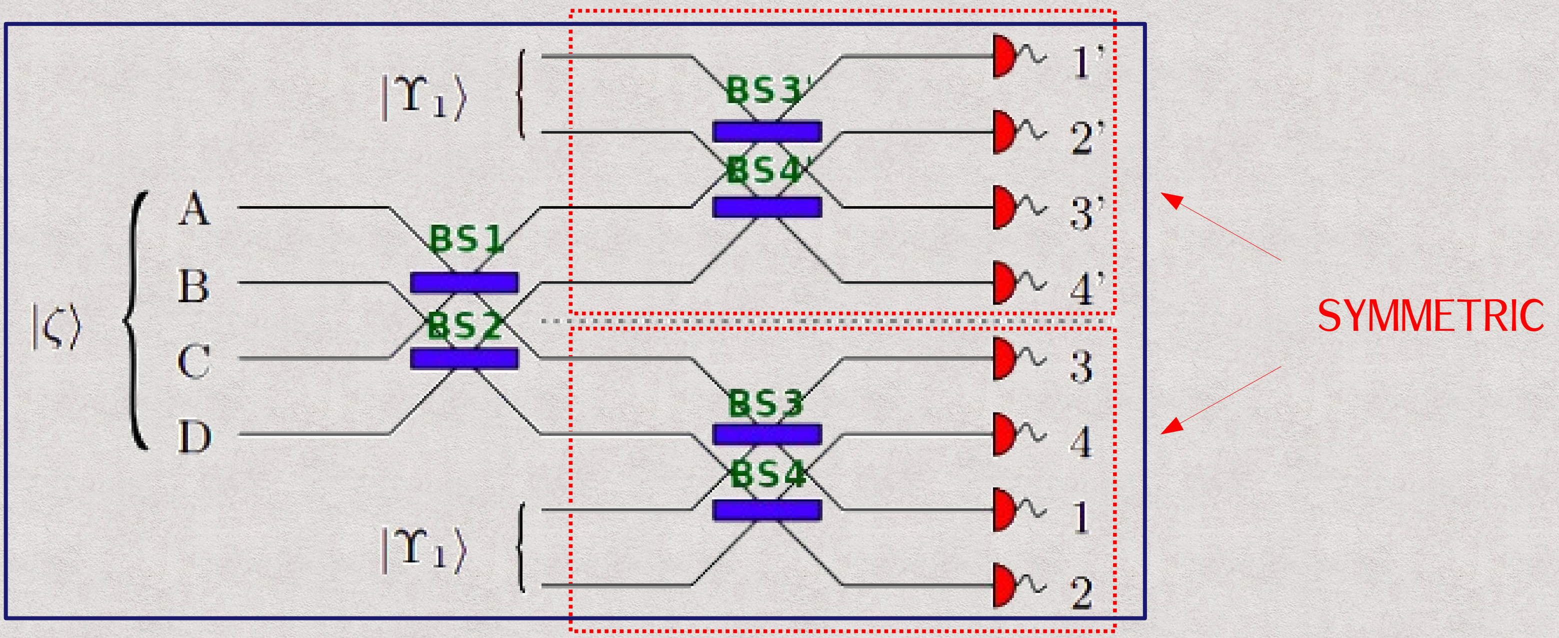


SLIDE 12

$$|\Psi_+\rangle \longrightarrow \frac{i}{\sqrt{2}}\Big(|1100\rangle + |0011\rangle\Big)$$

$$|\Psi_-\rangle \longrightarrow \frac{1}{\sqrt{2}}\Big(|1001\rangle - |0110\rangle\Big)$$

$$|\Phi_\pm\rangle \longrightarrow \frac{i}{2}\Big(|2000\rangle + |0020\rangle \pm |0200\rangle \pm |0002\rangle\Big)$$



This symmetry allows to take into account only half apparatus.

¾ EFFICIENT BELL-MEASUREMENT (2)

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SINGLET:

$$|\,\Psi_-\rangle \longrightarrow \frac{1}{\sqrt{2}}\Big(\,|\,1001\rangle - |\,0110\rangle\Big)$$

Is the only state that sends one photon in each side of the apparatus.

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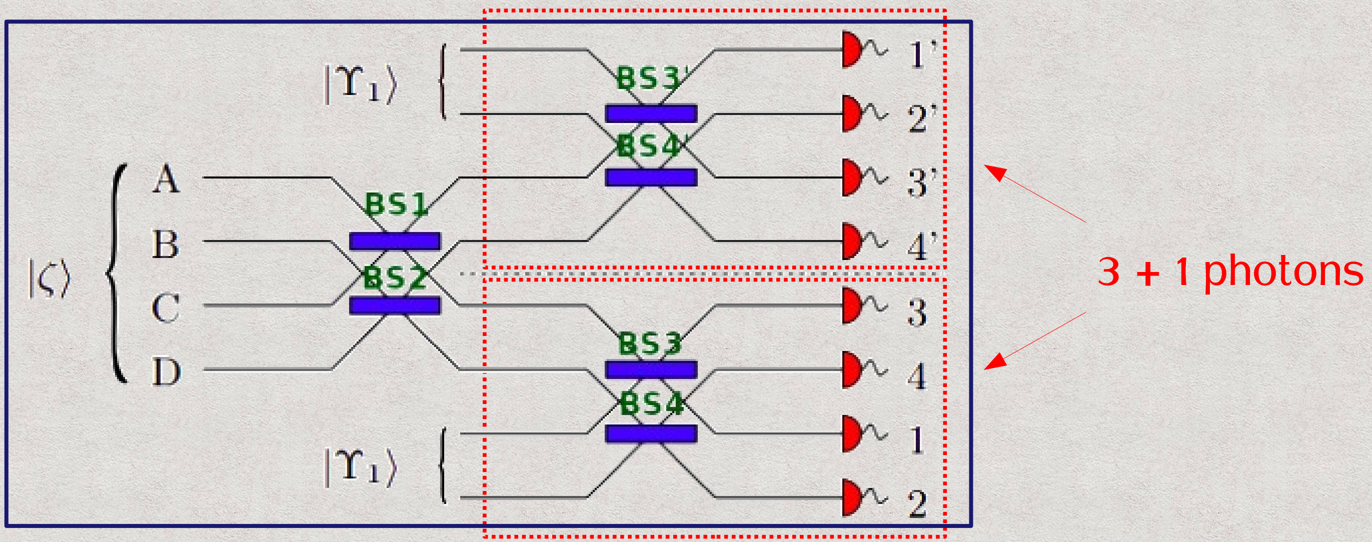
$$|\Psi_-\rangle \longrightarrow \frac{1}{\sqrt{2}}\Big(|1001\rangle - |0110\rangle\Big)$$

Is the only state that sends one photon in each side of the apparatus.



⇒ Always unambiguously discriminated from other Bell states, which instead send 0 or 2.

¾ EFFICIENT BELL-MEASUREMENT (2)

TRIPLET:

$$\begin{tikzcd}
 & \boxed{\mathbf{n_{odd}}} \arrow[dl, "\text{ODD}"'] \arrow[dr, "\text{EVEN}"] & \\
\boxed{\,|\,\Psi^{+}\rangle\,|\,\Upsilon_1\rangle} & & \boxed{\,|\,\Phi^{\pm}\rangle\,|\,\Upsilon_1\rangle}
\end{tikzcd}$$

TRIPLET:

$$\mathbf{n_{odd}}$$

ODD

$$|\,\Psi^{+}\rangle\,|\,\Upsilon_{1}\rangle$$

EVEN

$$|\,\Phi^{\pm}\rangle\,|\,\Upsilon_{1}\rangle$$

clearly detectable!

TRIPLET:

n_{odd}

ODD → $|\Psi^{+}\rangle\,|\Upsilon_1\rangle$

EVEN → $|\Phi^{\pm}\rangle\,|\Upsilon_1\rangle$

clearly detectable! ✓

50% cases:

photons equally distributed on even and odd modes ✓

50% cases:

photons all in modes with the same parity ✗

¾ EFFICIENT BELL-MEASUREMENT (2)

TRIPLET:

$$\mathbf{n_{odd}}$$

ODD

$$|\,\Psi^+\rangle\,|\,\Upsilon_1\rangle$$

EVEN

$$|\,\Phi^\pm\rangle\,|\,\Upsilon_1\rangle$$

$$\frac{1}{8}\Big[-\sqrt{6}\Big(\,|4000\rangle + |0040\rangle \pm |0400\rangle \pm |0004\rangle\,\Big) - 2\Big(\,|2020\rangle \pm |0202\rangle\,\Big)+$$

$$\xrightarrow{\Phi^+} \quad +2\Big(\,|2002\rangle - |2200\rangle - |0022\rangle + |0220\rangle - 2\,|1111\rangle\,\Big)\Big]$$

$$\xrightarrow{\Phi^-} \quad +2i\sqrt{2}\Big(\,|2101\rangle - |0121\rangle + |1012\rangle - |1210\rangle\,\Big)\Big]$$

(example: below part of the apparatus)

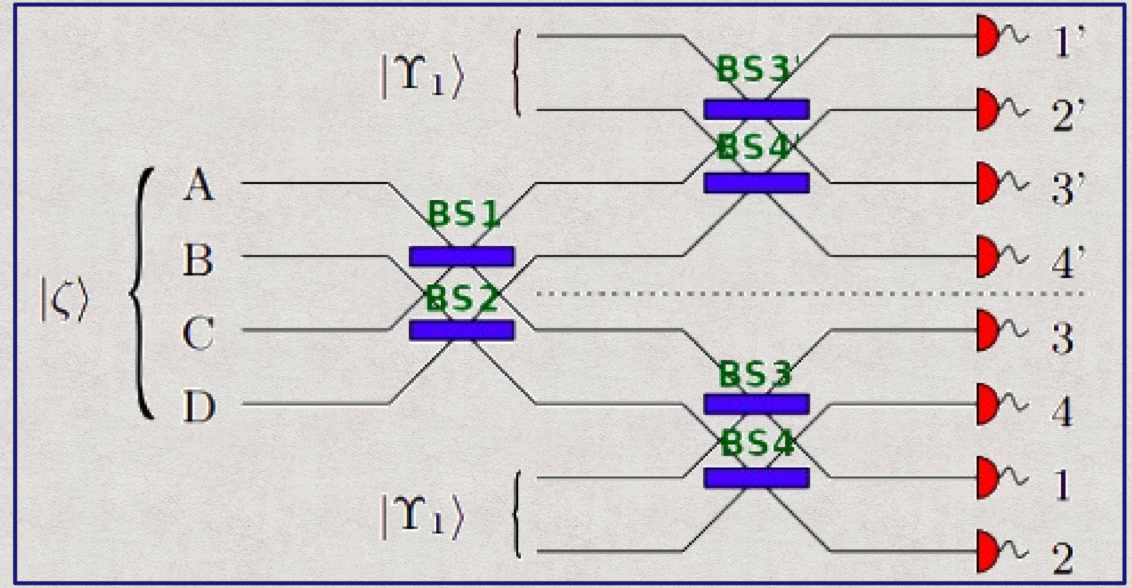
50% cases:

photons equally distributed on even and odd modes ✔

50% cases:

photons all in modes with the same parity ✘

IN CONCLUSION:



$$\mathcal{P}_{succ} = \frac{1}{4}\Big(1 + 1 + 0.5 + 0.5\Big) = \frac{3}{4}$$

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Surpassing the 75% limit

Through a method similar to Grice's general scheme, they showed that, from a theoretical point of view:

$$\boxed{\mathcal{P}_{succ}^{(N)} = \frac{1+1+2(1-2^{-N})}{4} = 1 - 2^{-1-N} \to 1 \quad \text{for} \quad N \to \infty}$$

This result holds only for highly entangled ancillary states:

$$|\,\Upsilon_j\rangle \quad j \geq 2$$

which cannot be obtained from single-photon states using passive linear optics.

Surpassing the 75% limit with unentangled ancillae

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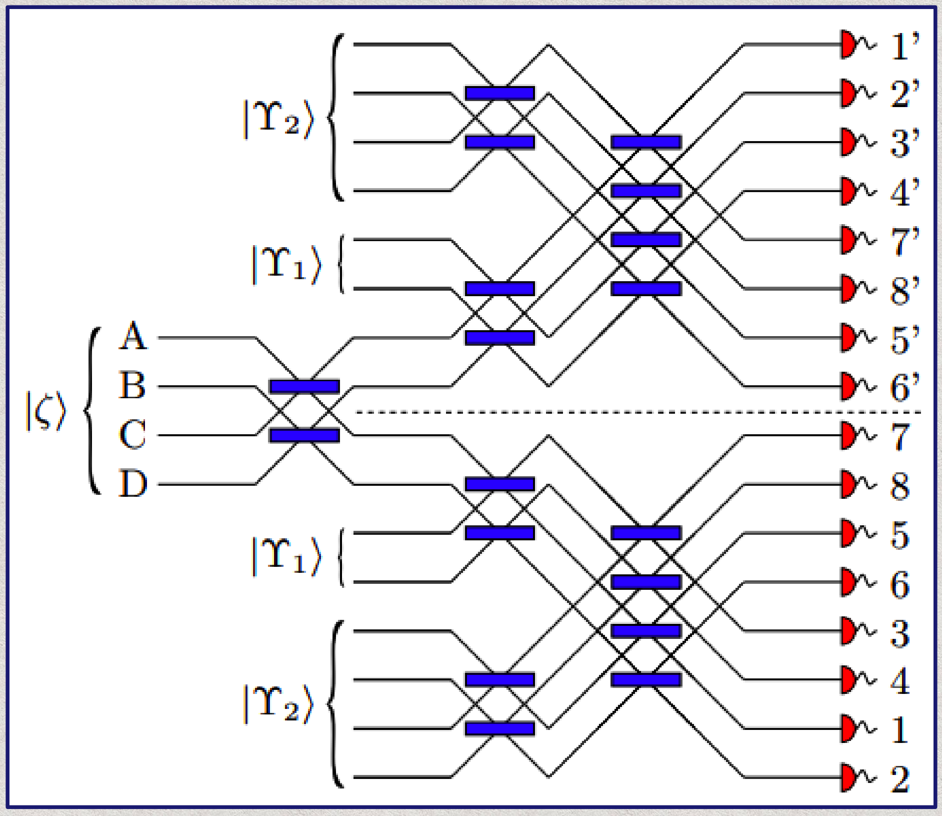
SETUP FOR j = 2



Surpassing the 75% limit with unentangled ancillae

SETUP FOR j = 2

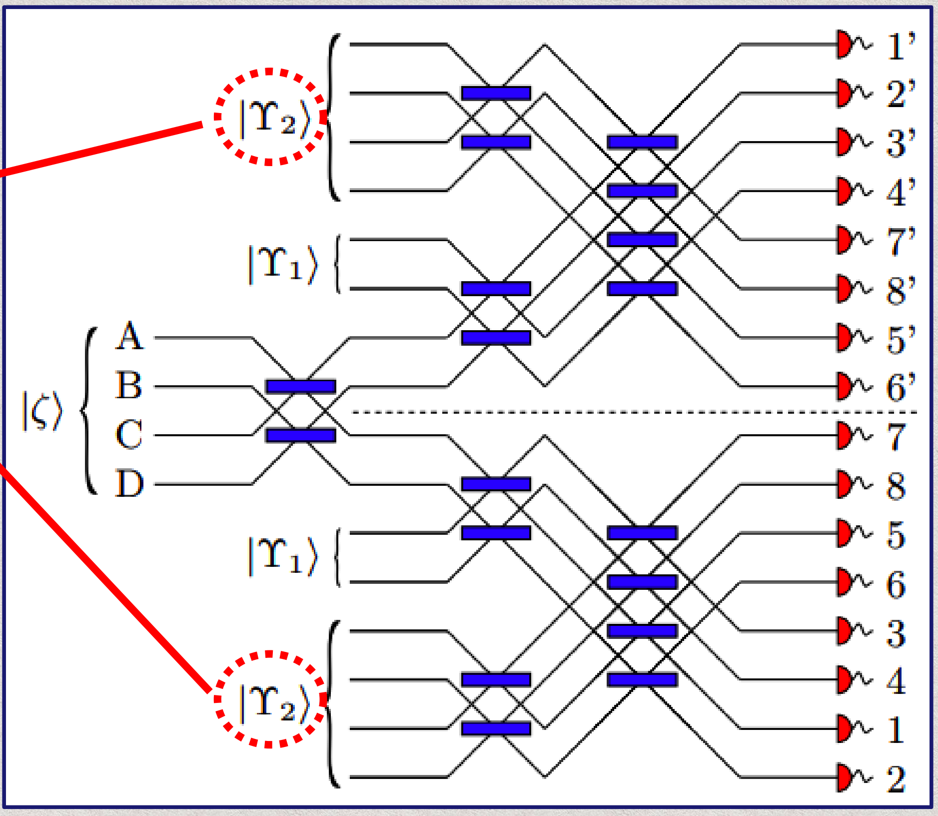
Entangled ancillae:

$$\boxed{|\Upsilon_2\rangle = \frac{1}{\sqrt{2}}\Big(|2020\rangle + |0202\rangle\Big)}$$

replaced with

$$\boxed{|\Upsilon_1\rangle\,|\Upsilon_1\rangle}$$

obtained by sending single photons $|1111\rangle$ through two BSs



Surpassing the 75% limit with unentangled ancillae

They found:

$$\boxed{\mathcal{P}_{succ}^{(N)} = \frac{1+1+\frac{3}{8}+\frac{3}{4}}{4} = \frac{25}{32} \sim 78\%}$$

Although the gain success probability surely is not worth the experimental cost, this shows that no conceptual limit has been found yet!

With analytical and numerical investigation, they found strong hints that the success probability does NOT increase by adding more $|\Upsilon_1\rangle$

BOOSTED
TYPE-II FUSION

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Reference:

From three-photon GHZ states to ballistic universal quantum computation

Mercedes Gimeno-Segovia,[1] Pete Shadbolt,[1] Dan E. Browne,[2] and Terry Rudolph[1]

[1]*Department of Physics, Imperial College London, London SW7 2AZ, United Kingdom*
[2]*Department of Physics and Astronomy, University College London, London WC1E 6BT, United Kingdom*

(Dated: July 17, 2015)

Single photons, manipulated using integrated linear optics, constitute a promising platform for universal quantum computation. A series of increasingly efficient proposals have shown linear-optical quantum computing to be formally scalable. However, existing schemes typically require extensive adaptive switching, which is experimentally challenging and noisy, thousands of photon sources per renormalized qubit, and/or large quantum memories for repeat-until-success strategies. Our work overcomes all these problems. We present a scheme to construct a cluster state universal for quantum computation, which uses no adaptive switching, no large memories, and which is at least an order of magnitude more resource-efficient than previous passive schemes. Unlike previous proposals, it is constructed entirely from loss-detecting gates and offers a robustness to photon loss. Even without the use of an active loss-tolerant encoding, our scheme naturally tolerates a total loss rate $\sim 1.6\%$ in the photons detected in the gates. This scheme uses only 3-GHZ states as a resource, together with a passive linear-optical network. We fully describe and model the iterative process of cluster generation, including photon loss and gate failure. This demonstrates that building a linear optical quantum computer need be less challenging than previously thought.

USEFUL NOTIONS

PERCOLATION PHENOMENON

Prototype phase transition on graphs that, through to a randomized process

1 – p → Lose bonds and/or sites

p → Create bonds and/or sites

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USEFUL NOTIONS

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$\Downarrow$

PERCOLATION THRESHOLD P_C

For $p>p_c$

there exists at least one spanning path from one side of the lattice to the other.

In the context of Fusion-Based Quantum Computation:

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PERCOLATION GRAPH

It defines a **cluster state**, whose bonds/sites are effectively removed due to failure of probabilistic entangling gates together with photon loss.

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In the context of Fusion-Based Quantum Computation:

PERCOLATION GRAPH

It defines a **cluster state**, whose bonds/sites are effectively removed due to failure of probabilistic entangling gates together with photon loss.

PERCOLATION THRESHOLD

It marks a **phase transition** in the computational power of the resource state generated, which distinguishes the states that can be used for **universal quantum computation** from those which cannot.

New percolation approach

3-photon GHZ cluster states are fused together to form a lattice

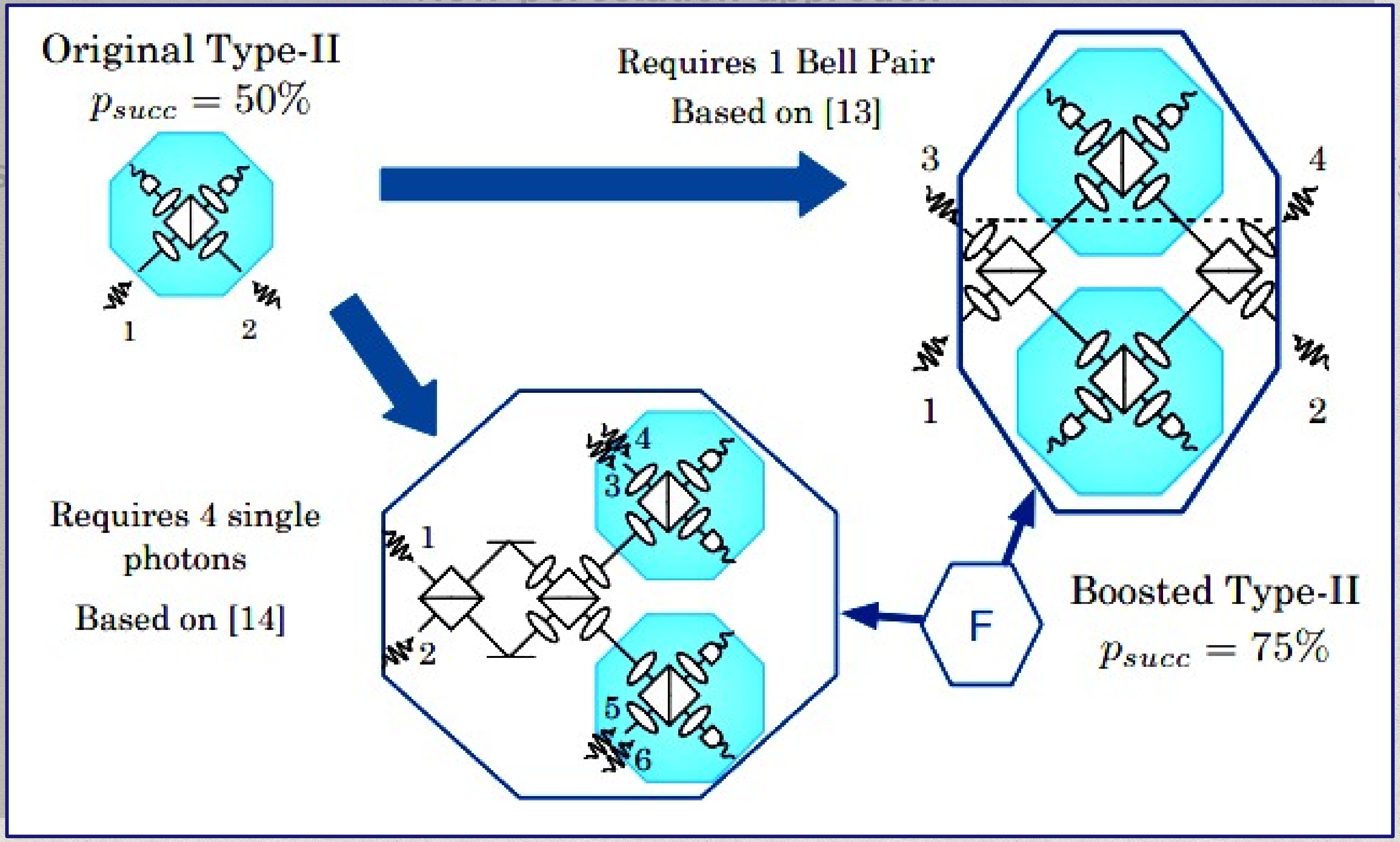
New percolation approach

3-photon GHZ cluster states are fused together to form a lattice

- **Boosted gates:** → Same success and failure outcomes as the original Type-II;
  → Higher success probability.

New percolation approach

▶ Boos … ype-II;



[13] Arbitrarily complete Bell-state measurement using only linear optical elements, *Grice*

[14] ¾ efficient Bell measurement with passive linear optics and unentangled ancillae, *Ewert and van Loock*

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New percolation approach

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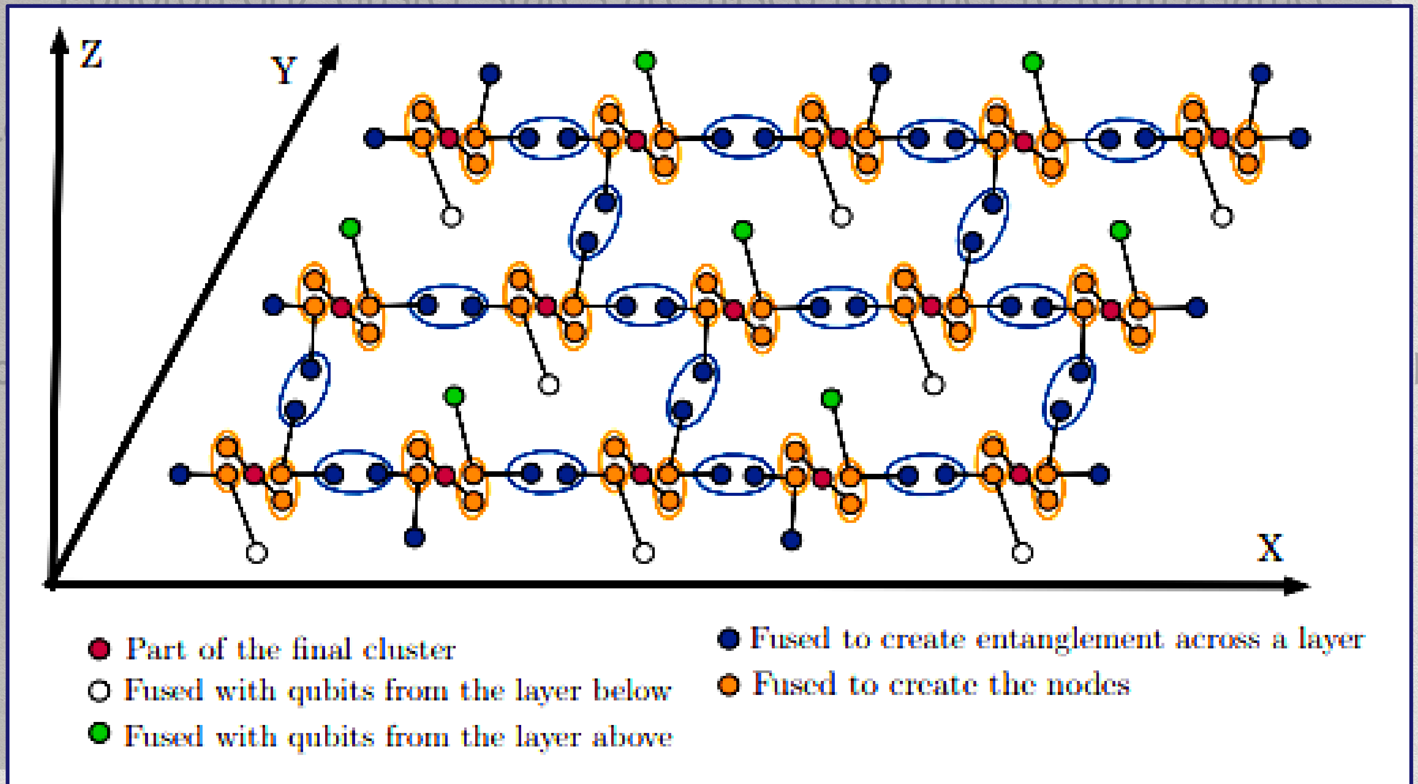
- Diamo... D lattices;



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  → 5-star created for both successful outcomes;
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New percolation approach

- Booste… Type-II;
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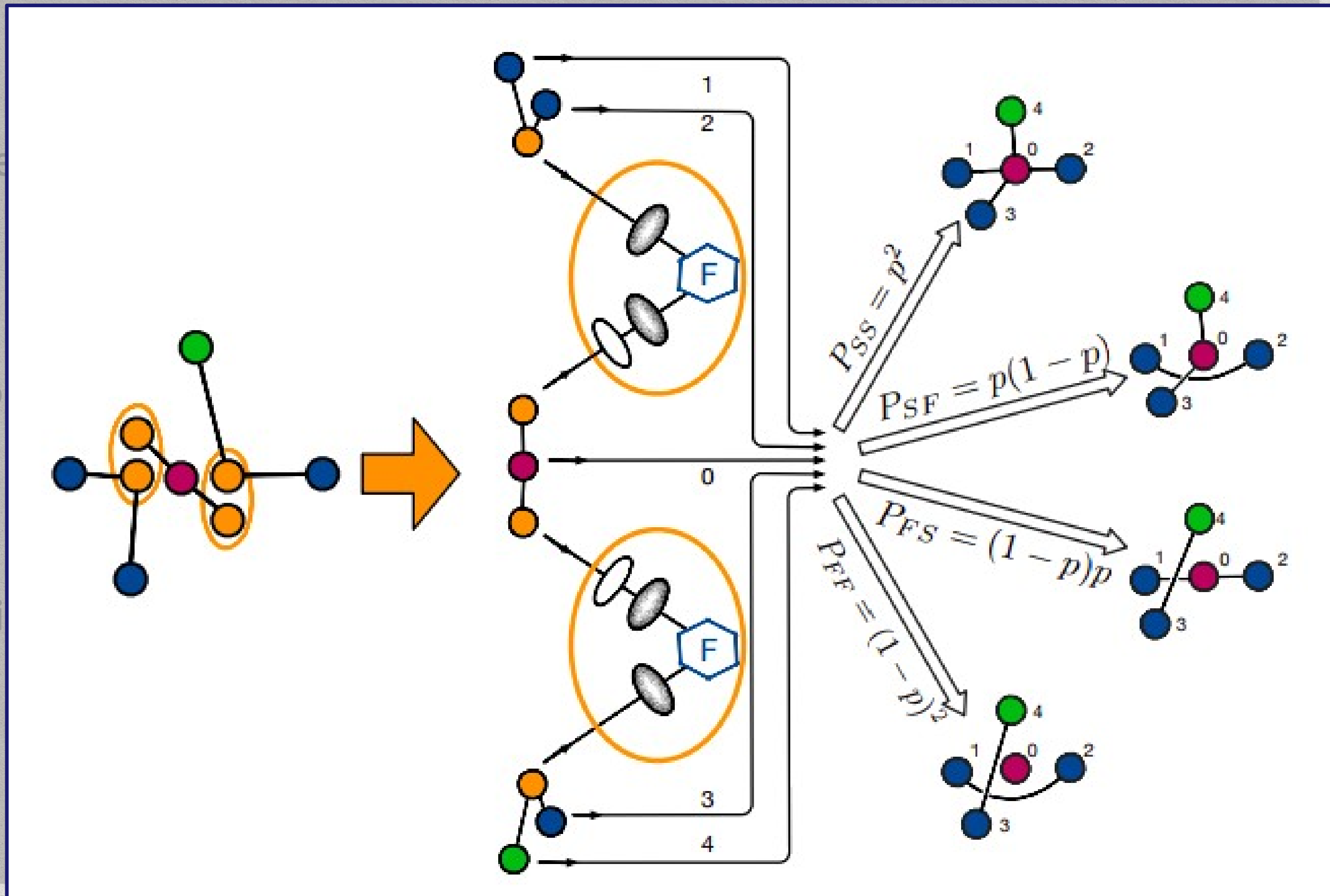

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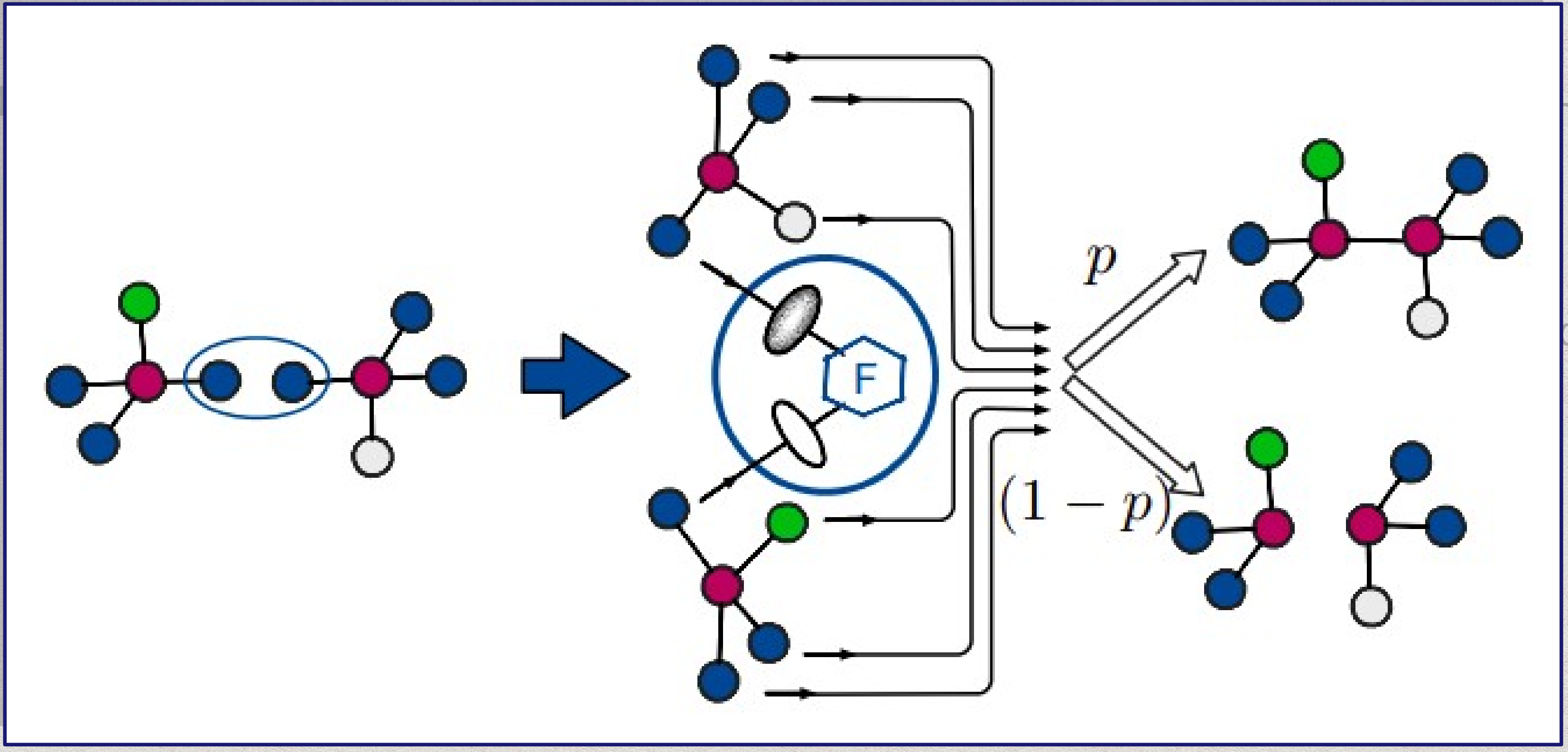


▶ …-II;

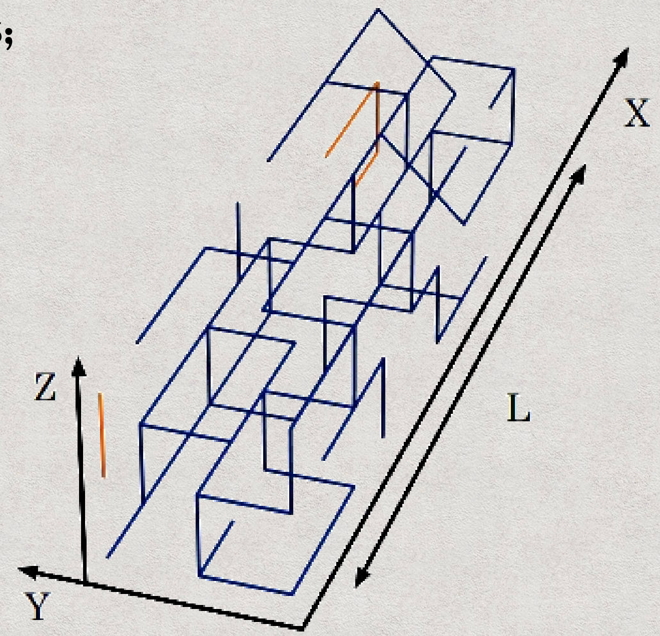
▶ …ces;

▶

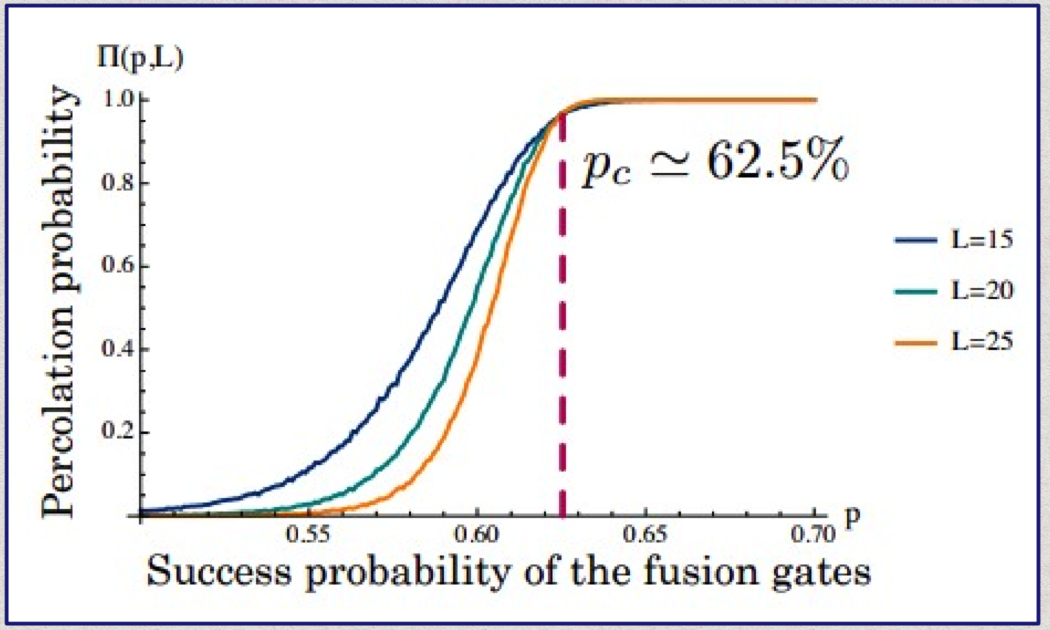
▶ Fusion of 5-qubit microcluster to form the final lattice.

MONTECARLO SIMULATIONS TO ASSESS PERCOLATION PROPERTIES OF THE LATTICE

- Sequential building instead of deleting nodes;
- Diagonal bonds result from failures;
- Lattices can be inferred by a simple classical algorithm.



$$\Pi(p, L) = \text{percolation probability, with: } \begin{cases} p = \text{fusion success probability} \\ L = \text{linear dimension of lattice} \end{cases}$$



Simulations on a bulk of cluster of L= 15, 20, 25. Each cluster contain L^3 sites and has been generated from $3 \cdot L^3$ GHZ states.

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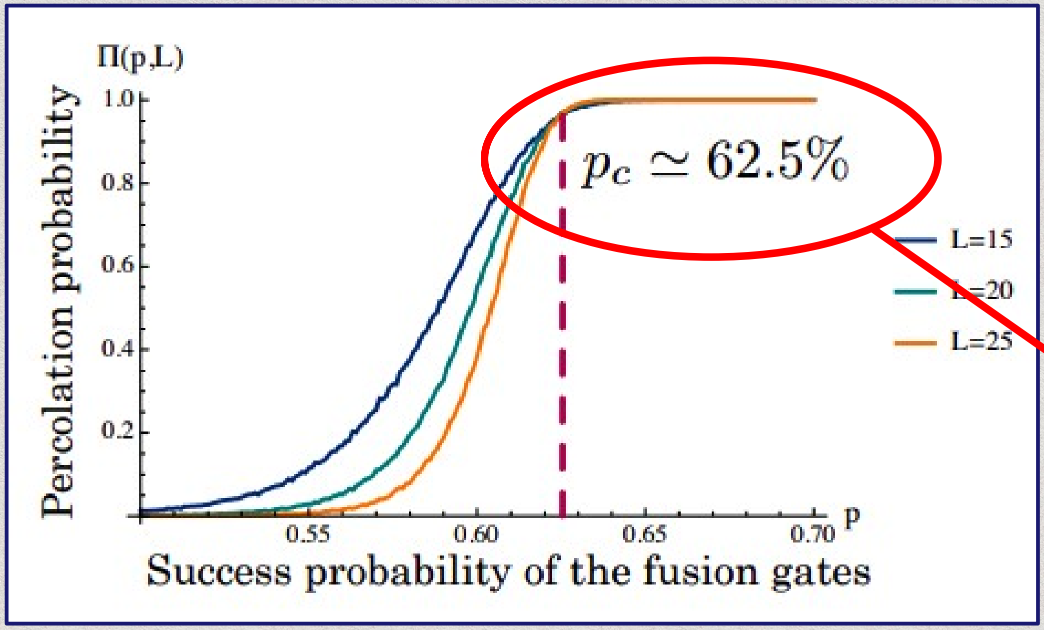

PERCOLATION THRESHOLD

Simulations on a bulk of cluster of L= 15, 20, 25. Each cluster contain L^3 sites and has been generated from $3 \cdot L^3$ GHZ states.

$$\boxed{\begin{cases} p_c \simeq 62.5\% \\ p_{boosted} = 75\% \end{cases} \implies p_{boosted} > p_c}$$

⇒ UNIVERSAL FOR QUANTUM COMPUTATION!

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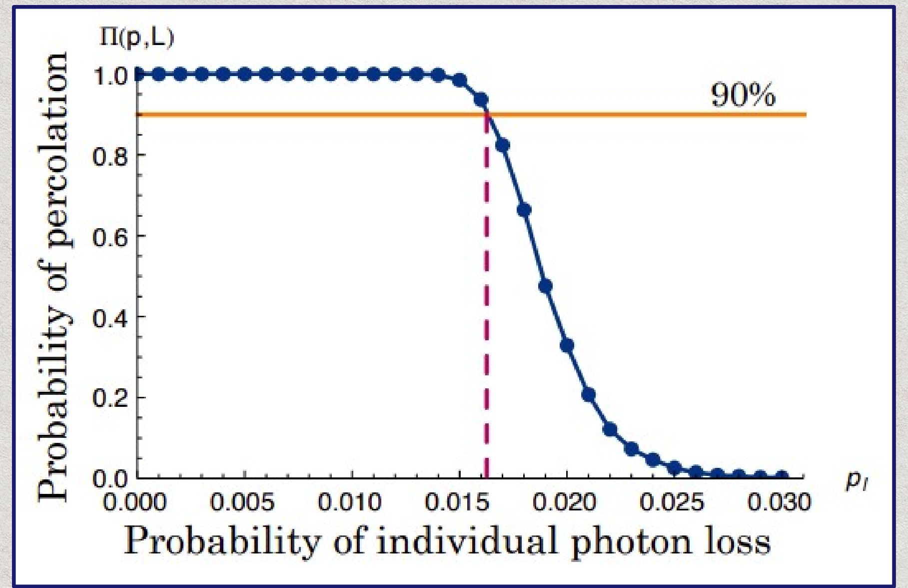
⇒ UNIVERSAL FOR QUANTUM COMPUTATION!

⇒ TOLERANCE TO PHOTON LOSS:

- Boosted fusion gates can detect losses
- This scheme works with a presence of photon loss up to 1.6%



Thanks for the Attention!