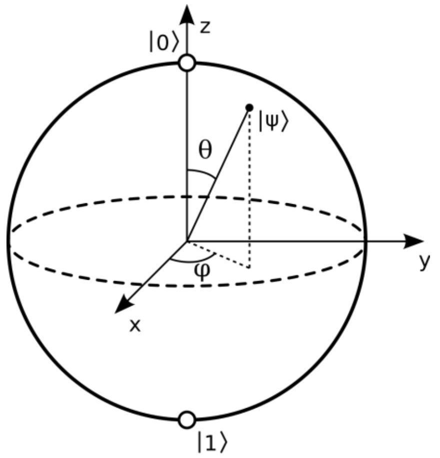


Quantum teleportation (and variations)

Ernesto F. Galvão (INL, UFF)

INTERNATIONAL IBERIAN NANOTECHNOLOGY LABORATORY

Instituto de Física
Universidade Federal Fluminense

LOQC group seminar, 23/7/2020

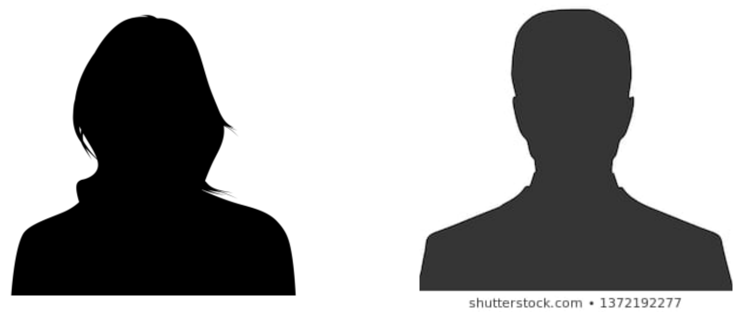
Quantum and Linear-Optical Computation (QLOC)

Ernesto Galvão
(from July 2019)
Group leader

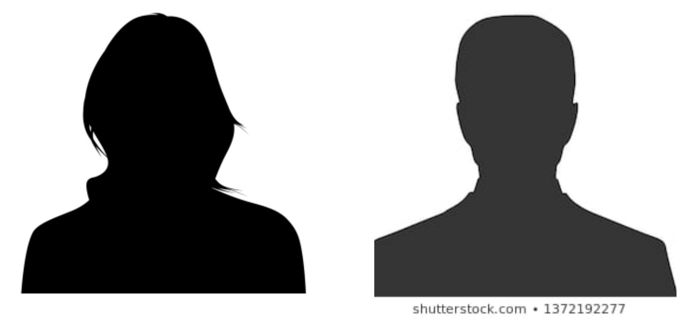
Rui Soares Barbosa
Staff Researcher

Raffaele Santagati
Staff Researcher

Carlos Diogo Fernandes
PhD student
co-supervised with Nuno Peres



+ 2-4 PhD students
(2020/2021)



+ 2 postdocs, starting in
2020/2021

+ 5 Master's students
(2020/2021)

Outline

- Original teleportation protocol
- Post-selected teleportation
- One-bit teleportation and gate teleportation
- Port-based teleportation

Teletransporte quântico

Precisamos recriar à distância estado original, destruindo-o e sem obter nenhuma informação sobre ele.

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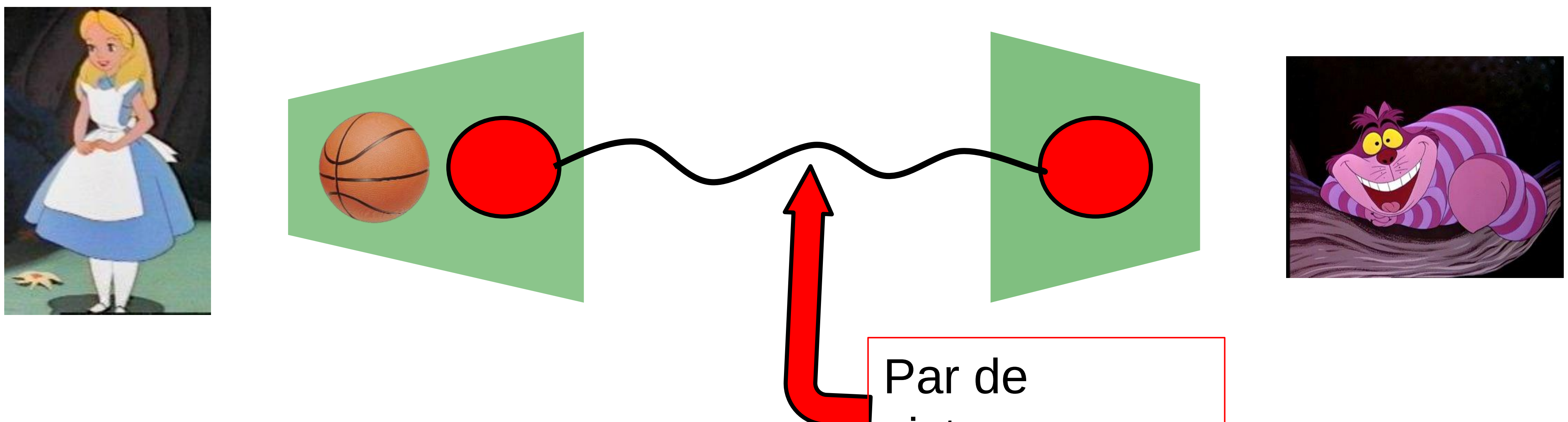

Protocolo de teletransporte: (Bennett et al., 1993)

1- A e B dispõem de par de partículas emaranhadas.

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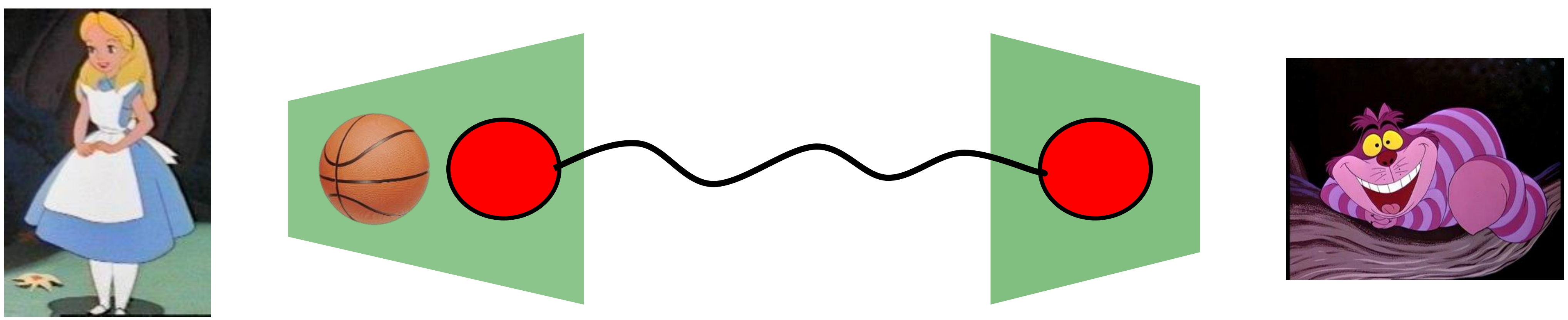

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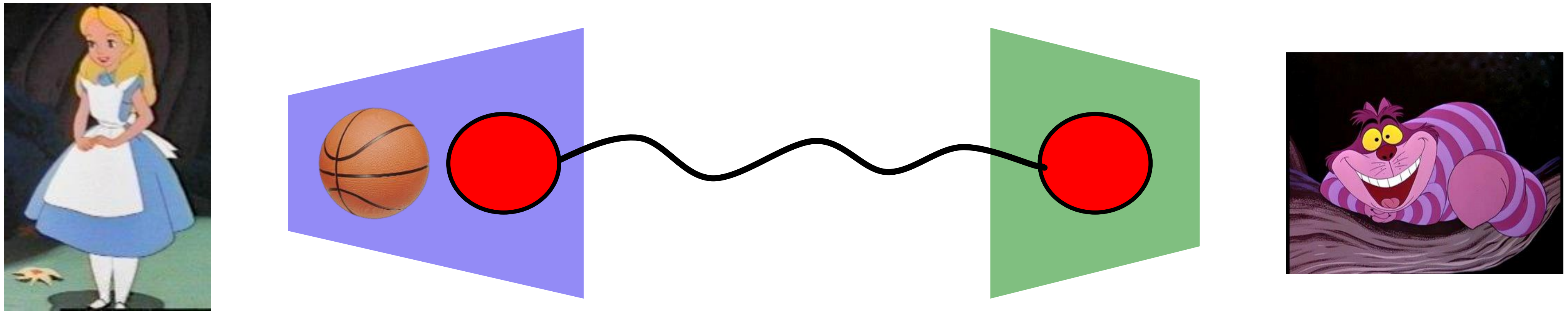

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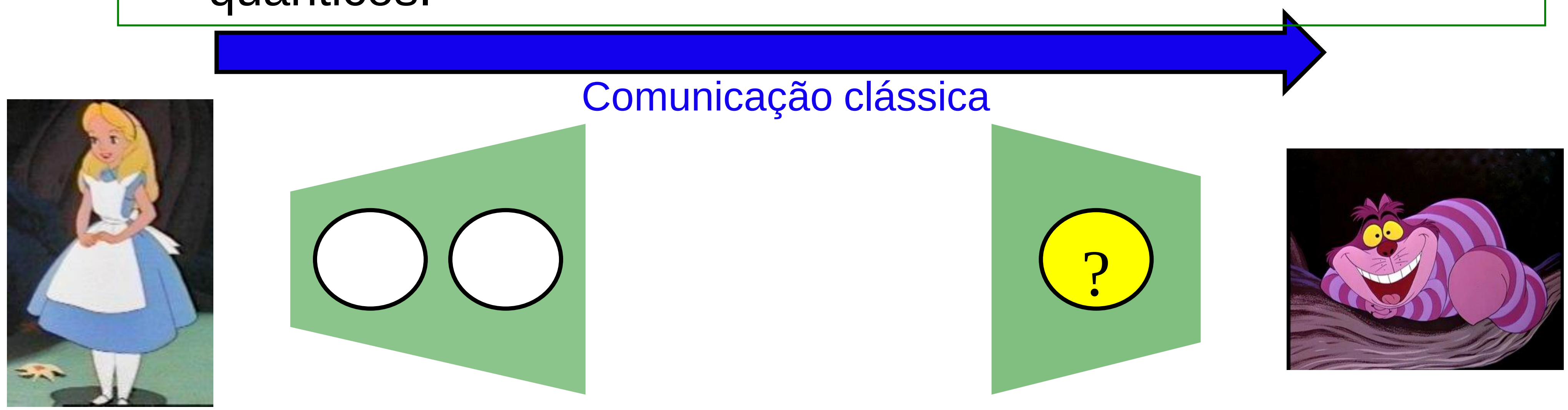

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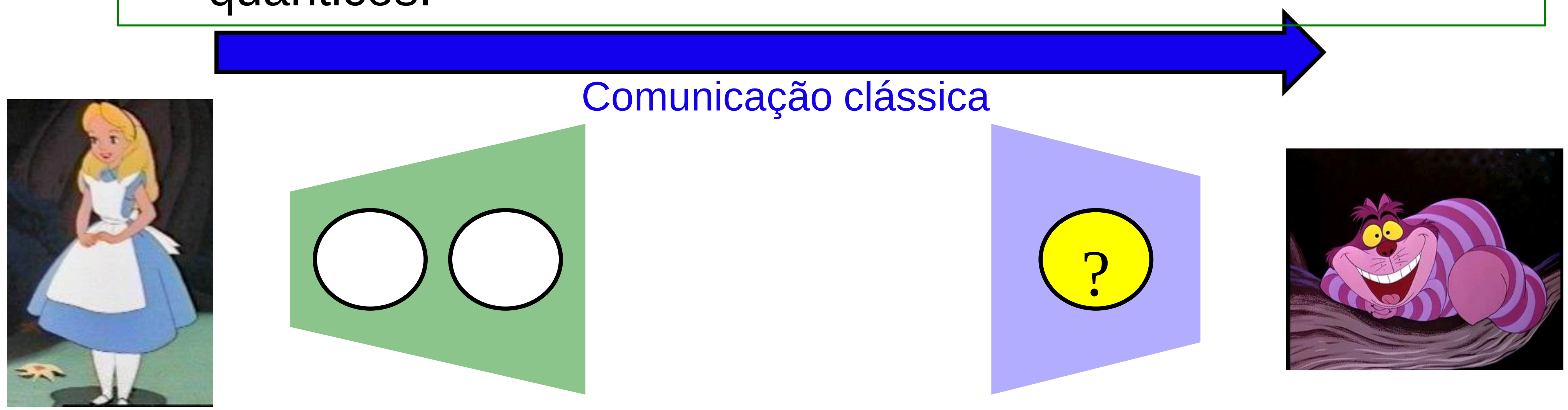

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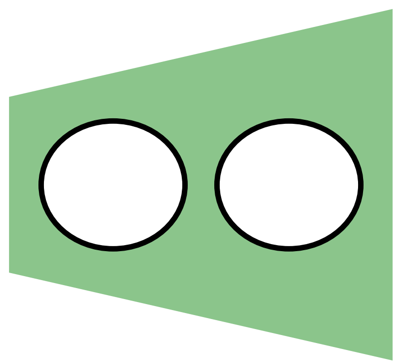

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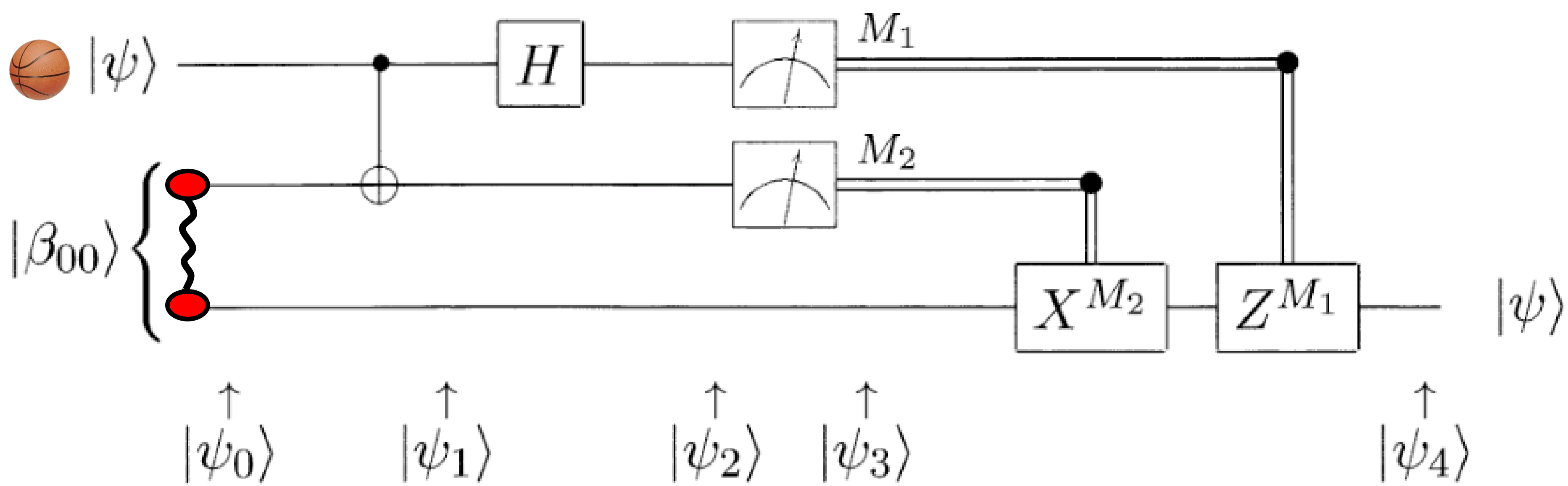
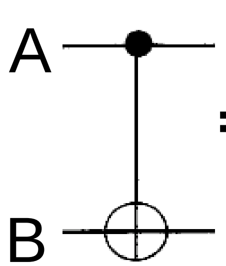
Teletransporte quântico, passo a passo



- Estado inicial:

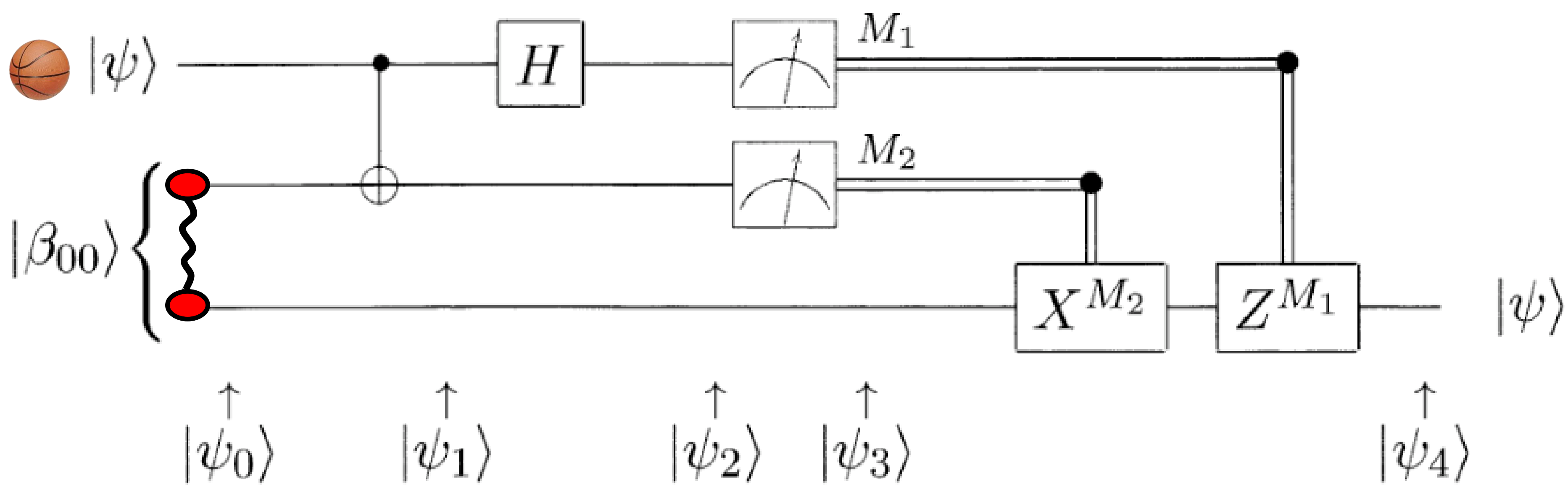
$$|\psi_0\rangle = \underbrace{\left(\alpha|0\rangle + \beta|1\rangle\right)}_{|\psi\rangle}\underbrace{\frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)}_{|\beta_{00}\rangle} = \frac{1}{\sqrt{2}}\left[\alpha|0\rangle(|00\rangle + |11\rangle) + \beta|1\rangle(|00\rangle + |11\rangle)\right]$$



$$= CNOT_{A\rightarrow B}$$

$$\xrightarrow{CNOT} |\psi_1\rangle = \frac{1}{\sqrt{2}}\left[\alpha|0\rangle(|00\rangle + |11\rangle) + \beta|1\rangle(|10\rangle + |01\rangle)\right]$$

Teletransporte quântico, passo a passo



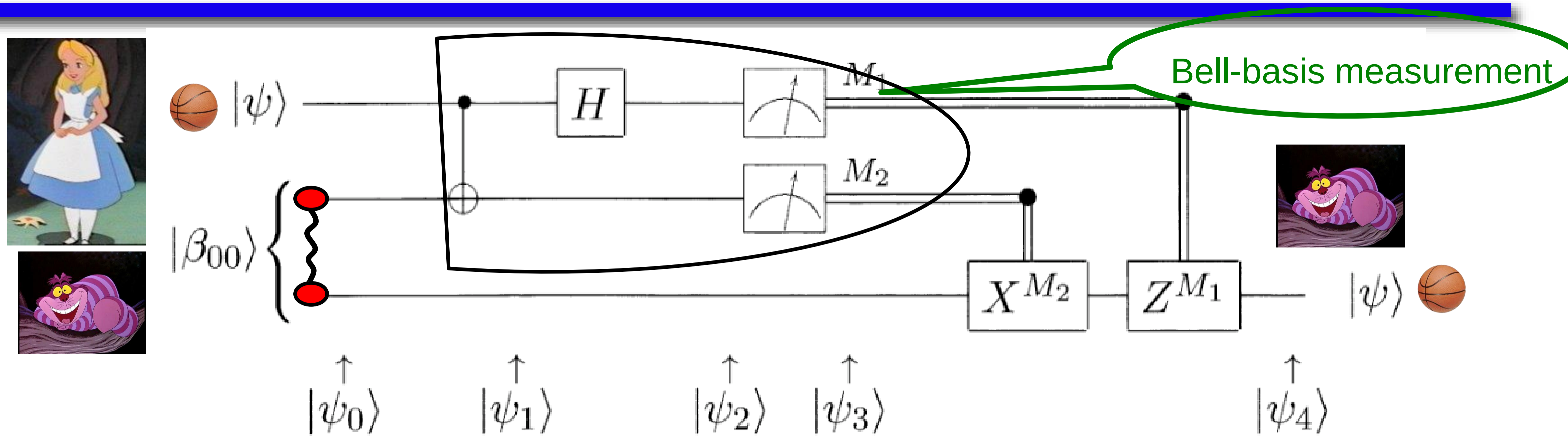
$$|y_1\rangle = \frac{1}{\sqrt{2}}\left[a|0\rangle(|00\rangle + |11\rangle) + b|1\rangle(|10\rangle + |01\rangle)\right]$$

$$-\boxed{H}- = Hadamard = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$\xrightarrow{H} |\psi_2\rangle = \frac{1}{\sqrt{2}}\left[\alpha(|0\rangle + |1\rangle)(|00\rangle + |11\rangle) + \beta(|0\rangle - |1\rangle)(|10\rangle + |01\rangle)\right]$$

$$= \frac{1}{2}\begin{bmatrix} |00\rangle(\alpha|0\rangle + \beta|1\rangle) + |01\rangle(\alpha(|1\rangle + \beta|0\rangle) \\ +|10\rangle(\alpha(|0\rangle - \beta|1\rangle) + |11\rangle(\alpha(|1\rangle - \beta|0\rangle) \end{bmatrix}$$

Teletransporte quântico, passo a passo

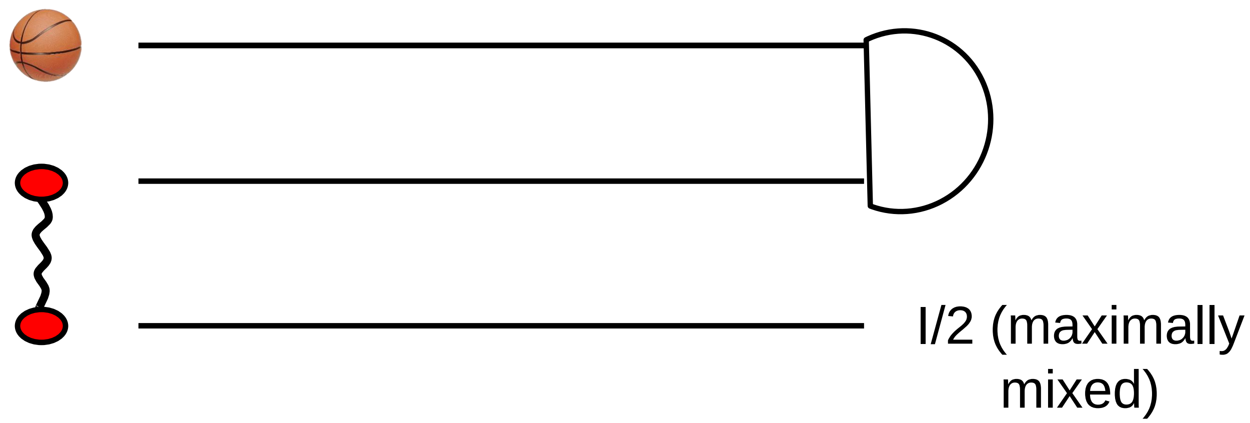


$$|y_2\rangle = \frac{1}{2}\Big[|00\rangle(a|0\rangle + b|1\rangle) + |01\rangle(a(|1\rangle + b|0\rangle)) + |10\rangle(a(|0\rangle - b|1\rangle)) + |11\rangle(a(|1\rangle - b|0\rangle))\Big]$$

- Medidas: resultados M_1M_2 e estados $|y_3(M_1M_2)\rangle$ em cada caso:

$$\left.\begin{array}{l}
00 \longmapsto |\psi_3(00)\rangle \equiv \big[\alpha|0\rangle + \beta|1\rangle\big] \xrightarrow{1} |\psi\rangle \\
01 \longmapsto |\psi_3(01)\rangle \equiv \big[\alpha|1\rangle + \beta|0\rangle\big] \xrightarrow{X} |\psi\rangle \\
10 \longmapsto |\psi_3(10)\rangle \equiv \big[\alpha|0\rangle - \beta|1\rangle\big] \xrightarrow{Z} |\psi\rangle \\
11 \longmapsto |\psi_3(11)\rangle \equiv \big[\alpha|1\rangle - \beta|0\rangle\big] \xrightarrow{ZX} |\psi\rangle
\end{array}\right\} = Z^{M_1} X^{M_2} |\psi_3\rangle =$$

What if we don't correct?



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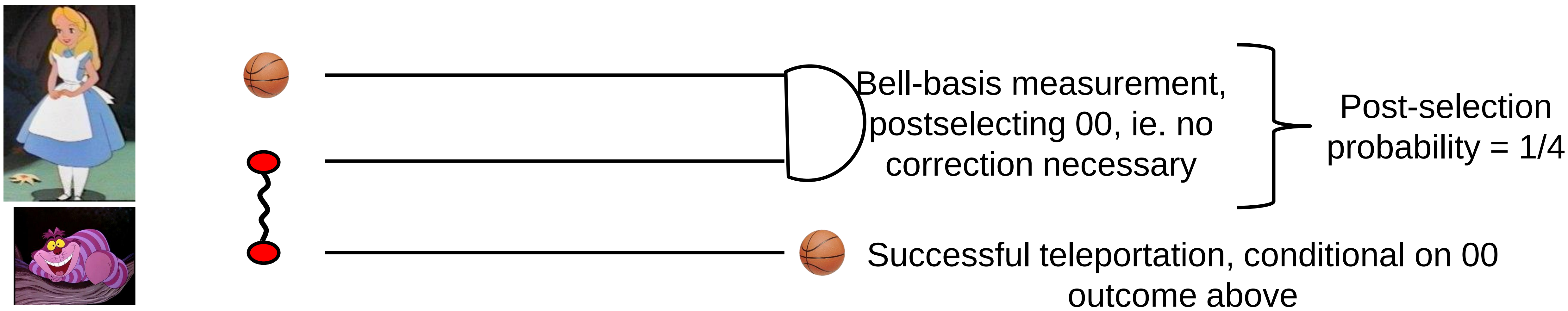
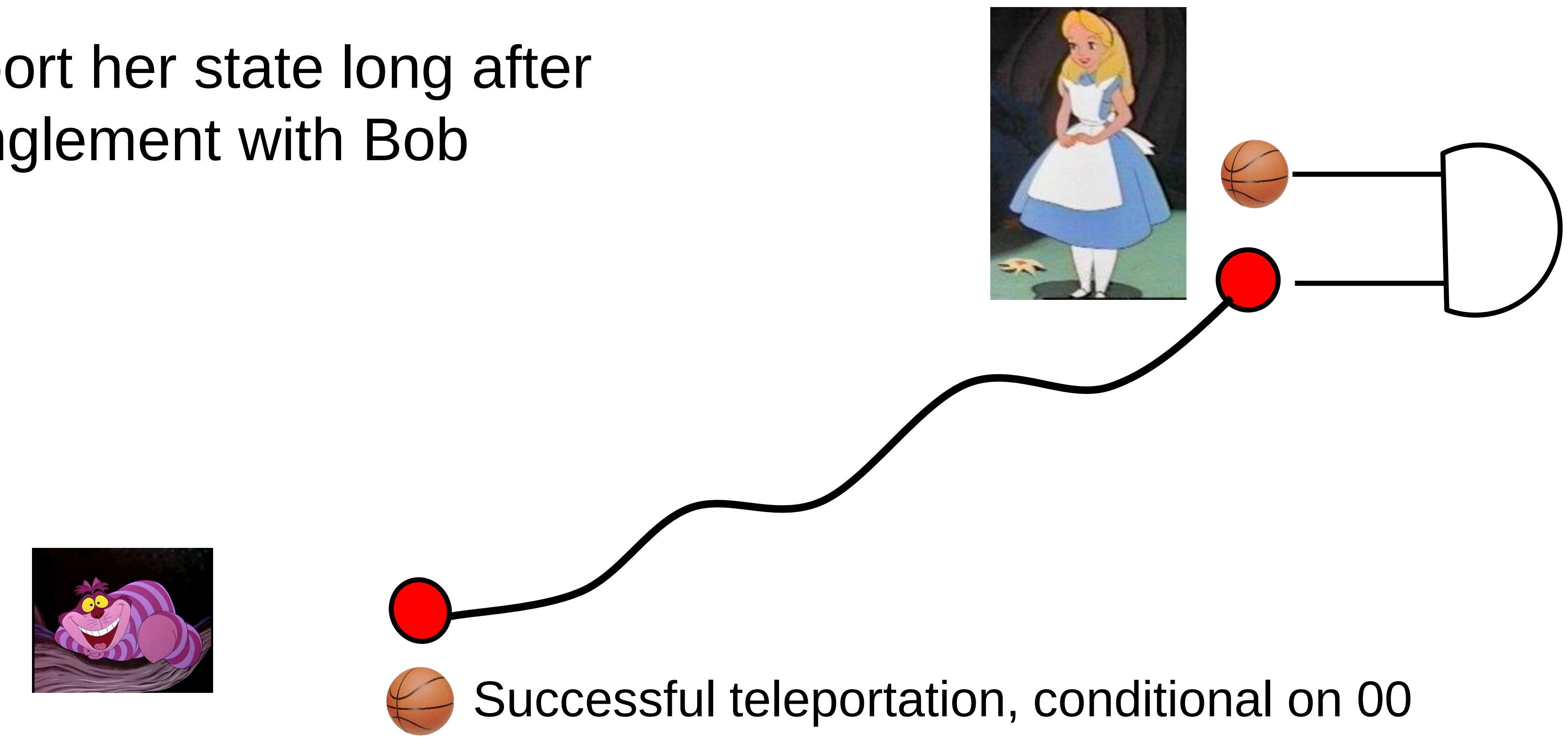


What if we don't correct?

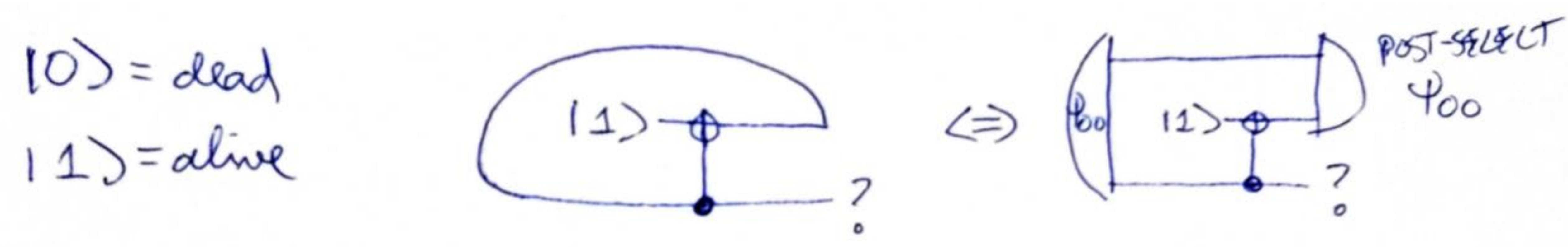
- Alice can try to teleport her state long after having shared entanglement with Bob



Successful teleportation, conditional on 00 outcome **of later measurement**

- Post-selected teleportation is a model for time-travel in quantum theory
  - It simulates (with limited success) what a "real" closed timelike curve (CTC) would do deterministically
  - Possible interactions between time-travelling and time-respecting "twins" may limit the prob. of success

Grandfather paradox with postselected teleportation



Grandfather paradox: travel back in time to kill your ancestors.

- Above: simulation via postselected teleportation
- In general, input-output map is non-linear, due to the postselection – nonlinear extension of QM
- What's the prediction for the output of this paradoxical situation?

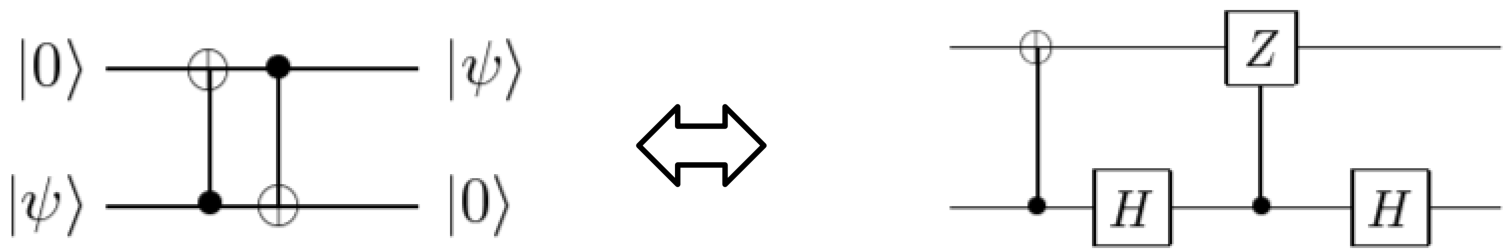
- Postselection happens with probability **zero** – quantum mechanics refuses to say what would happen!
  - Quantum theory automatically identifies paradoxes, yielding null postselection probability.
- Quantum mechanics + postselection would be computationally powerful, solving problems in computational complexity class PP (including NP-complete problems)

[S. Aaronson, Proc. Roy. Soc. A 461, 3473 (2005)]

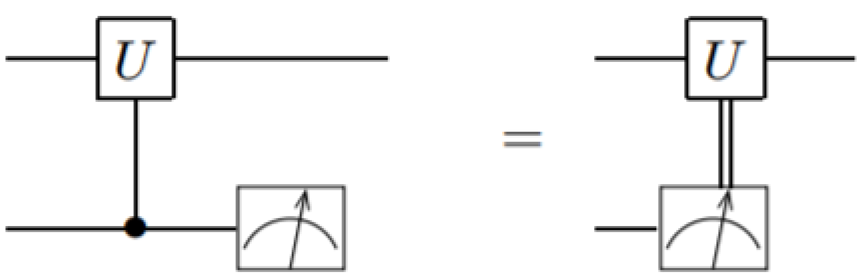
1-bit teleportation

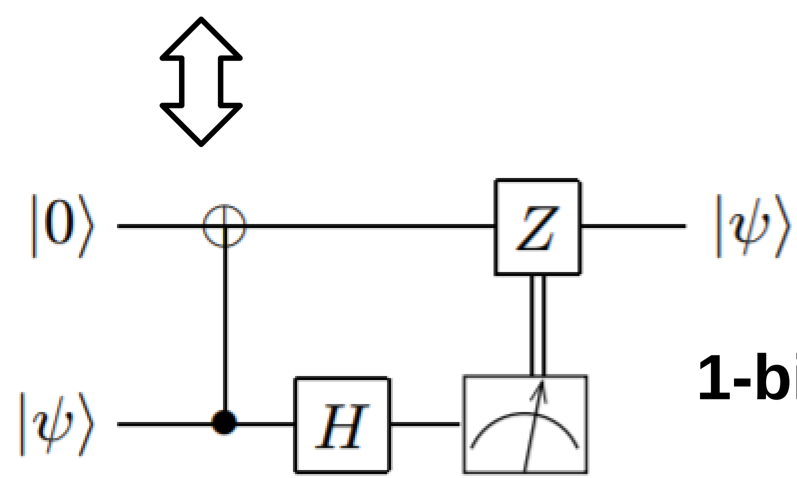
[Zhou, Leung, Chuang PRA 2000]

- Variant of teleportation that is helpful in the description of gate teleportation and measurement-based quantum computation



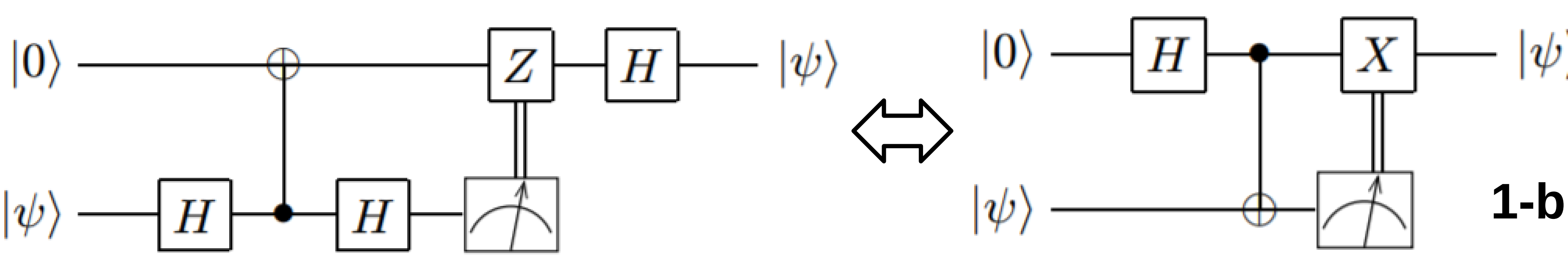
Quantum-controlled gate replaced by measurement + classical control:





1-bit Z teleportation

Now Z-teleport the original state rotated by H (and undo at the end):

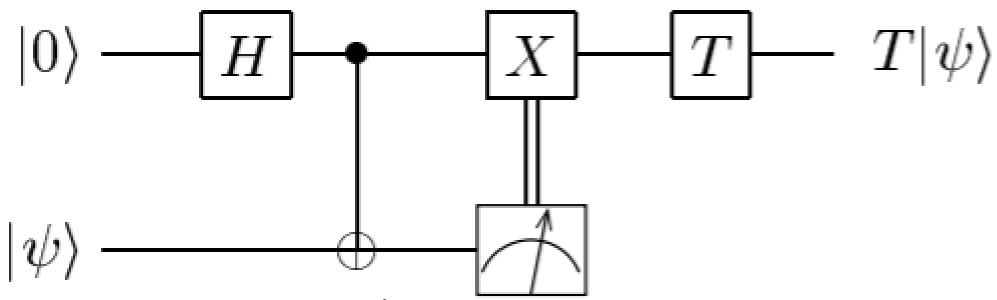


1-bit X teleportation

1-bit (gate) teleportation

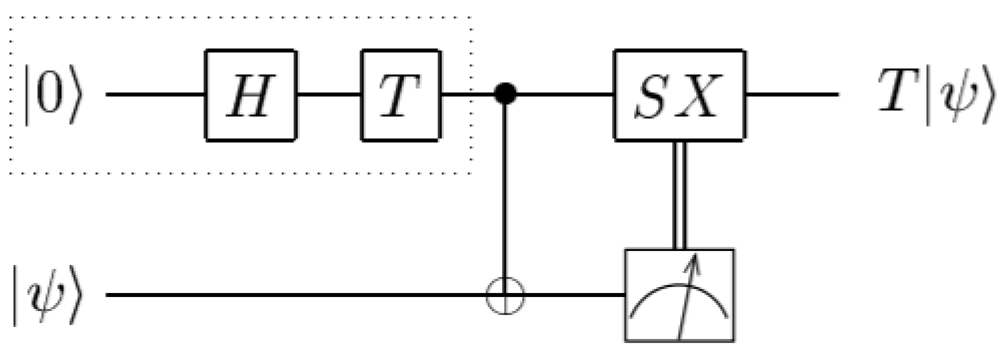
[Zhou, Leung, Chuang PRA 2000]

- Let's introduce gate teleportation as a variation of the 1-bit X teleportation circuit:



$$T = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$$

$$TXT^\dagger = e^{-i\pi/4} SX$$



- Alternatively, replace box by magic state auxiliary state:

$$|\phi_+\rangle = TH|0\rangle = \frac{|0\rangle + e^{i\pi/4}|1\rangle}{\sqrt{2}}$$

- Note: by using only Clifford unitaries (and classically-controlled Clifford unitaries), we can implement the T gate, thus simulating a universal quantum computer => magic state injection model of QC.

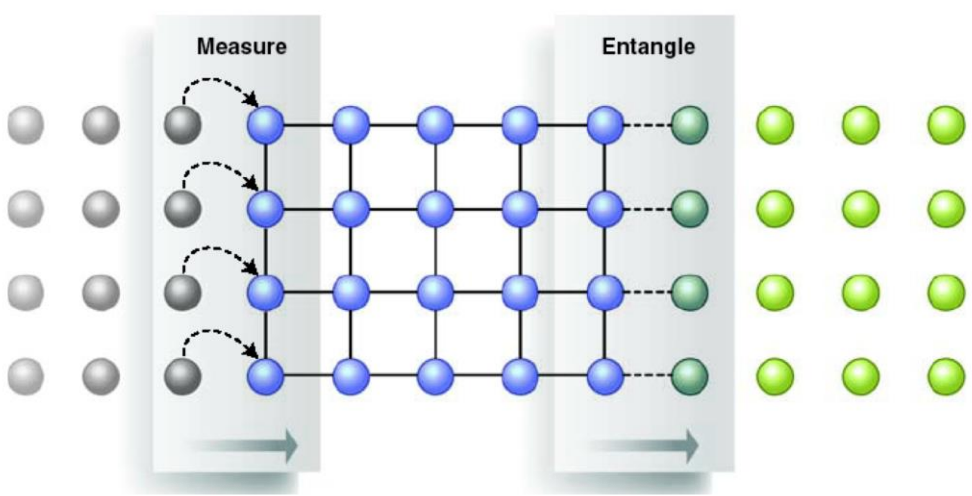
From gate teleportation to MBQC

- Gate teleportation is the key idea enabling measurement-based quantum computation (MBQC)

MBQC can proceed either by:

- Alternating entangling gates, adaptive measurements

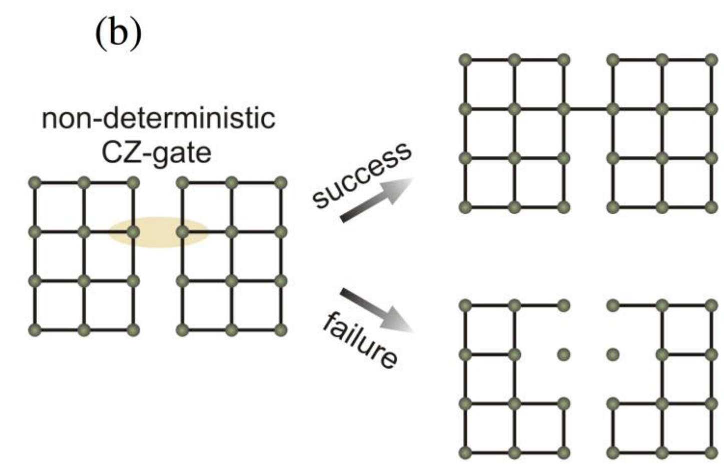
- Advantages: flying qubits, little time for decoherence



from: O'Brien, Science 318, 1467 (2007)

- All entangling gates first (creating highly entangled states)
- Followed by adaptive measurements

- Advantages: small depth, suitable to e.g. atoms in optical lattices



from: Briegel *et al., Nat. Phys.* 5 (1), 19 (2009)

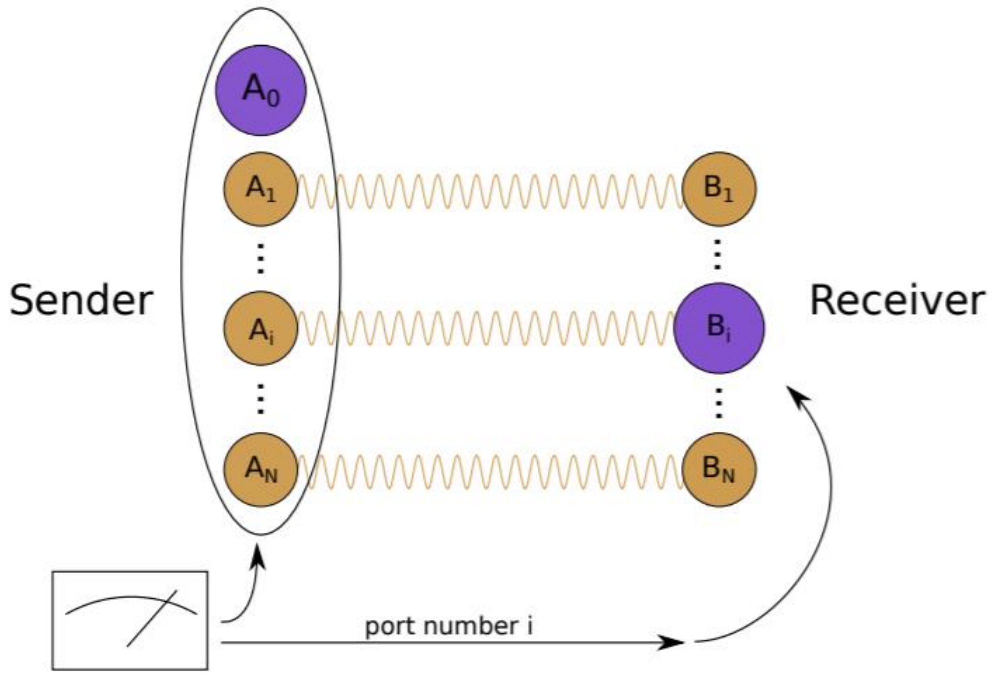
Port-based teleportation

[Ishizaka, Hiroshima PRL 2008]

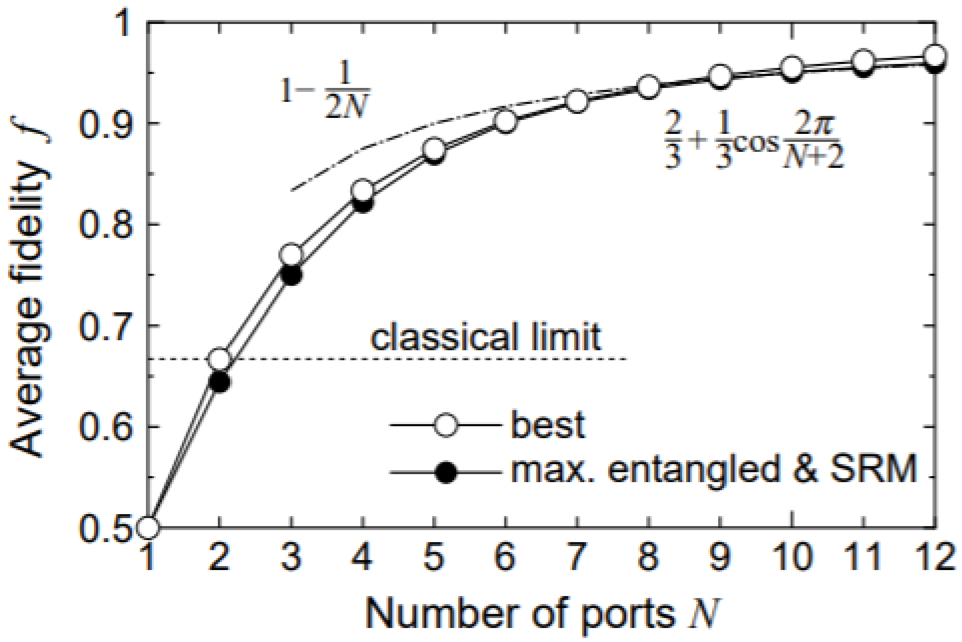
- Joint measurement of Alice's original system + part of an entangled state
- Alice sends outcome to Bob

- Bob picks the subsystem indicated by Alice, and there is the teleported state

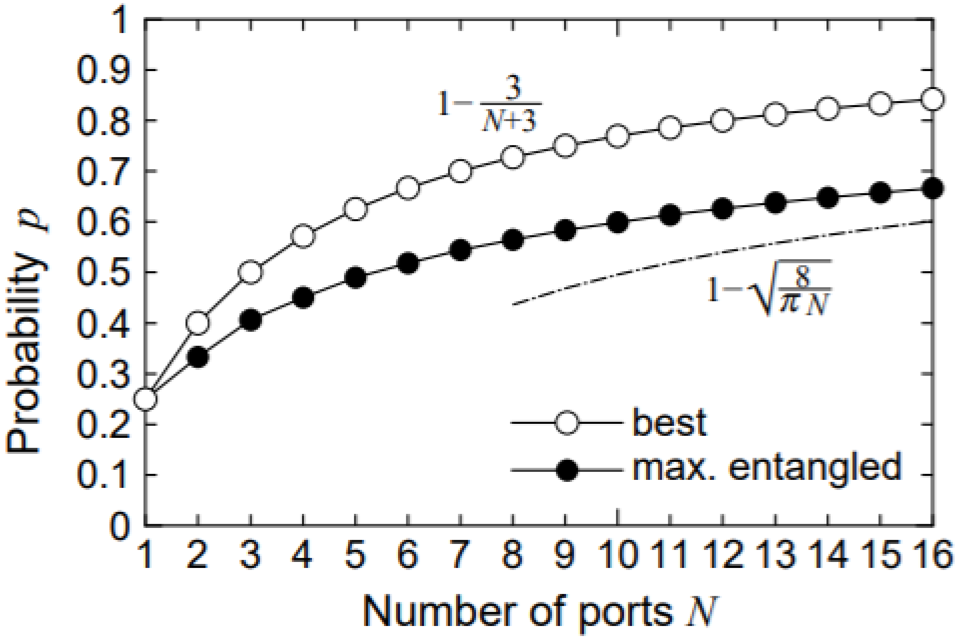
=>"correction" consists in choosing the appropriate subsystem



- Deterministic case: fidelity increases with number of EPR pairs N



- Probabilistic scheme: heralded failure, but when successful fidelity =1.



Thank you for your attention!