# Noncontextuality Inequalities from Antidistinguishability

Journal Club–QLOC
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# Outline I



Necessary Definitions

- Contextuality scenarios
- Value functions
- Quantum models
- States
- Antidistinguishability
- Pairwise antisets

Main Result: Leifer-Duarte inequalities

- Example: Yu-Oh

Antidistinguishability vs. Coherence-free

# Necessary Definitions

# Contextuality Scenarios

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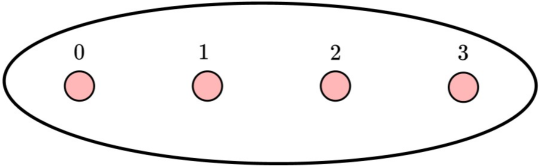
A contextuality scenario is then the triple $\mathfrak{C} := (X, \mathscr{M}, \mathscr{N})$.

# Example: $\mathfrak{C} = (X, X, \emptyset)$

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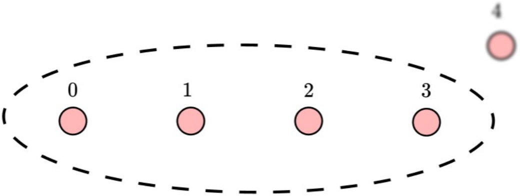


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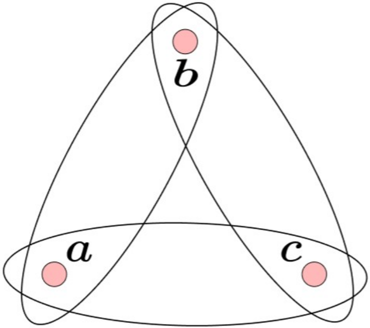
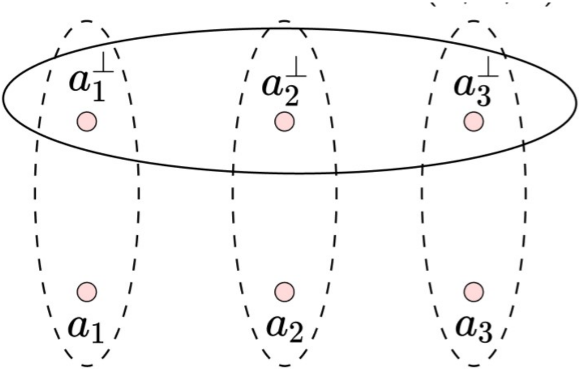
# Example: Specker's triangle



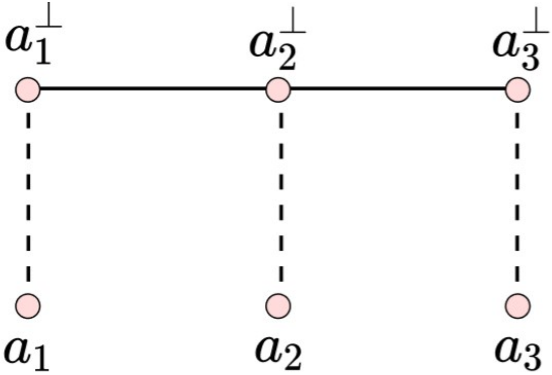



$$\mathfrak{C}_{Sp} = (\{a, b, c\}, \{\{a, b\}, \{a, c\}, \{b, c\}\}, \emptyset)$$

# Examples: Antidistinguishability scenario




$$\mathfrak{C}_{AntD} = (\{a_1, a_2, a_3, a_1^\perp, a_2^\perp, a_3^\perp\},$$
$$\{\{a_1^\perp, a_2^\perp, a_3^\perp\}\}, \{\{a_1, a_a^\perp\}, \{a_2, a_2^\perp\}, \{a_3, a_3^\perp\}\})$$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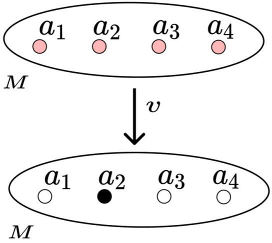

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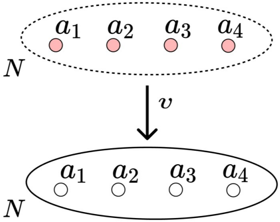

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For instance, take $X = \{a, b, c\}$ this implies that there are $2^3$ functions for this set:

$$(0,0,0)\ \ (0,0,1)\ \ (0,1,0)\ \ (0,1,1)$$
$$(1,0,0)\ \ (1,0,1)\ \ (1,1,0)\ \ (1,1,1)$$

# Value functions are defined for scenarios!



In the scenario $\mathscr{C} = (X, X, \emptyset)$, with $X = \{a, b, c\}$ we have that $\mathscr{M} = X$ so the possible value functions can only be the following:

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We define the set of all possible value functions $v$ in a given $\mathfrak{C}$ by the set $V_{\mathfrak{C}}$.

For the scenario $\mathfrak{C}_{Sp}$ the possible value functions are,

$$\cancel{(0,0,0)}\ \cancel{(0,0,1)}\ \cancel{(0,1,0)}\ \cancel{(0,1,1)}$$
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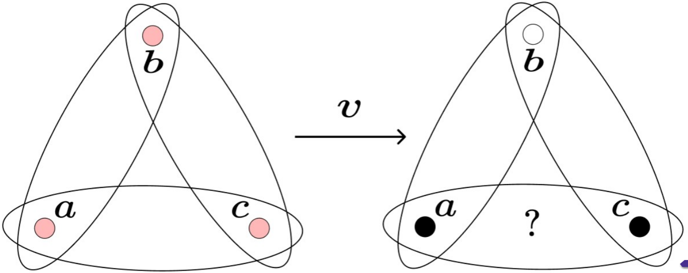

# Quantum models

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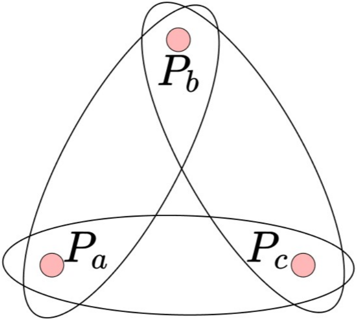
  (b2) $\forall N \in \mathscr{N}$ if $a, b \in N$ then $a \neq b$ implies $P_a P_b = 0$.




If this is possible then we must have that, $P_a + P_b = \mathbb{1}$, $P_a + P_c = \mathbb{1}$ and $P_b + P_c = \mathbb{1}$. But then $P_a = P_b \implies P_a = \mathbb{1}/2$ which is not a projection.

# States: Quantum & Classical

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The set of all states in $\mathfrak{C}$ is denoted $S_{\mathfrak{C}}$.

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A quantum state on a contextuality scenario $\mathfrak{C} = (X, \mathscr{M}, \mathscr{N})$ isa **state** such that there exists a **quantum model** for the scenario together with a state $\rho$ in the Hilbert state of the same model such that

$$\omega(a) = \mathrm{Tr}\{\rho P_a\}.$$

We denote the set of quantum states of $\mathfrak{C}$ by $Q_{\mathfrak{C}}$.

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where $0 \leq p_v \leq 1$, $\sum_{n \in V_{\mathfrak{C}}} p_v = 1$.

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1. The set of all KSNC states is denoted by $C_{\mathfrak{C}}$.
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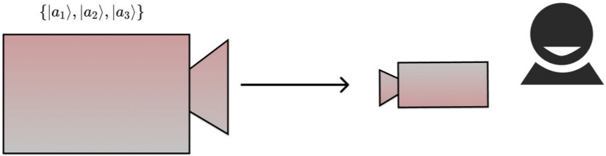
3. For all $\mathfrak{C}$ we have $C_{\mathfrak{C}} \subset Q_{\mathfrak{C}} \subset S_{\mathfrak{C}}$.

# Antidistinguishability

# Antidistinguishability: idea






$$|a_1\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad |a_2\rangle = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad |a_3\rangle = \frac{1}{\sqrt{3}} \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \tag{2}$$

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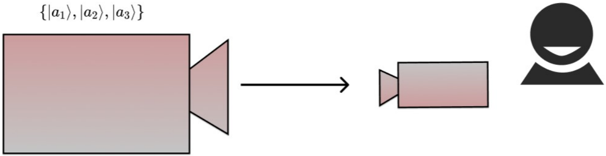


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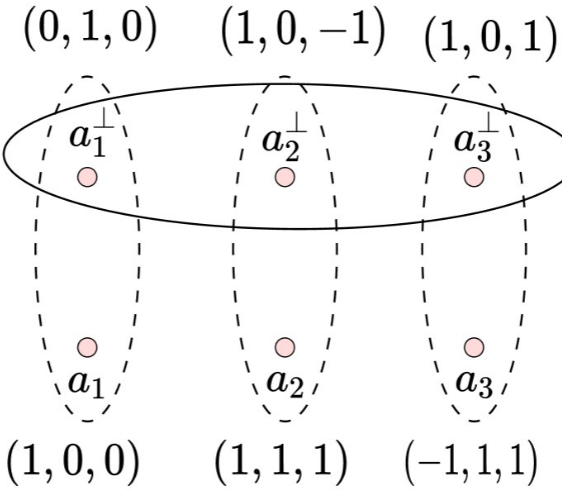

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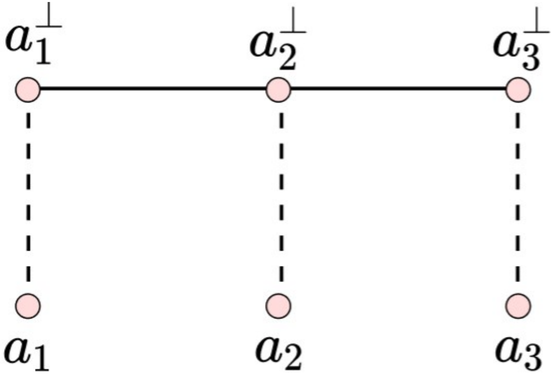
3. For each outcome $a \in M \setminus \{a_1^\perp, \ldots, a_n^\perp\}$, and each $a_j$ there exists a context or a maximal partial context $N$ such that $\{a, a_j\} \subseteq N$. Here for technical necessity.


$a_1^\perp$
$a_2^\perp$
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$a_1$
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$a_3$

**Theorem (Caves-Fuchs-Schack)**

*Let* $X = \{|a_1\rangle, |a_2\rangle, |a_3\rangle\}$. *Then, the set* $X$ *is antidistinguishable* ***if, and only if***

$$r_{12} + r_{13} + r_{23} < 1 \quad (4)$$

$$(r_{12} + r_{13} + r_{23} - 1)^2 \geq 4 r_{12} r_{13} r_{23} \quad (5)$$

*where* $r_{ij} := |\langle a_i | a_j \rangle|^2$.

Corollary

*Every triple of antidistinguishable outcomes* $|a_1\rangle, |a_2\rangle, |a_3\rangle$ *has overlap vectors inside the coherence-free polytope.*

# Pairwise Antisets

# Pairwise antisets



A weak pairwise antiset $W$ would be a set of outcomes such that there exists another outcome $c \in X$ such that any pair $a, b \in W$ forms an antidistinguishable triple with $c$. In other words, $\{a, b, c\}$ is again an antidistinguishable triple.

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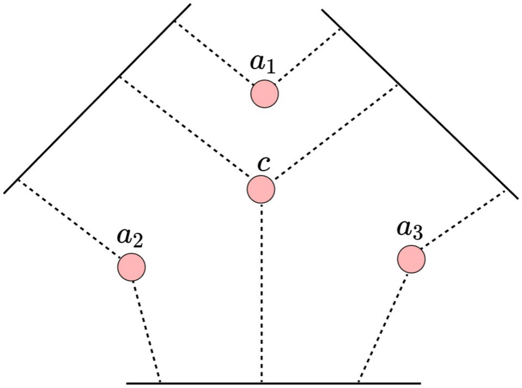

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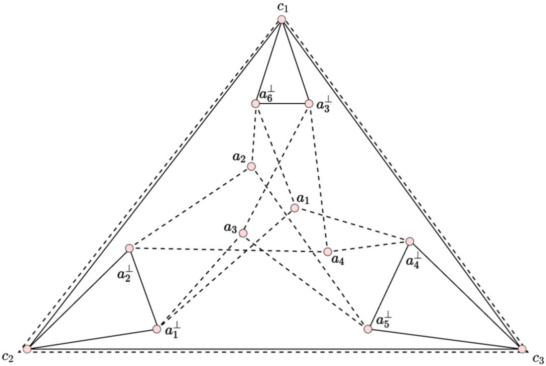
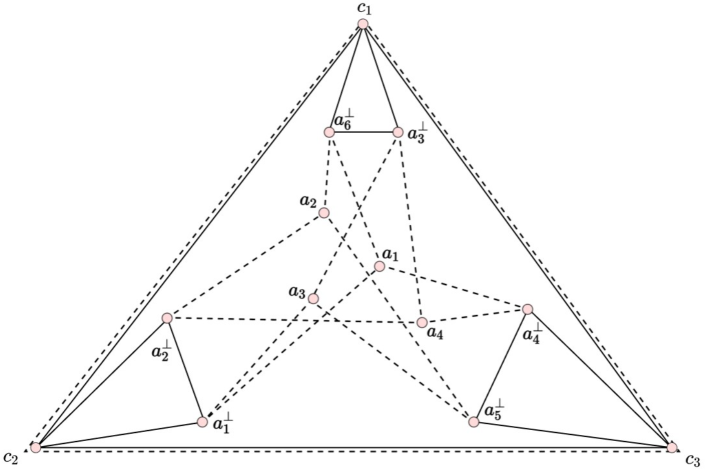


# Strong Pairwise Antiset: Yu-Oh





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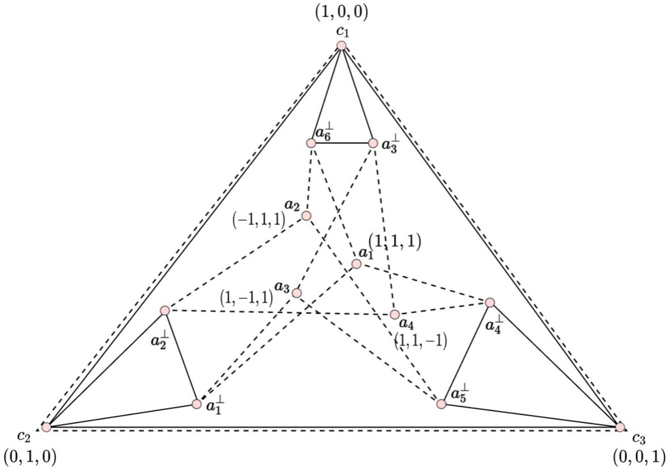

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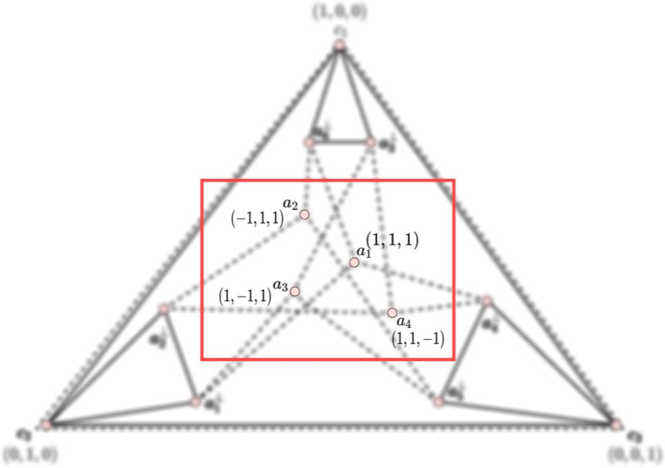

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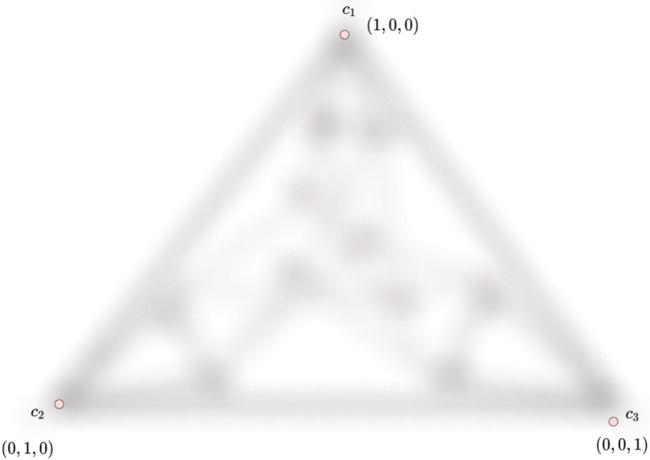

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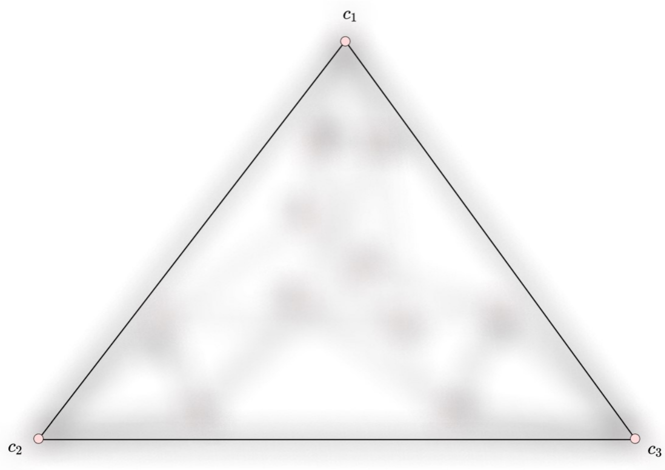

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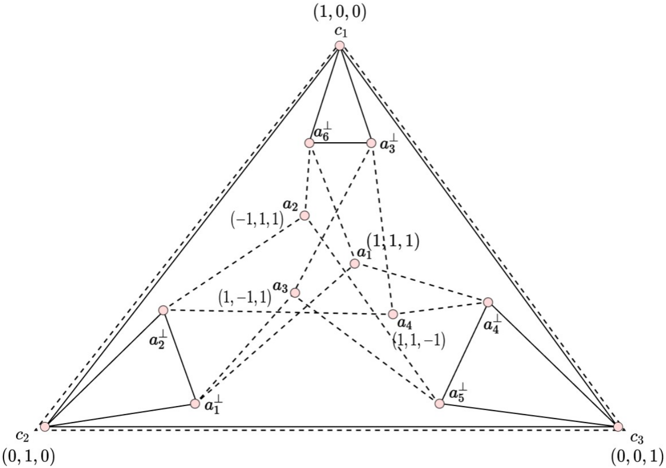

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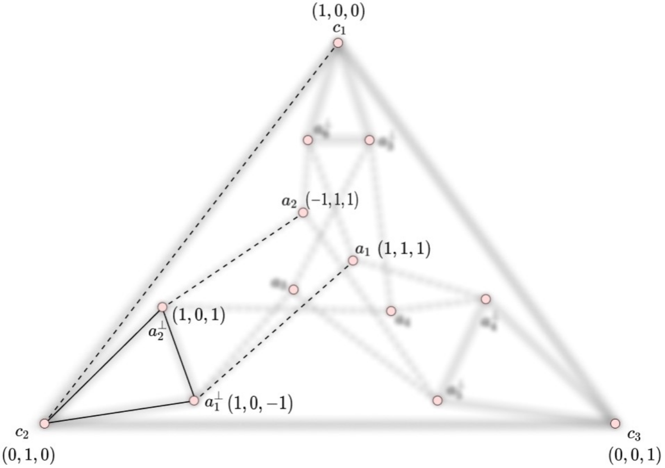
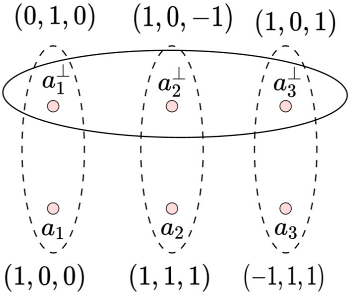
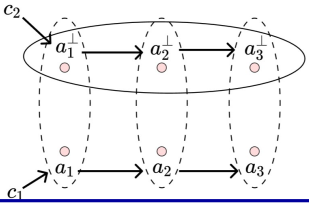




(0, 1, 0)
(1, 0, −1)
(1, 0, 1)
$a_1^\perp$
$a_2^\perp$
$a_3^\perp$
$a_1$
$a_2$
$a_3$
(1, 0, 0)
(1, 1, 1)
(−1, 1, 1)

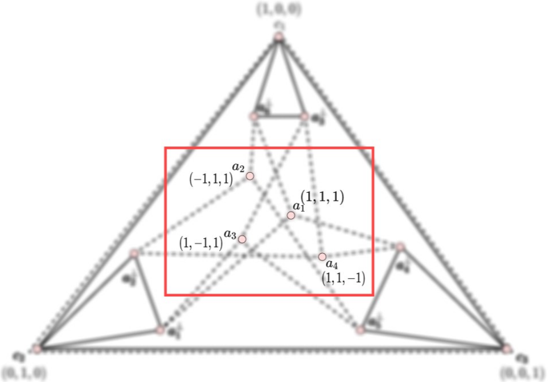



$c_2$
$a_1^\perp$
$a_2^\perp$
$a_3^\perp$
$a_1$
$a_2$
$a_3$
$c_1$

# Strong Pairwise Antiset: Yu-Oh



$$|a_1\rangle = \frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\1\end{pmatrix},\ |a_2\rangle = \frac{1}{\sqrt{3}}\begin{pmatrix}-1\\1\\1\end{pmatrix},\ |a_3\rangle = \frac{1}{\sqrt{3}}\begin{pmatrix}1\\-1\\1\end{pmatrix},\ |a_4\rangle = \frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\-1\end{pmatrix}.$$



# Main Result: Leifer-Duarte inequalities

Theorem (Leifer-Duarte)

*Let $W$ be a strong pairwise antiset in a contextuality scenario $\mathfrak{C}$. Then, for every state $\omega \in C_{\mathfrak{C}}$*

$$\sum_{a \in W} \omega(a) \leq 1 \tag{6}$$

*If $W$ is only a weak pairwise antiset in the contextuality scenario and the principle outcome $c$ is such that $\omega(c) = 1$ then the same inequality holds.*

We have $W = \{a_1, a_2, a_3, a_4\}$ be a strong pairwise antiset of scenario $\mathfrak{C}_{Yu-Oh}$ with principle context given by $\{c_1, c_2, c_3\}$.

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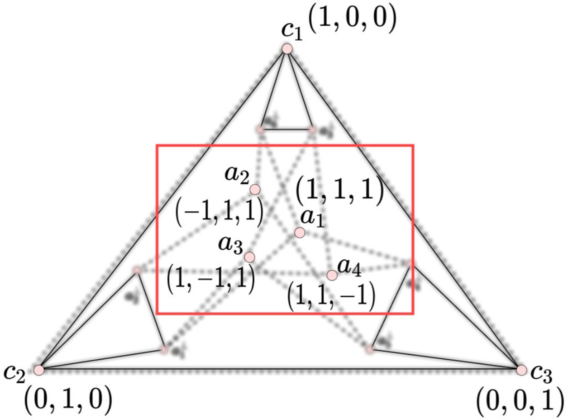

According to the Leifer-Duarte theorem we have that,

$$\sum_{a \in W} \omega(a) = \sum_{i=1}^{4} \omega(a_i) \leq 1 \tag{7}$$

For KSNC states.

$$|a_1\rangle = \frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\1\end{pmatrix},\ |a_2\rangle = \frac{1}{\sqrt{3}}\begin{pmatrix}-1\\1\\1\end{pmatrix},\ |a_3\rangle = \frac{1}{\sqrt{3}}\begin{pmatrix}1\\-1\\1\end{pmatrix},\ |a_4\rangle = \frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\-1\end{pmatrix}.$$

$$c_1 = \begin{pmatrix}1\\0\\0\end{pmatrix},\ c_2 = \begin{pmatrix}0\\1\\0\end{pmatrix},c_3 = \begin{pmatrix}0\\0\\1\end{pmatrix}, \tag{8}$$

$$\sum_{i=1}^{4} |a_i\rangle\langle a_i| =$$

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$$\frac{1}{3}\begin{pmatrix}1\\1\\1\end{pmatrix}\begin{pmatrix}1 & 1 & 1\end{pmatrix} + \frac{1}{3}\begin{pmatrix}-1\\1\\1\end{pmatrix}\begin{pmatrix}-1 & 1 & 1\end{pmatrix} +$$

$$\frac{1}{3}\begin{pmatrix}1\\-1\\1\end{pmatrix}\begin{pmatrix}1 & -1 & 1\end{pmatrix} + \frac{1}{3}\begin{pmatrix}1\\1\\-1\end{pmatrix}\begin{pmatrix}1 & 1 & -1\end{pmatrix} =$$

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$$\frac{1}{3}\begin{pmatrix}1 & 1 & 1\\1 & 1 & 1\\1 & 1 & 1\end{pmatrix} + \frac{1}{3}\begin{pmatrix}1 & -1 & -1\\-1 & 1 & 1\\-1 & 1 & 1\end{pmatrix} + \frac{1}{3}\begin{pmatrix}1 & -1 & 1\\-1 & 1 & -1\\1 & -1 & 1\end{pmatrix} + \frac{1}{3}\begin{pmatrix}1 & 1 & -1\\1 & 1 & -1\\-1 & -1 & 1\end{pmatrix} =$$

$$\frac{4}{3}\begin{pmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{pmatrix}$$

This implies that this pairwise antiset of states reach the following quantum bound, for any state $\rho$.

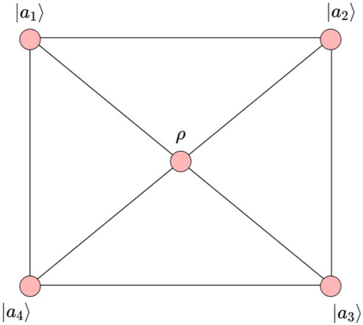
$$\sum_{i=1}^{4} \omega(a_i) = \sum_i \mathrm{Tr}\{\rho|a_i\rangle\langle a_i|\} = \mathrm{Tr}\left\{\rho \sum_i |a_i\rangle\langle a_i|\right\} = \frac{4}{3} > 1 \quad (9)$$

- They show that the same holds for other sets and other dimensions.
- They have proved that this kind of antidistinguishability inequalities of overlaps is associated to noncontextuality inequalities.
- They introduced an interesting framework with new types of scenarios that mixes CSW and AFLS notions.

# Antidistinguishability vs. Coherence-free

# Antidistinguishability vs Coherence-free






$$\sum_{i=1}^{|W|} \omega(a_i) \equiv \sum_i r_{\rho i} \leq 1 \tag{10}$$

# Evidence from $K_5$ graph



$$+r_{\rho a_1}+r_{\rho a_2}+r_{\rho a_3}+r_{\rho a_4}-r_{a_1 a_2}-r_{a_1 a_3}-r_{a_1 a_4}-r_{a_2 a_3}-r_{a_2 a_4}-r_{a_3 a_4} \leq 1$$

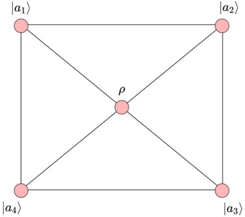
$$\begin{aligned} &+2r_{\rho a_1} + 2r_{\rho a_2} + 2r_{\rho a_3} + 2r_{\rho a_4} - r_{a_1 a_2} \\ &-r_{a_1 a_3} - r_{a_1 a_4} - r_{a_2 a_3} - r_{a_2 a_4} - r_{a_3 a_4} \leq 3 \end{aligned}$$

# Evidence from $K_5$ graph



$$+r_{\rho a_1} + r_{\rho a_2} + r_{\rho a_3} + r_{\rho a_4} - r_{a_1 a_2} - r_{a_1 a_4} - r_{a_2 a_3} - r_{a_3 a_4} \leq 1$$

$$+2r_{\rho a_1} + 2r_{\rho a_2} + 2r_{\rho a_3} + 2r_{\rho a_4} - r_{a_1 a_2} - r_{a_1 a_4} - r_{a_2 a_3} - r_{a_3 a_4} \leq 3$$



# Evidence from $K_5$

$$+r_{\rho a_1} + r_{\rho a_2} + r_{\rho a_3} + r_{\rho a_4} - r_{a_1 a_2} - r_{a_1 a_4} - r_{a_2 a_3} - r_{a_3 a_4} \leq 1$$

$$+$$

$$+r_{\rho a_1} + r_{\rho a_2} + r_{\rho a_3} + r_{\rho a_4} \leq 1$$

$$=$$

$$+2r_{\rho a_1} + 2r_{\rho a_2} + 2r_{\rho a_3} + 2r_{\rho a_4} - r_{a_1 a_2} - r_{a_1 a_4} - r_{a_2 a_3} - r_{a_3 a_4} \leq 1 + 1$$